Домашняя работа по алгебре и началам математического анализа за 10 класс

к задачнику «Алгебра и начала математического анализа. 10–11 классы. В 2 ч. Ч. 2. Задачник для учащихся общеобразовательных учреждений (базовый уровень) / [А.Г. Мордкович и др.]; под ред. А.Г. Мордковича. — 10-е изд., стер. — М.: Мнемозина, 2009»

2010

Глава 1. Числовые функции

§ 1. Определение числовой функции и способы ее задания

1.1. а) $y=3-\frac{3}{4}x$ б) $y=\frac{-7}{2x+1}$

в) $y=\frac{5}{6}x-\frac{1}{6}$ г) $y=\frac{9}{3x^2+4x}$

1.2. а) $f(1)=1^3-5\cdot1^2+7=3$ в) $f(-2)=(-2)^3-5\cdot(-2)^2+7=-21$

б) $f(3)=3^3-5\cdot3^2+7=-11$ г) $f(1,5)=\left(\frac{3}{2}\right)^3-5\cdot\left(\frac{3}{2}\right)^2+7=-\frac{7}{8}$

1.3. а) $f(x-2)=\frac{2(x-2)^2+3(x-2)-4}{3(x-2)+3}=\frac{2x^2-8x+8+3x-6-4}{3x-6+3}=$

$=\frac{2x^2-5x-2}{3x-3}$

б) $f(-x^3)=\frac{3x^6-3x^3-4}{-3x^3+3}$

в) $f\left(\frac{1}{x}\right)=\frac{\frac{2}{x^2}+\frac{3}{x}-4}{\frac{3}{x}+3}=\frac{-4x^2+2+3x}{3x^2+3x}$

г) $f(2x^2+3x+5)=\frac{2(2x^2+3x+5)^2+3(2x^2+3x+5)-4}{3x+3}=$

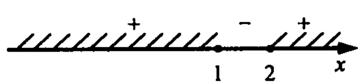
$=\frac{8x^4+24x^3+64x^2+69x+61}{6x^2+9x+18}$

1.4. а) $5x+3\neq0\Rightarrow x\neq-0,6$ в) $2x-4\neq0\Rightarrow x\neq2$

б) $x^2-16\neq0\Rightarrow x\neq\pm4$ г) $25-x^2\neq0\Rightarrow x\neq\pm5$

1.5. а) $x^2-3x+2\geq0$;

$x^2-3x+2=0$.



По теореме Виета: $x_1=1, x_2=2$.

Тогда $x^2-3x+2\geq0\Leftrightarrow x\in(-\infty;1]\cup[2;+\infty)$

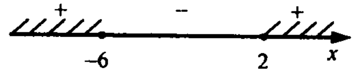
б) $x^2-4>0\Leftrightarrow|x|>2\Leftrightarrow x\in(-\infty;-2)\cup(2;+\infty)$

в) $x^2+4x-12\geq0$;

$x^2+4x-12=0$.

По теореме Виета:

$x_1=-6,\ x_2=2$.



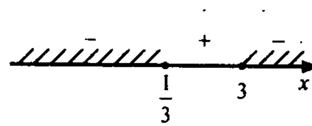
Тогда

$x^2+4x-12\ge 0 \Leftrightarrow x\in(-\infty;-6]\cup[2;+\infty)$

г) $49-x^2>0 \Leftrightarrow |x|<7 \Leftrightarrow x\in(-7;7)$

1.6. а) $\begin{cases}2x-4\ge 0\\ 10-2,5x>0\end{cases} \Leftrightarrow \begin{cases}x\ge 2\\ x<4\end{cases} \Leftrightarrow x\in(2;4)$

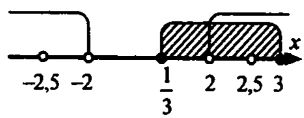
б) $\begin{cases}10x-3x^2-3\ge 0\\ x^2-4>0\\ 25-4x^2\ne 0\end{cases} \Leftrightarrow\quad 10x-3x^2-3=0$



$D=100-36=64 \Rightarrow x_{1,2}=\dfrac{-10\pm 9}{-6} \Rightarrow$

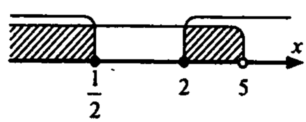
$x_1=3$ и $x_2=\dfrac{1}{3} \quad\Leftrightarrow$

$\Leftrightarrow \begin{cases}\dfrac{1}{3}\le x\le 3\\ \left[\begin{array}{l}x>2\\ x<-2\end{array}\right.\\ x\ne\pm 2,5\end{cases} \Leftrightarrow \left[\begin{array}{l}2<x<2,5\\ 2,5<x\le 3\end{array}\right.$



в) $\begin{cases}2x^2-5x+2\ge 0\\ 10-2x>0\end{cases} \Leftrightarrow \begin{cases}\left[\begin{array}{l}x\le\dfrac{1}{2}\\ x\ge 2\end{array}\right.\\ x<5\end{cases} \Leftrightarrow$

$\Leftrightarrow \left[\begin{array}{l}x\le\dfrac{1}{2}\\ 2\le x<5\end{array}\right.$;

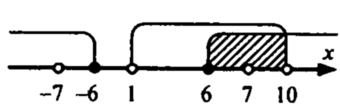


$2x^2-5x+2=0$

$D=25-16=9 \Rightarrow x_{1,2}=\dfrac{5\pm 3}{4} \Rightarrow\quad x_1=2$ и $x_2=\dfrac{1}{2}$

г) $\begin{cases}x^2-36\ge 0\\ 11x-x^2-10>0\\ x^4-2401\ne 0\end{cases} \Leftrightarrow \begin{cases}|x|\ge 6\\ 1<x<10\\ x\ne\pm 7\end{cases} \Leftrightarrow$

$\Leftrightarrow \left[\begin{array}{l}6\le x<7\\ 7<x<10\end{array}\right.$

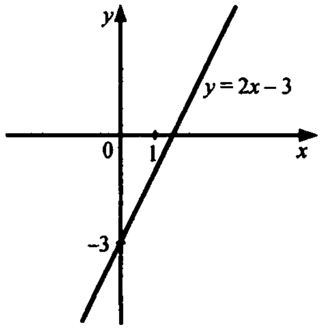


$x^2-11x+10=0.$

По теореме Виета: $x_1=1$ и $x_2=10$.

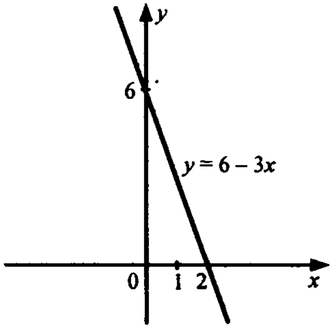
1.7. а) $y = 2x - 3$;

$D(f) = \mathbb{R}$; $E(f) = \mathbb{R}$



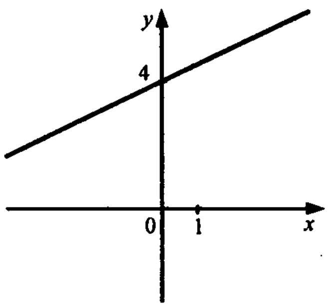
б) $y = 6 - 3x$;

$D(f) = \mathbb{R}$; $E(f) = \mathbb{R}$



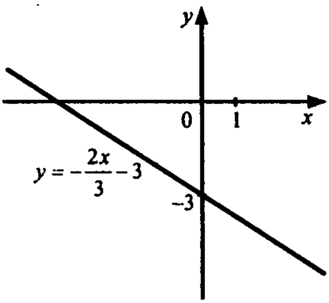
в) $y = \frac{x}{2} + 4$;

$D(f) = \mathbb{R}$; $E(f) = \mathbb{R}$



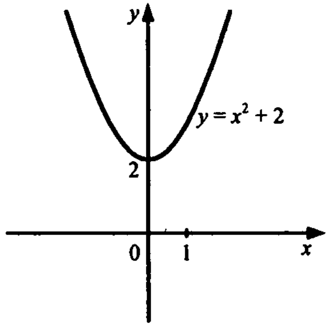
г) $y = -\frac{2x}{3} - 3$;

$D(f) = \mathbb{R}$; $E(f) = \mathbb{R}$



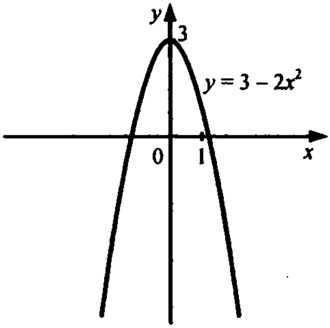
1.8. а) $y = x^2 + 2$;

$D(f) = \mathbb{R}$, $E(f) = [2; +\infty)$



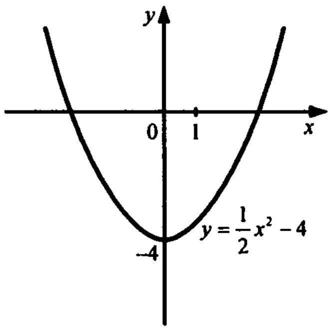
б) $y = 3 - 2x^2$;

$D(f) = \mathbb{R}$, $E(f) = (-\infty; 3]$



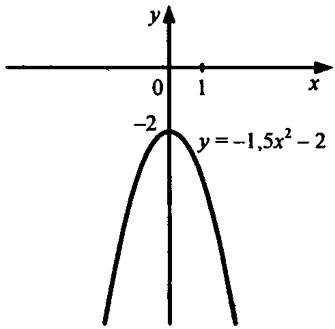
в) $y=\frac{1}{2}x^2-4$;

$D(f)=\mathbb{R}$, $E(f)=[-4;+\infty)$



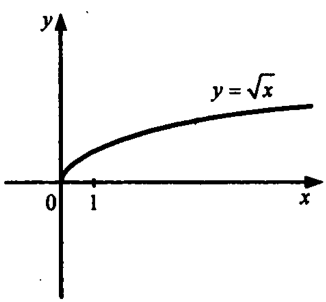
г) $y=-1{,}5x^2-2$;

$D(f)=\mathbb{R}$, $E(f)=(-\infty;-2]$



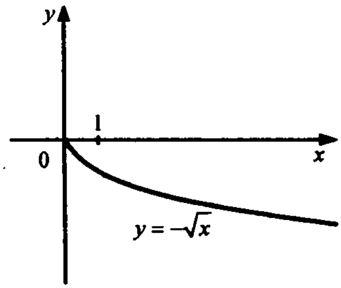
1.9. а) $y=\sqrt{x}$;

$D(f)=[0;+\infty)$, $E(f)=(0;+\infty)$



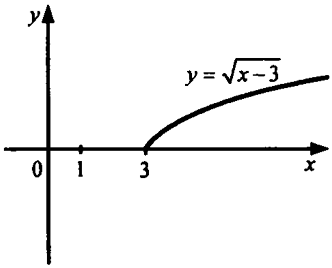
в) $y=-\sqrt{x}$;

$D(f)=(0;+\infty)$, $E(f)=(-\infty;0]$



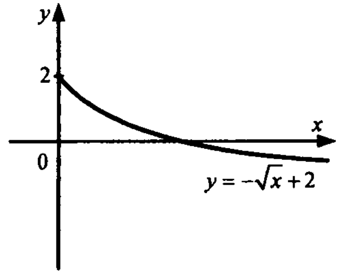
б) $y=\sqrt{x-3}$;

$D(f)=[3;+\infty)$, $E(f)=[0;+\infty)$



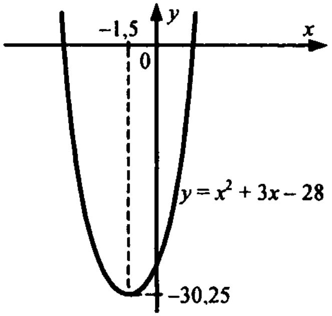
г) $y=-\sqrt{x}+2$;

$D(f)=(0;+\infty)$, $E(f)=(-\infty;2]$



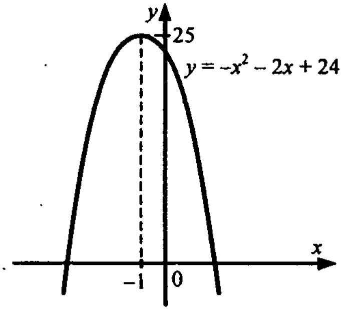
1.10. а) $y = x^2 + 3x - 28 =$
$= (x + 1{,}5)^2 - 30{,}25$

$D(f) = \mathbb{R}$; $E(f) = (-30{,}25; +\infty)$



б) $y = -x^2 - 2x + 24 =$
$= -(x + 1)^2 + 25$

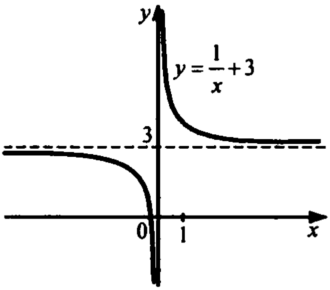
$D(f) = \mathbb{R}$; $E(f) = (-\infty; 25]$



1.11. а) $y = \dfrac{1}{x} + 3$;

$D(f) = (-\infty; 0) \cup (0; +\infty)$

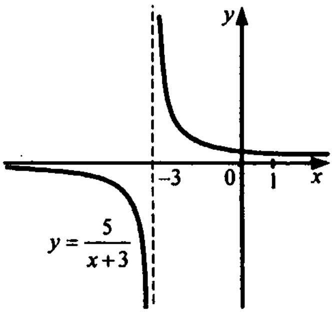
$E(f) = (-\infty; 3) \cup (3; +\infty)$



б) $y = \dfrac{5}{x + 3}$;

$D(f) = (-\infty; -3) \cup (-3; +\infty)$

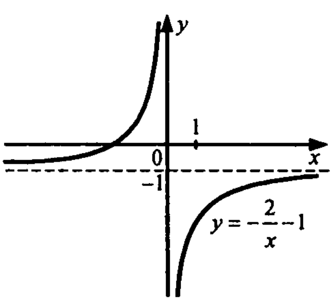
$E(f) = (-\infty; 0) \cup (0; +\infty)$



в) $y = \dfrac{-2}{x} - 1$;

$D(f) = (-\infty; 0) \cup (0; +\infty)$

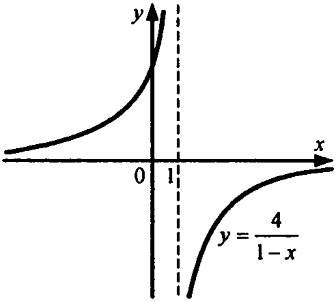
$E(f) = (-\infty; -1) \cup (-1; +\infty)$



г) $y = \dfrac{4}{1 - x}$;

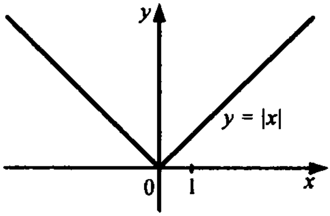
$D(f) = (-\infty; 1) \cup (1; +\infty)$

$E(f) = (-\infty; 0) \cup (0; +\infty)$



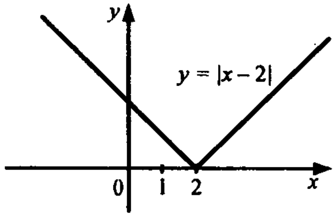
1.12. а) $y=|x|$;

$D(f)=\mathbb{R}$; $E(f)=(0;+\infty)$



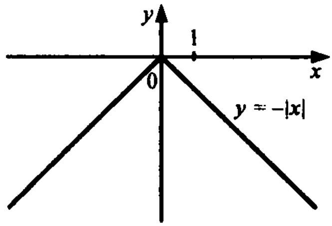
б) $y=|x-2|$;

$D(f)=\mathbb{R}$; $E(f)=(0;+\infty)$



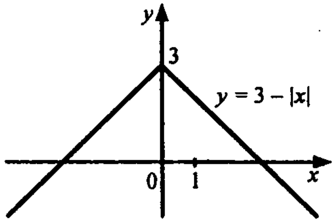
в) $y=-|x|$;

$D(f)=\mathbb{R}$, $E(f)=(-\infty;0]$



г) $y=3-|x|$;

$D(f)=\mathbb{R}$, $E(f)=(-\infty;3]$



1.13. . а) $y=\dfrac{1}{16x^2-49}$

$D(f)=\left\{x \mid 16x^2-49\neq 0\right\}=\left(-\infty;-\dfrac{7}{4}\right)\cup\left(\dfrac{7}{4};+\infty\right)$

$E(16x^2-49)=[-49;+\infty)\Rightarrow E(f)=\left(-\infty;-\dfrac{1}{49}\right]\cup(0;+\infty)$

б) $y=\sqrt{x^2+4x+3}$

$D(f)=\left\{x \mid x^2+4x+3\geq 0\right\}=(-\infty;-3]\cup[-1;+\infty)$

$x^2+4x+3=0$. По теореме Виета: $x_1=-1, x_2=-3$.

$E(f)=[0;+\infty)$

в) $y=\dfrac{1}{9-25x^2}$

$D(f)=\left\{x \mid 9-25x^2\neq 0\right\}=\left(-\infty;-\dfrac{3}{5}\right)\cup\left(\dfrac{3}{5};+\infty\right)$

$E(9-25x^2)=(-\infty;9)\Rightarrow(-\infty;0)\cup\left[\dfrac{1}{9};+\infty\right)$

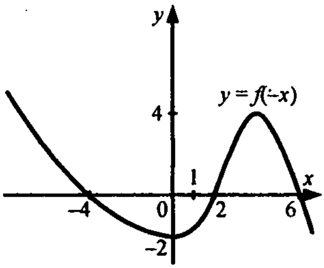
г) $y=\sqrt{3x-x^2+18}$

$D(f)=\left\{x \mid 3x-x^2+18\geq 0\right\}=[-3;6]$

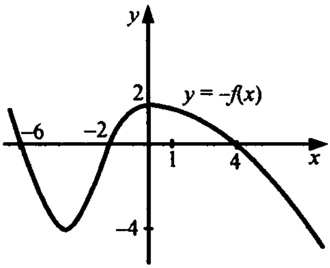
$3x-x^2+18=0$. По теореме Виета: $x_1=6, x_2=-3$.

$E(f)=\left\{y \mid y=\sqrt{3x-x^2+18};-3\leq x\leq 6\right\}=[0;4,5]$

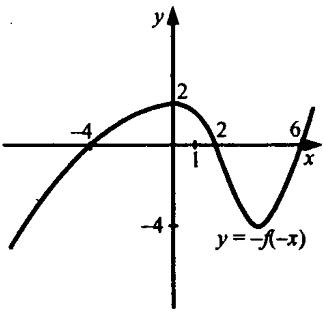
1.14. а) $y=f(-x)$



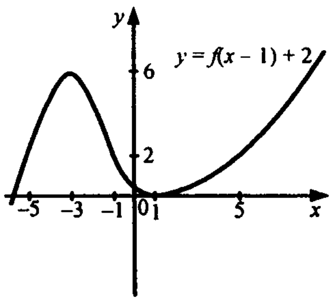
б) $y=-f(x)$



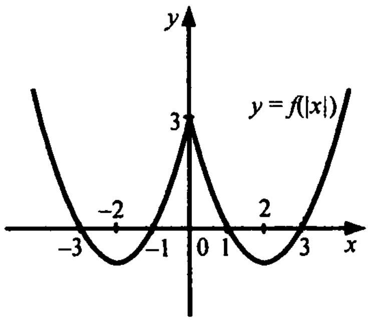
в) $y=-f(-x)$



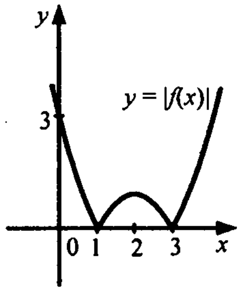
г) $y=f(x-1)+2$



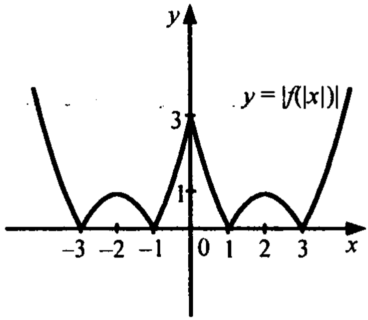
1.15: а) $y=f\left(|x|\right)=x^2-4|x|+3$



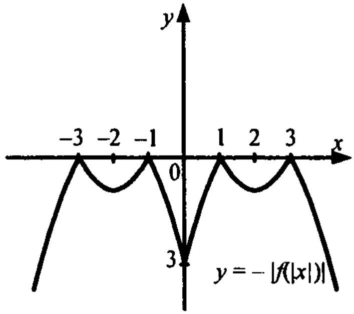
б) $y=|f(x)|=\left|x^2-4x+3\right|$



в) $y=\left|f\left(|x|\right)\right|=\left|x^2-4|x|+3\right|$



г) $y=-\left|f\left(|x|\right)\right|=-\left|x^2-4|x|+3\right|$



1.16.

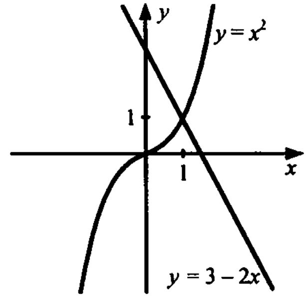
а)

$x^3=3-2x$.

Построим графики обеих частей

$y=x^3$ и $y=3-2x$

Точка пересечения (1; 1) $\Rightarrow x=1$.



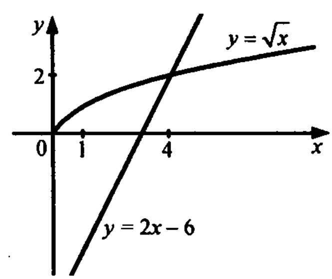
б)

$\sqrt{x}=2x-6$.

Построим графики обеих частей:

$y=\sqrt{x}$ и $y=2x-6$.

Точка пересечения (4; 2) $\Rightarrow x=4$.



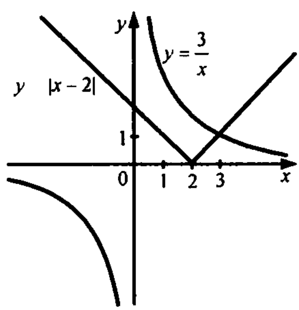
в)

$|x-2|=\dfrac{3}{x}$.

Построим графики обеих частей:

$y=|x-2|$ и $y=\dfrac{3}{x}$.

Точка пересечения (3; 1) $\Rightarrow x=3$.



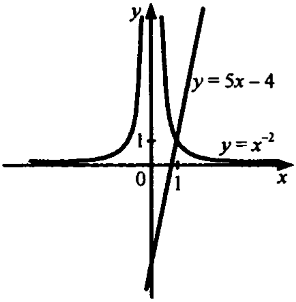
г)

$x^{-2} = 5x - 4$.

Построим графики обеих частей:

$y = x^{-2}$ и $y = 5x - 4$.

Точка пересечения (1; 1) $\Rightarrow x = 1$.



1.17. а) $f\left(\frac{1}{4}\right) = 5$ б) $f\left(\sqrt{2}\right) = 1$

в) $f\left(1\frac{1}{6}\right) = 6$ г) $f\left(\left(\sqrt{5}\right)^2\right) = 0$

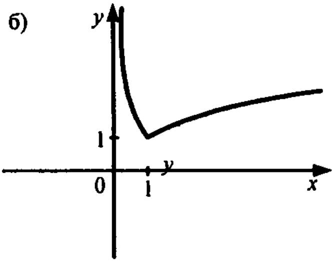
1.18.

а) $f(6{,}25) = 2{,}5$;

$f(0{,}01) = 100$;

$f(-3)$ не определено

б)



в) $D(f) = (0; +\infty)$;

г) $E(f) = [1; +\infty)$

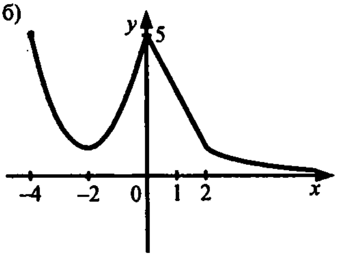
1.19.

а) $f(-5)$ — не определено;

$f(-3) = 2$; $f(0) = 5$;

$f(4) = \frac{1}{2}$.

б)



в) $D(f) = [-4; +\infty)$;

г) $E(f) = (0; 5]$

§ 2. Свойства функции

2.1. а) $y = 8x + 3$;

$x_1 < x_2 \Rightarrow 8x_1 + 3 < 8x_2 + 3 \Rightarrow y_1 < y_2 \Rightarrow$ функция возрастает.

б) $y = 5 - 2x$;

$x_1 < x_2 \Rightarrow 5 - 2x_1 < 5 - 2x_2 \Rightarrow y_1 > y_2 \Rightarrow$ функция убывает.

в) $y = \frac{x}{3} + 1$;

$x_1 < x_2 \Rightarrow \frac{x_1}{3} + 1 < \frac{x_2}{3} + 1 \Rightarrow y_1 < y_2 \Rightarrow$ функция возрастает.

г) $y=\frac{1}{3}-\frac{2x}{5}$;

$x_1<x_2 \Rightarrow \frac{1}{3}-\frac{2x_1}{5}>\frac{1}{3}-\frac{2x_2}{5} \Rightarrow y_1>y_2 \Rightarrow$ функция убывает.

2.2. а) $y=2x^3-3$;

$x_1<x_2 \Rightarrow x_1^3<x_2^3 \Rightarrow 2x_1^3-3<2x_2^3-3 \Rightarrow y_1<y_2 \Rightarrow$ функция возрастает.

б) $y=7-\frac{x^3}{2}$;

$x_1<x_2 \Rightarrow x_1^3<x_2^3 \Rightarrow 7-\frac{x_1^3}{2}>7-\frac{x_2^3}{2} \Rightarrow y_1>y_2 \Rightarrow$ функция убывает.

в) $y=\frac{2}{3}-x^3$;

$x_1<x_2 \Rightarrow x_1^3<x_2^3 \Rightarrow \frac{2}{3}-x_1^3>\frac{2}{3}-x_2^3 \Rightarrow y_1>y_2 \Rightarrow$ функция убывает.

г) $y=4+x^3$;

$x_1<x_2 \Rightarrow x_1^3<x_2^3 \Rightarrow 4+x_1^3<4+x_2^3 \Rightarrow y_1<y_2 \Rightarrow$ функция возрастает.

2.3. а) $y=x^2+2x+1, x\geq -1$;

$y=x^2+2x+1=(x+1)^2$;

$-1\leq x_1<x_2 \Rightarrow 0\leq x_1+1<x_2+1 \Rightarrow$

$(x_1+1)^2<(x_2+1)^2 \Rightarrow y_1<y_2 \Rightarrow$ функция возрастает.

б) $y=\frac{1}{x+2}$, $x<-2$;

$x_1<x_2<-2 \Rightarrow x_1+2<x_2+2<0 \Rightarrow$

$\frac{1}{x_1+2}>\frac{1}{x_2+2} \Rightarrow y_1>y_2 \Rightarrow$ функция убывает.

в) $y=-x^2+6x-12, x\geq 3$;

$y=-x^2+6x-12=-(x-3)^2-3$

$3\leq x_1<x_2 \Rightarrow 0\leq x_1-3<x_2-3 \Rightarrow (x_1-3)^2<(x_2-3)^2 \Rightarrow$

$-(x_1-3)^2-3>-(x_2-3)^2-3 \Rightarrow y_1>y_2 \Rightarrow$ функция убывает.

г) $y=\frac{-2}{x+5}, x>-5$;

$-5<x_1<x_2 \Rightarrow 0<x_1+5<x_2+5 \Rightarrow$

$\frac{1}{x_1+5}>\frac{1}{x_2+5} \Rightarrow \frac{-2}{x_1+5}<\frac{-2}{x_2+5} \Rightarrow y_1<y_2 \Rightarrow$ функция возрастает.

2.4. а) $y=x^3+2x$;

$x_1<x_2 \Rightarrow x_1^3<x_2^3 \Rightarrow x_1^3+2x_1<x_2^3+2x_2 \Rightarrow y_1<y_2 \Rightarrow$ функция возрастает.

б) $y=5-x^3-6x^9$;

$x_1<x_2 \Rightarrow x_1^3<x_2^3$ и $x_1^9<x_2^9 \Rightarrow 5-x_1^3-6x_2^9>5-x_2^3-6x_2^9 \Rightarrow y_1>y_2 \Rightarrow$ функция убывает.

в) $y=4-x^5$

$x_1<x_2 \Rightarrow x_1^5<x_2^5 \Rightarrow 4-x_1^5>4-x_2^5 \Rightarrow y_1>y_2 \Rightarrow$ функция убывает

г) $y=x^7+x^5-3$

$x_1<x_2 \Rightarrow x_1^7<x_2^7$ и $x_1^5<x_2^5 \Rightarrow x_1^7+x_1^5-3<x_2^7+x_2^5-3 \Rightarrow y_1<y_2 \Rightarrow$ функция возрастает.

2.5. а) $y=\sqrt{x^2+1}$

$-1\le x_1<x_2 \Rightarrow -1\le x_1^3<x_2^3 \Rightarrow 0\le x_1^3+1<x_2^3+1 \Rightarrow$

$\Rightarrow \sqrt{x_1^3+1}<\sqrt{x_2^3+1} \Rightarrow y_1<y_2 \Rightarrow$ функция возрастает.

б) $y=5-x^5-\sqrt{2x^3}$

$0\le x_1<x_2 \Rightarrow x_1^5<x_2^5$ и $\sqrt{2x_1^3}<\sqrt{2x_2^3} \Rightarrow 5-x_1^5-\sqrt{2x_1^3}>5-x_2^5-\sqrt{2x_2^3} \Rightarrow$ $y_1>y_2 \Rightarrow$ функция убывает.

в) $y=2-\sqrt{x}$;

$0\le x_1<x_2 \Rightarrow \sqrt{x_1}<\sqrt{x_2} \Rightarrow 2-\sqrt{x_1}>2-\sqrt{x_2} \Rightarrow y_1>y_2 \Rightarrow$ функция убывает.

г) $y=\sqrt{x^7}+x-1$;

$0\le x_1<x_2 \Rightarrow \sqrt{x_1^7}<\sqrt{x_2^7} \Rightarrow \sqrt{x_1^7}+x_1-1<\sqrt{x_2^7}+x_2-1 \Rightarrow y_1<y_2 \Rightarrow$ функция возрастает.

2.6. а) $y=x^2-8x+1=(x-4)^2-15\ge-15 \Rightarrow$ функция ограничена снизу.

б) $y=\dfrac{2x-4}{x}=2-\dfrac{4}{x}<2 \Rightarrow$ функция ограничена сверху.

в) $y=-2x^2-6x+15=-2\left(x+\dfrac{3}{2}\right)^2+19{,}5\le19{,}5 \Rightarrow$ функция ограничена сверху.

г) $y=\dfrac{5-2x}{1-x}=\dfrac{3}{1-x}+2>2 \Rightarrow$ функция ограничена снизу.

$x<1 \Rightarrow 1-x>0 \Rightarrow \dfrac{1}{1-x}>0$.

2.7. а) $y=\sqrt{-x^2+4x-5}=\sqrt{-(x-2)^2-1}$; $-(x-2)^2-1<0 \Rightarrow$ функция нигде не ограничена.

б) $y=\sqrt{\dfrac{x^2-4x+1}{5}}=\dfrac{1}{\sqrt5}\sqrt{(x-2)^2-3}\ge0 \Rightarrow$ функция ограничена снизу.

в) $y=\sqrt{-2x^2+8x+9}=\sqrt{-2(x-2)^2+17}\geq 0\Rightarrow$ функция ограничена снизу.

$17-2(x-2)^2\leq 17\Rightarrow y=\sqrt{17-2(x-2)^2}\leq\sqrt{17}\Rightarrow$ функция ограничена сверху.

г) $\sqrt{\dfrac{5}{2x^2-4x+2}}=\sqrt{\dfrac{5}{2(x-1)^2}}=\dfrac{\sqrt{5}}{\sqrt{2}|x-1|}>0\Rightarrow$ функция ограничена снизу.

2.8. а) $y=3-2x, x\in(-1;3]$;

$-1\leq x\leq 3\Rightarrow -6\leq -2x\leq 2\Rightarrow -3\leq 3-2x\leq 5\Rightarrow$ $y_{наиб}=5, y_{наим}=-3.$

б) $y=-2x^2+2x, x\in[-3;2;$

$$y=-2x^2+2x=-2\left(x-\frac{1}{2}\right)^2+\frac{1}{2}\leq\frac{1}{2}\Rightarrow y_{наиб}=\frac{1}{2}$$

$y(-3)=-24, y(2)=-4\Rightarrow y_{наиб}=-24$

в) $y=3-4x, x\in(-\infty;3]$;

$x\leq 3\Rightarrow -4x\geq -12\Rightarrow 3-4x\geq -9\Rightarrow y\geq -9\Rightarrow$

$y_{наиб}=-9$; $y_{наиб}$ не существует.

г) $y=x^2+4x+5, x\in(0;1]$;

$y=x^2+4x+5=(x+2)^2+1\geq 1\Rightarrow y_{наим}=1$;

y — возрастает на $(0;1]$ $\Rightarrow$ $y_{наиб}=y(1)=10$

2.9. а) $y=\sqrt{x}$, $x\in(2;+\infty)$;

$x\geq 2\Rightarrow y=\sqrt{x}\geq\sqrt{2}\Rightarrow y_{наим}=\sqrt{2}$; $y_{наиб}$ не существует.

б) $y=-\sqrt{x}$, $x\in[1;9]$;

y — убывающая функция $\Rightarrow$ $y_{наиб}=y(1)=-1$; $y_{наим}=y(9)=-3$

в) $y=\sqrt{x}$, $x\in[1,44;6,25]$;

$1,44\leq x\leq 6,25\Rightarrow 1,2\leq y=\sqrt{x}\leq 2,5\Rightarrow$ $y_{наим}=1,2$; $y_{наиб}=2,5$

г) $y=-\sqrt{x}$, $x\in(0;1,69]$;

$0<x\leq 1,69\Rightarrow -1,3\leq y=-\sqrt{x}<0\Rightarrow$

$\Rightarrow y_{наим}=-1,3$; $y_{наиб}$ не существует.

2.10. а) $y=2|x|-1, x\in(-3;2]$;

$-3\leq x\leq 2\Rightarrow 0\leq|x|\leq 3\Rightarrow -1\leq y=2|x|-1\leq 5\Rightarrow$ $y_{наиб}=5$; $y_{наим}=-1$

б) $y=3-|2x|, x\in(-5;4]$;

$-5<x\leq 4\Rightarrow -10<2x\leq 8\Rightarrow 0\leq|2x|<10\Rightarrow$; $-7<y=3-|2x|\leq 3\Rightarrow$

$y_{наиб}=3$, $y_{наим}$ не существует.

в) $y = 1,5 - |5x|,\ x \in [-8; 2]$;

$-8 \le x \le 2 \Rightarrow -40 \le 5x \le 10 \Rightarrow 0 \le |5x| \le 40 \Rightarrow -38,5 \le y = 1,5 - |5x| \le 1,5 \Rightarrow$

$y_{наиб} = 1,5;\ y_{наим} = -38,5$

г) $y = 6|x| - 2,\ x \in (-10; 4)$;

$-10 \le x < 4 \Rightarrow 0 \le |x| \le 10 \Rightarrow -2 \le y = 6|x| - 2 \le 58 \Rightarrow y_{наиб} = 58,\ y_{наим} = -2$

2.11. а) $y = x^2 + 2x^4 + 1$;

$y(-x) = (-x)^2 + 2(-x)^4 + 1 = x^2 + 2x^4 + 1 = y(x) \Rightarrow$ функция четная.

б) $y = \dfrac{x}{x^2+1}$;

$y(-x) = \dfrac{-x}{(-x)^2+1} = -\dfrac{x}{x^2+1} = -f(x) \Rightarrow$ функция нечетная.

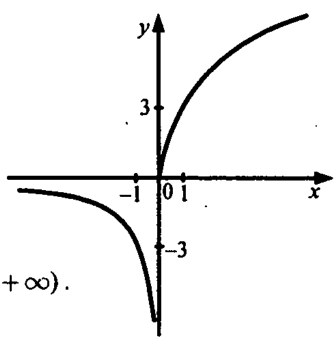
в) $y = \dfrac{-3x^2+1}{1-x^4}$;

$y(-x) = \dfrac{-3(-x)^2+1}{1-(-x)^4} = \dfrac{-3x^2+1}{1-x^4} = y(x) \Rightarrow$ функция четная.

г) $y = 5 - 3x^3$;

$y(-x) = 5 - 3(-x)^3 = 5 + 3x^3 \Rightarrow$ функция ни четная, ни нечетная.

2.12. $y = \begin{cases} \dfrac{3}{x}, \text{ если } x < 0 \\ 3\sqrt{x}, \text{ если } x \ge 0 \end{cases}$



1) $D(f) = \mathbb{R} = (-\infty; +\infty)$

2) Функция возрастает на $(0; +\infty)$;

функция убывает на $(-\infty; 0)$.

3) Функция не ограничена.

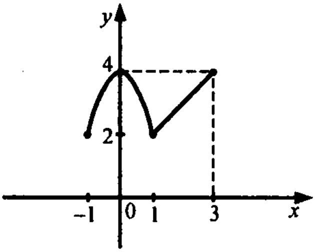
4) $y_{наиб}$ и $y_{наим}$ не существуют.

5) Функция непрерывна на $(-\infty; 0)$ и на $(0; +\infty)$.

В точке $x = 0$ функция имеет разрыв.

6) $E(f) = \mathbb{R} = (-\infty; +\infty)$.

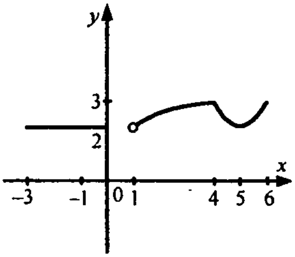
2.13. $y = \begin{cases} 4 - 2x^2, \text{ если } -1 \le x \le 1 \\ x + 1, \text{ если } 1 < x \le 3 \end{cases}$



1) $D(f) = (-1; 3]$

2) Функция возрастает на $[-1; 0]$ и $[1; 3]$;
функция убывает на $[0; 1]$.
3) Функция ограничена и снизу, и сверху.
4) $y_{наиб} = 4$ (достигает в $x = 0$), $y_{наим} = 2$
(достигает в $x = 1$).
5) Функция непрерывна на $[-1; 3]$. 6) $E(f) = [2; 4]$.

2.14. $y=\begin{cases}2,\text{ если } -3\le x\le 0\\ \sqrt{x}+1,\text{ если } 1<x\le 4\\ (x-5)^2+2,\text{ если } 4<x\le 6\end{cases}$

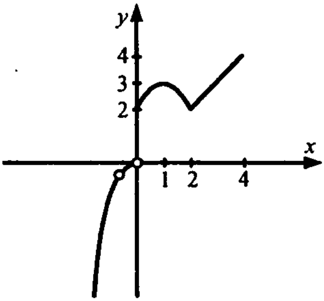


1) $D(f)=[-3;0]\cup(1;6]$

2) Функция возрастает на $(1;4]$ и $[5;6]$;
функция убывает на $[4;5]$;
функция постоянна на $[-3;0]$.
3) Функция ограничена и сверху, и снизу.
4) $y_{наиб}=3$ (достигается в $x=4$), $y_{наим}=2$ (достигается в $x=0$).
5) Функция непрерывна на $[-3;0]\cup(1;6]$.

6) $E(f)=[2;3]$.

2.15. $y=\begin{cases}x^3,\text{ если } x<0\\ -x^2+2x+2,\text{ если } 0\le x\le 2\\ x,\text{ если } 2<x\le 4\end{cases}$



1) $D(f)=(-\infty;4)$.

2) Функция возрастает на $(-\infty;1]$ и $(2;4]$.

3) Функция ограничена сверху и
неограничена снизу.
4) $y_{наиб}=4$ (достигается в $x=4$).
5) Функция непрерывна на
$(-\infty;0)\cup(0;4]$. в точке $x=0$ функция имеет разрыв.

6) $E(f)=(-\infty;0)\cup(2;4]$.

§ 3. Обратная функция

3.1. а) $y=3x-1\Rightarrow x=\dfrac{y+1}{3}=\dfrac{1}{3}y+\dfrac{1}{3}$

в) $y=5x+2\Rightarrow x=\dfrac{y-2}{5}=0,2y-0,4$

б) $y=2+4x\Rightarrow x=\dfrac{y-2}{4}=0,25y-0,5$

г) $y=3-x\Rightarrow x=3-y$

3.2. а) $y=\dfrac{x+1}{2x-3}=\dfrac{1}{2}\left(\dfrac{2x+2}{2x-3}\right)=\dfrac{1}{2}\left(1+\dfrac{5}{2x-3}\right)$

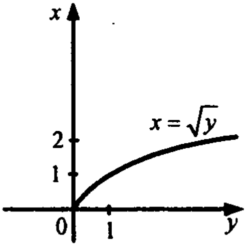
$\Rightarrow 2x-3=\dfrac{5}{2y-1}\Rightarrow x=\dfrac{1}{2}\left(3+\dfrac{5}{2y-1}\right)=\dfrac{3y+1}{2y-1}$

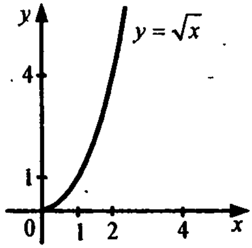
б) $y=\dfrac{4-3x}{1+x}=\dfrac{7}{1+x}-3\Rightarrow 1+x=\dfrac{7}{y+3}\Rightarrow x=\dfrac{y-4}{y+3}$;

в) $y=\dfrac{3-2x}{5x+1}\Rightarrow 5yx+y=3-2x\Rightarrow x=\dfrac{3-y}{5y+2}$

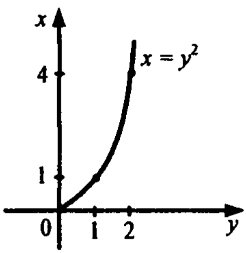
г) $y=\dfrac{2x-5}{1+2x}=1-\dfrac{6}{1+2x}\Rightarrow 1+2x=\dfrac{6}{1-y}\Rightarrow x=\dfrac{y+5}{2-2y}$

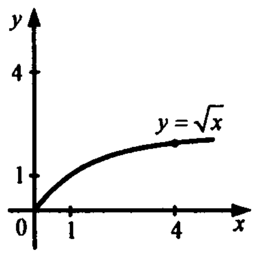
3.3. а) $y=x^2,\ x\ge 0\Rightarrow x=\sqrt{y}$



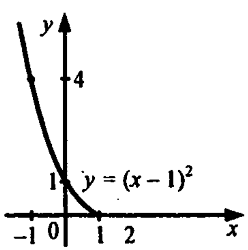


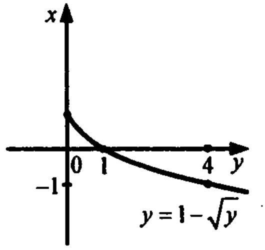
б) $y=\sqrt{x}\Rightarrow x=y^2,\ y\ge 0$



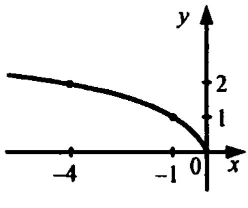


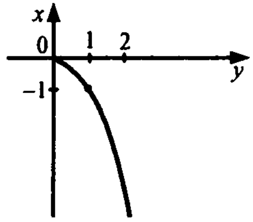
в) $y=(x-1)^2,\ x\le 1\Rightarrow x=1-\sqrt{y}$



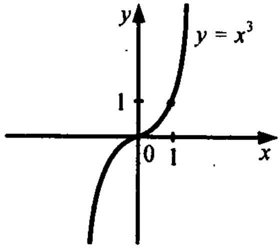


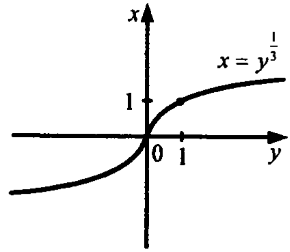
г) $y=\sqrt{-x}\Rightarrow x=-y^2,\ y\ge 0$



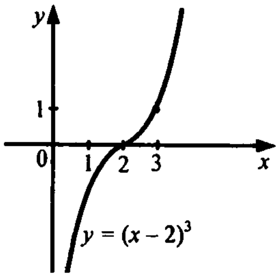


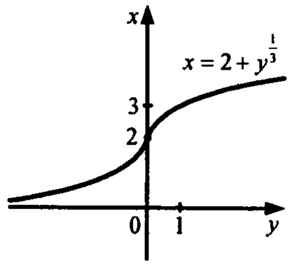
3.4. а) $y = x^3 \Rightarrow x = y^{\frac{1}{3}}$



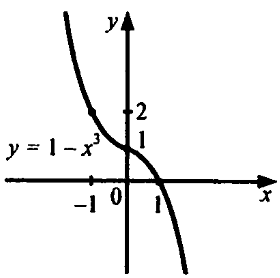


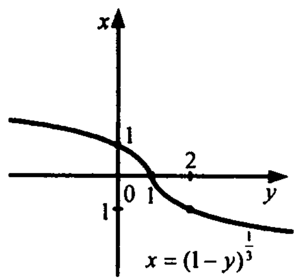
б) $y = (x-2)^3 \Rightarrow x = 2 + y^{\frac{1}{3}}$



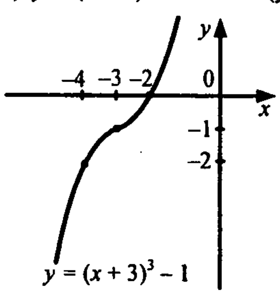


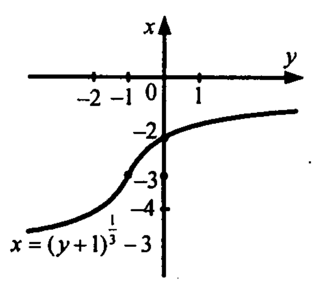
в) $y = 1 - x^3 \Rightarrow x = (1-y)^{\frac{1}{3}}$





г) $y = (x+3)^3 - 1 \Rightarrow x = (y+1)^{\frac{1}{3}} - 3$





3.5.

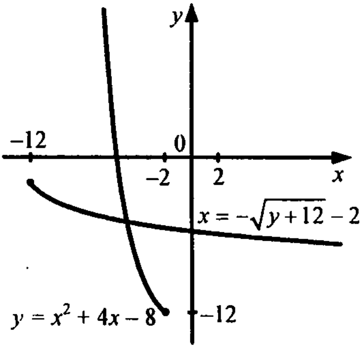
а) $y = x^2 + 4x - 8, x \in [-3; 0]$

$y = (x+2)^2 - 12$

$x \in [-2; 0] \Rightarrow x = \sqrt{y+12} - 2$

$x \in (-3; -1] \Rightarrow x = -\sqrt{y+12} - 2$

$\Rightarrow$ обратная функция не существует.



б) $y = x^2 + 4x - 8, \ x \in (-\infty; -2)$

$\Rightarrow x = -\sqrt{y+12} - 2$

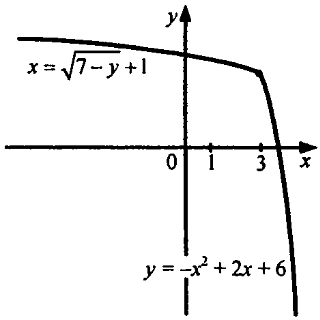
в) $y = -x^2 + 2x + 6, x \in (0; 3]$

$y = -(x-1)^2 + 7$

$x \in [0; 1] \Rightarrow x = -\sqrt{7-y} + 1$

$x \in (1; 3] \Rightarrow x = \sqrt{7-y} + 1$

$\Rightarrow$ обратная функция не существует



г) $y = -x^2 + 2x + 6, \ x \in (3; +\infty)$

$\Rightarrow x = \sqrt{7-y} + 1$

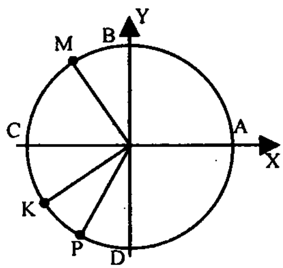
Глава 2. Тригонометрические функции

§ 4. *Числовая окружность*

В задачах 4.1–4.4 требуется найти длину дуги. Она находится по формуле: $\ell = \alpha \cdot r$, где α – радианная мера дуги, r – радиус окружности. Так как рассматривается единичная окружность, то $r = 1$, т.е. $\ell = \alpha$. Переход от градусной к радианной мере:
$\alpha = \frac{\beta}{180} \cdot \pi$, где β – мера угла в градусах.

4.1. $\overset{\cup}{AM} = 90° + 45° = 135°$, $l_{AM} = \frac{3}{4}\pi$;

$\overset{\cup}{BK} = 90° + 30° = 120°$, $l_{BK} = \frac{2}{3}\pi$;



$\overset{\cup}{MP} = 45° + 60° = 105°$, $l_{MP} = \frac{7}{12}\pi$;

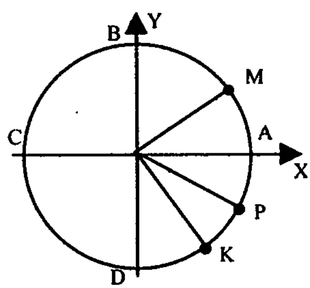
$\overset{\cup}{DC} = 270°$, $l_{DC} = \frac{3}{2}\pi$;

$\overset{\cup}{KA} = 150°$, $l_{KA} = \frac{5}{6}\pi$;

$\overset{\cup}{BP} = 150°$, $l_{BP} = \frac{5}{6}\pi$;

$\overset{\cup}{CB} = 270°$, $l_{CB} = \frac{3}{2}\pi$; $\overset{\cup}{BC} = 90°$, $l_{BC} = \frac{1}{2}\pi$.

4.2. $\overset{\cup}{AM} = 45°$, $l_{AM} = \frac{\pi}{4}$;



$\overset{\cup}{BD} = 180°$, $l_{BD} = \pi$;

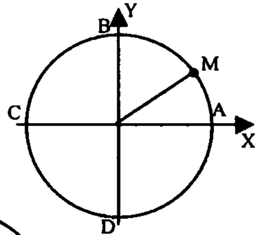
$\overset{\cup}{CK} = 120°$, $l_{CK} = \frac{2}{3}\pi$;

$\overset{\cup}{MP} = 285°$, $l_{MP} = \frac{19}{12}\pi$;

$\overset{\cup}{DM} = 135°$, $l_{DM} = \frac{3}{4}\pi$;

$\overset{\cup}{MK} = 360° - 105° = 255°$, $l_{MK} = \frac{17}{12}\pi$;

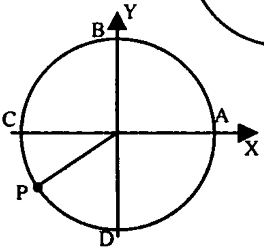
$\overset{\cup}{CP} = 150°$, $l_{CP} = \frac{5}{6}\pi$; $\overset{\cup}{PC} = 210°$ $l_{PC} = \frac{7}{6}\pi$.



4.3. $l_{AM} = \frac{\pi}{2} \cdot \frac{2}{2+3} = \frac{\pi}{5}$;

$l_{MB} = \frac{\pi}{2} - \frac{\pi}{5} = \frac{3}{10}\pi$; $l_{DM} = \frac{\pi}{2} + \frac{\pi}{5} = \frac{7}{10}\pi$;

$l_{MC} = \frac{3}{10}\pi + \frac{\pi}{2} = \frac{4}{5}\pi$.



4.4. $l_{EP} = \frac{\pi}{2} \cdot \frac{1}{1+5} = \frac{\pi}{12}$;

$l_{PD} = \frac{\pi}{2} \cdot \frac{5}{1+5} = \frac{5\pi}{12}$

$l_{AP} = l_{AC} + l_{CP} = \frac{13}{12}\pi$.

4.5. а) $\frac{\pi}{2}$; (0; 1). б) π; (– 1; 0).

в) $\frac{3\pi}{2}$; (0; – 1). г) 2π; (1; 0).

4.6. а) 7π; (– 1; 0). б) 4π; (1; 0). в) 10π; (1; 0). г) 3π; (–1; 0).

4.7. а) $\frac{\pi}{3}$; $(\frac{1}{2}; \frac{\sqrt{3}}{2})$. б) $\frac{\pi}{4}$; $(\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2})$. в) $\frac{\pi}{6}$; $(\frac{\sqrt{3}}{2}; \frac{1}{2})$.

г) $\frac{\pi}{8}$; $\cos^2\frac{\pi}{8} = \frac{1+\cos\frac{\pi}{4}}{2}$; $\cos\frac{\pi}{8} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}} = \frac{\sqrt{2+\sqrt{2}}}{2}$; $\sin^2\frac{\pi}{8} = \frac{1-\cos\frac{\pi}{4}}{2}$;

$\sin\frac{\pi}{8} = \frac{\sqrt{2-\sqrt{2}}}{2}$; $(\frac{\sqrt{2+\sqrt{2}}}{2}; \frac{\sqrt{2-\sqrt{2}}}{2})$.

4.8. а) $\frac{2\pi}{3}$, $(-\frac{1}{2}; \frac{\sqrt{3}}{2})$; б) $\frac{3\pi}{4}$, $(-\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2})$;

в) $\frac{5\pi}{6}$, $(-\frac{\sqrt{3}}{2}; \frac{1}{2})$; г) $\frac{5\pi}{4}$, $(-\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2})$.

4.9. а) $\frac{4\pi}{3}$, $(-\frac{1}{2}; -\frac{\sqrt{3}}{2})$; б) $\frac{5\pi}{3}$, $(\frac{1}{2}; -\frac{\sqrt{3}}{2})$;

в) $\frac{7\pi}{6}$, $(-\frac{\sqrt{3}}{2}; -\frac{1}{2})$; г) $\frac{11\pi}{6}$, $(\frac{\sqrt{3}}{2}; -\frac{1}{2})$.

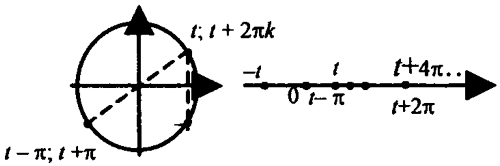
4.10. а) $-\frac{\pi}{2}$, (0; –1); б) $-\frac{2\pi}{3}$, $(-\frac{1}{2}; -\frac{\sqrt{3}}{2})$;

в) -2π, (1; 0); г) $-\frac{3\pi}{4}$, $(-\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2})$.

4.11. а) $\frac{25\pi}{4}$, $(\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2})$; б) $-\frac{26\pi}{3}$, $(-\frac{1}{2}; -\frac{\sqrt{3}}{2})$;

в) $-\frac{25\pi}{6}$, $(\frac{\sqrt{3}}{2}; -\frac{1}{2})$; г) $\frac{16\pi}{3}$, $(-\frac{1}{2}; -\frac{\sqrt{3}}{2})$.

4.12.



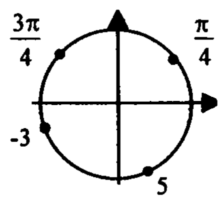
а) t; $-t$. на прямой: симметрично относительно нуля; на окружности: симметрично относительно оси x.
б) t; $t + 2\pi k$. на прямой: стоят с периодом $2\pi k$; на окружности: совпадают.
в) t; $t + \pi$. на прямой: стоят на расстоянии в π; на окружности: диаметрально противоположны.
г) $t + \pi$, $t - \pi$. на прямой: стоят на расстоянии в 2π; на окружности: совпадают

4.13. а) $M_1\left(\frac{\pi}{4}\right)$: $\frac{\pi}{4} + 2\pi k, k \in Z$;

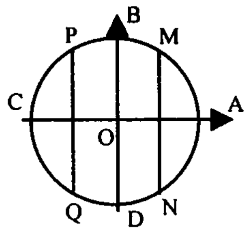
б) M_2 (5): $5+2\pi k$, $k \in Z$;

в) $M_3\left(\frac{3\pi}{4}\right)$: $\frac{3\pi}{4} + 2\pi k, k \in Z$

г) M_4 (– 3): $-3+\pi k$, $k \in Z$.



4.14. а) A: $2\pi k$;
б) C: $\pi + 2\pi k$;
в) A и C : πn



4.16. а) 1: (0, 540; 0,841);
б) – 5: (0,284; 0,950);
в) 4,5: (–0,211; –0,978);
г) (– 3): (– 0,990; – 0,141).
4.17. а) 6: IV; б) 2: II; в) 3: II; г) 4: III.
4.18. а) 5: IV; б) – 5: I; в) 8: II; г) – 8: III.

4.19. а) AM: $t \in (2\pi k; \frac{\pi}{4} + 2\pi k)$; б) CM: $t \in (-\pi+2\pi k; \frac{\pi}{4} + 2\pi k)$

в) MA: $t \in (\frac{\pi}{4} + 2\pi k; 2\pi + 2\pi k)$ г) MC $t \in (\frac{\pi}{4} + 2\pi k; \pi + 2\pi k)$

4.20. а) DM: $t \in (-\frac{\pi}{2} + 2\pi k; \frac{3\pi}{4} + 2\pi k)$

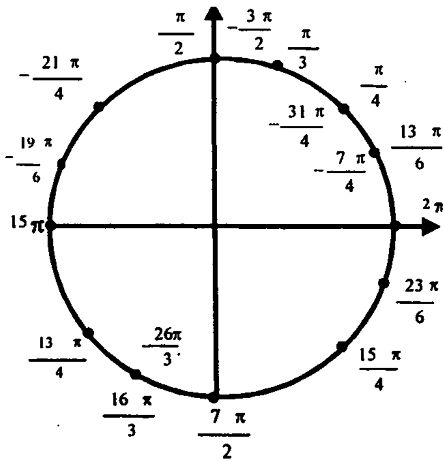
в) MD: $t \in (\frac{3\pi}{4} + 2\pi k;\ \frac{3\pi}{2} + 2\pi k)$;

б) BD: $t \in (\frac{\pi}{2} + 2\pi k;\ \frac{3\pi}{2} + 2\pi k)$;

г) DB: $t \in (-\frac{\pi}{2} + 2\pi k;\ \frac{\pi}{2} + 2\pi k)$.

§ 5. Числовая окружность на координатной плоскости.

с № 5.1 по 5.3 см. рис.



5.1. а) $M(\frac{\pi}{4}) = (\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2})$;

б) $M(\frac{\pi}{3}) = (\frac{1}{2}; \frac{\sqrt{3}}{2})$;

в) $M(\frac{\pi}{6}) = (\frac{\sqrt{3}}{2}; \frac{1}{2})$;

г) $M(\frac{\pi}{2}) = (0; 1)$.

5.2. а) $M(2\pi) = (1; 0)$;

б) $M(\frac{7\pi}{2}) = (0; -1)$;

в) $M(-\frac{3\pi}{2}) = (0; 1)$;

г) $M(15\pi) = (-1; 0)$.

5.3. а) $M(\frac{15\pi}{4}) = (\frac{\sqrt{2}}{2}; -\frac{\sqrt{2}}{2})$;

б) $M(\frac{16\pi}{3}) = (-\frac{1}{2}; -\frac{\sqrt{3}}{2})$;

в) $M(-\frac{31\pi}{4}) = (\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2})$

г) $M(-\frac{26\pi}{3}) = (-\frac{1}{2}; -\frac{\sqrt{3}}{2})$

5.4. а) $M(\frac{\sqrt{3}}{2}; \frac{1}{2})$: min положит.$=\frac{\pi}{6}$, max отрицат.$=-\frac{11\pi}{6}$

б) $M(-\frac{\sqrt{3}}{2}; \frac{1}{2})$: min положит.$=\frac{5\pi}{6}$, max отрицат.$=-\frac{7\pi}{6}$

в) $M(\frac{\sqrt{3}}{2}; -\frac{1}{2})$: min положит.$=\frac{11\pi}{6}$, max отрицат.$=-\frac{\pi}{6}$;

г) $M(-\frac{\sqrt{3}}{2}; -\frac{1}{2})$: min положит.$=\frac{7\pi}{6}$, max отрицат.$=-\frac{5\pi}{6}$

5.5. а) M ($\frac{1}{2}$; $\frac{\sqrt{3}}{2}$):min положит.=$\frac{\pi}{3}$, max отрицат.=$-\frac{5\pi}{3}$

б) M ($-\frac{1}{2}$; $\frac{\sqrt{3}}{2}$): min положит.=$\frac{2\pi}{3}$, max отрицат.=$-\frac{4\pi}{3}$

в) M ($-\frac{1}{2}$;$-\frac{\sqrt{3}}{2}$): min положит.=$\frac{4\pi}{3}$, max отрицат.=$-\frac{2\pi}{3}$

г) M ($\frac{1}{2}$;$-\frac{\sqrt{3}}{2}$): min положит.=$\frac{5\pi}{3}$, max отрицат.=$-\frac{\pi}{3}$

5.6. а) $y=\frac{\sqrt{2}}{2}$; $\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$ и $\left(-\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$; $t=\frac{\pi}{4}+2\pi n$; $\frac{3\pi}{4}+2\pi n$;

б) $y=\frac{1}{2}$; $\frac{1}{2}$ и $\left(-\frac{\sqrt{3}}{2};\frac{1}{2}\right)$; $t=\frac{\pi}{6}+2\pi n$; $\frac{5\pi}{6}+2\pi n$;

в) $y=0$; $(1;0)$ и $(-1;0)$ $t=\pi n$;

г) $y=\frac{\sqrt{3}}{2}$; $\left(\frac{1}{2};\frac{\sqrt{3}}{2}\right)$ и $\left(-\frac{1}{2};\frac{\sqrt{3}}{2}\right)$ $t=\frac{\pi}{3}+2\pi n$, $\frac{2\pi}{3}+2\pi n$.

5.7. а) $y=-\frac{\sqrt{3}}{2}$, $\left(\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$ и $\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$ $x=(-1)^{k+1}\frac{\pi}{3}+\pi k$;

б) $y=1$, $(0;1)$, $x=\frac{\pi}{2}+2\pi k$;

в) $y=-\frac{\sqrt{2}}{2}$, $\left(\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$ и $\left(-\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$ $x=(-1)^{k+1}\frac{\pi}{4}+\pi k$;

г) $y=-1$, $(0;-1)$ $x=-\frac{\pi}{2}+2\pi k$.

5.8. а) $x=\frac{\sqrt{3}}{2}$, $\left(\frac{\sqrt{3}}{2};\frac{1}{2}\right)$ и $\left(\frac{\sqrt{3}}{2};-\frac{1}{2}\right)$ $x=\pm\frac{\pi}{6}+2\pi k$;

б) $x=\frac{1}{2}$, $\left(\frac{1}{2};\frac{\sqrt{3}}{2}\right)$ и $\left(\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$ $y=\pm\frac{\pi}{3}+2\pi k$;

в) $x=1$, $(1;0)$ $y=2\pi n$;

г) $x=\frac{\sqrt{2}}{2}$, $\left(\frac{\sqrt{2}}{2};\frac{\sqrt{2}}{2}\right)$ и $\left(\frac{\sqrt{2}}{2};-\frac{\sqrt{2}}{2}\right)$ $y=\pm\frac{\pi}{4}+2\pi k$.

5.9. а) $x=0$,$(0;1)$ и $(0;-1)$ $y=\pi/2+\pi n$;

б) $x=-\frac{1}{2}$, $\left(-\frac{1}{2};\frac{\sqrt{3}}{2}\right)$ и $\left(-\frac{1}{2};-\frac{\sqrt{3}}{2}\right)$ $y=\pm\frac{2\pi}{3}+2\pi k$;

в) $x = -\frac{\sqrt{3}}{2}$, $\left(-\frac{\sqrt{3}}{2}; \frac{1}{2}\right)$ и $\left(-\frac{\sqrt{3}}{2}; -\frac{1}{2}\right)$ $y = \pm\frac{5\pi}{6} + 2\pi k$;

г) $x = -1$, $(-1; 0)$. $y = \pi + 2\pi n$.

5.10. см. ответ

5.11. а) $x > 0$, $t \in (-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n)$;

б) $x < \frac{1}{2}$, $t \in (\frac{\pi}{3} + 2\pi n; \frac{5\pi}{3} + 2\pi n)$;

в) $x > \frac{1}{2}$, $t \in (-\frac{\pi}{3} + 2\pi n; \frac{\pi}{3} + 2\pi n)$;

г) $x < 0$, $t \in (\frac{\pi}{2} + 2\pi n; \frac{3}{2}\pi + 2\pi n)$.

5.12. а) $x < \frac{\sqrt{2}}{2}$, $t \in (\frac{\pi}{4} + 2\pi k; \frac{7\pi}{4} + 2\pi k)$;

б) $x > -\frac{\sqrt{2}}{2}$, $t \in (-\frac{3\pi}{4} + 2\pi k; \frac{3\pi}{4} + 2\pi k)$;

в) $x \le -\frac{\sqrt{3}}{2}$, $t \in (\frac{5\pi}{6} + 2\pi k; \frac{7\pi}{6} + 2\pi k)$; г) $x \ge \frac{\sqrt{3}}{2}$, $t \in (-\frac{\pi}{6} + 2\pi k; \frac{\pi}{6} + 2\pi k)$;

5.13. а) $y > 0$, $t \in (2\pi k; \pi + 2\pi k)$; б) $y < \frac{1}{2}$, $t \in (\frac{5\pi}{6} + 2\pi k; \frac{\pi}{6} + 2\pi k)$;

в) $y > \frac{1}{2}$, $t \in (\frac{\pi}{6} + 2\pi k; \frac{5\pi}{6} + 2\pi k)$; г) $y < 0$, $t \in (-\pi + 2\pi k; 2\pi k)$.

5.14. а) $y < \frac{\sqrt{2}}{2}$, $-\frac{5\pi}{4} + 2\pi k < t < \frac{\pi}{4} + 2\pi k$; б) $y > -\frac{\sqrt{2}}{2}$, $-\frac{\pi}{4} + 2\pi k < t < \frac{5\pi}{4} + 2\pi k$;

в) $y \le -\frac{\sqrt{3}}{2}$, $\frac{4\pi}{3} + 2\pi k < t < \frac{5\pi}{3} + 2\pi k$; г) $y \ge \frac{\sqrt{3}}{2}$, $\frac{\pi}{3} + 2\pi k < t < \frac{2\pi}{3} + 2\pi k$;

§ 6. Синус и косинус. Тангенс и котангенс.

6.1. а) $\sin 0 = 0$, $\cos 0 = 1$, $\operatorname{tg} 0 = 0$,

б) $\sin\frac{\pi}{2} = 1$, $\cos\frac{\pi}{2} = 0$, $\operatorname{tg}\frac{\pi}{2} = +\infty$;

в) $\sin\frac{3\pi}{2} = -1$, $\cos = \frac{3\pi}{2} = 0$, $\operatorname{tg}\frac{3\pi}{2} = -\infty$;

г) $\sin\pi = 0$, $\cos\pi = -1$, $\operatorname{tg}\pi = 0$.

6.2. а) $\sin(-2\pi) = 0$, $\cos(-2\pi) = 1$, $\operatorname{tg}(-2\pi) = 0$;

б) $\sin\left(-\frac{\pi}{2}\right)=-1,\ \cos\left(-\frac{\pi}{2}\right)=0,\ \mathrm{tg}\left(-\frac{\pi}{2}\right)=-\infty$;

в) $\sin\left(-\frac{3\pi}{2}\right)=1,\ \mathrm{tg}\left(-\frac{3\pi}{2}\right)=+\infty,\ \cos\left(-\frac{3\pi}{2}\right)=0$;

г) $\sin(-\pi)=0,\ \cos(-\pi)=-1,\ \mathrm{tg}(-\pi)=0$.

6.3. а) $\sin\frac{5\pi}{6}=\frac{1}{2},\ \cos\frac{5\pi}{6}=-\frac{\sqrt{3}}{2},\ \mathrm{tg}\left(\frac{5\pi}{6}\right)=-\frac{1}{\sqrt{3}}$;

б) $\sin\frac{5\pi}{4}=-\frac{\sqrt{2}}{2},\ \cos\frac{5\pi}{4}=-\frac{\sqrt{2}}{2},\ \mathrm{tg}\left(\frac{5\pi}{4}\right)=1$;

в) $\sin\frac{7\pi}{6}=-\frac{1}{2},\ \cos\frac{7\pi}{6}=-\frac{\sqrt{3}}{2},\ \mathrm{tg}\left(\frac{7\pi}{6}\right)=\frac{1}{\sqrt{3}}$;

г) $\sin\frac{7\pi}{4}=-\frac{\sqrt{2}}{2},\ \cos\frac{7\pi}{4}=\frac{\sqrt{2}}{2},\ \mathrm{tg}\left(\frac{7\pi}{4}\right)=-1$.

6.4. а) $\sin\left(-\frac{7\pi}{4}\right)=\frac{\sqrt{2}}{2},\ \cos\left(-\frac{7\pi}{4}\right)=\frac{\sqrt{2}}{2},\ \mathrm{tg}\left(-\frac{7\pi}{4}\right)=1$;

б) $\sin\left(-\frac{4\pi}{3}\right)=\frac{\sqrt{3}}{2},\ \cos\left(-\frac{4\pi}{3}\right)=-\frac{1}{2},\ \mathrm{tg}\left(-\frac{4\pi}{3}\right)=-\sqrt{3}$;

в) $\sin\left(-\frac{5\pi}{6}\right)=-\frac{1}{2},\ \cos\left(-\frac{5\pi}{6}\right)=-\frac{\sqrt{3}}{2},\ \mathrm{tg}\left(-\frac{5\pi}{6}\right)=\frac{1}{\sqrt{3}}$;

г) $\sin\left(-\frac{5\pi}{3}\right)=\frac{\sqrt{3}}{2},\ \cos\left(-\frac{5\pi}{3}\right)=\frac{1}{2},\ \mathrm{tg}\left(-\frac{5\pi}{3}\right)=\sqrt{3}$.

6.5. а) $\sin\frac{13\pi}{6}=\frac{1}{2},\ \cos\frac{13\pi}{6}=\frac{\sqrt{3}}{2}\ \mathrm{tg}\left(\frac{13\pi}{6}\right)=\frac{1}{\sqrt{3}}$;

б) $\sin\left(-\frac{8\pi}{3}\right)=-\frac{\sqrt{3}}{2},\ \cos\left(-\frac{8\pi}{3}\right)=-\frac{1}{2}\ \mathrm{tg}\left(-\frac{8\pi}{3}\right)=\sqrt{3}$;

в) $\sin\frac{23\pi}{6}=-\frac{1}{2},\ \cos\frac{23\pi}{6}=\frac{\sqrt{3}}{2}\ \mathrm{tg}\left(\frac{23\pi}{6}\right)=-\frac{1}{\sqrt{3}}$;

г) $\sin\left(-\frac{11\pi}{4}\right)=-\frac{\sqrt{2}}{2},\ \cos\left(-\frac{11\pi}{4}\right)=-\frac{\sqrt{2}}{2}\ \mathrm{tg}\left(-\frac{11\pi}{4}\right)=1$.

6.6. а) $\sin\left(-\frac{\pi}{4}\right)+\cos\frac{\pi}{3}+\cos\left(-\frac{\pi}{6}\right)=-\frac{\sqrt{2}}{2}+\frac{1}{2}+\frac{\sqrt{3}}{2}+=\frac{\sqrt{3}+1-\sqrt{2}}{2}$;

б) $\cos\frac{\pi}{6}\cos\frac{\pi}{4}\cos\frac{\pi}{3}\cos\frac{\pi}{2}=0$, т.к. $\cos\frac{\pi}{2}=0$;

в) $\sin\left(-\frac{\pi}{2}\right)-\cos(-\pi)+\sin\left(-\frac{3\pi}{2}\right)=-1+1+1=1$;

г) $\sin\frac{\pi}{6}\sin\frac{\pi}{4}\sin\frac{\pi}{3}\sin\frac{\pi}{2}=\frac{1}{2}\cdot\frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\cdot 1=\frac{\sqrt{6}}{8}$.

6.7. а) $\sin\left(-\frac{3\pi}{4}\right)+\cos\left(-\frac{\pi}{4}\right)+\sin\frac{\pi}{4}\cos\frac{\pi}{2}+\cos 0\sin\frac{\pi}{2}=$

$=-\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}+1=1$;

б) $\cos\frac{5\pi}{3}+\cos\frac{4\pi}{3}+\sin\frac{3\pi}{2}\sin\frac{5\pi}{8}\cos\frac{3\pi}{2}=\frac{1}{2}-\frac{1}{2}=0$.

6.8. а) $\operatorname{tg}\frac{\pi}{4}+\operatorname{ctg}\frac{5\pi}{4}=1+1=2$;

б) $\operatorname{ctg}\frac{\pi}{3}\cdot\operatorname{tg}\frac{\pi}{6}=\frac{\sqrt{3}}{3}\cdot\frac{\sqrt{3}}{3}=\frac{1}{3}$;

в) $\operatorname{tg}\frac{\pi}{6}-\operatorname{ctg}\frac{\pi}{6}=\frac{\sqrt{3}}{3}-\sqrt{3}=\frac{-2\sqrt{3}}{3}$;

г) $\operatorname{tg}\frac{9\pi}{4}+\operatorname{ctg}\frac{\pi}{4}=1+1=2$.

6.9. а) $\operatorname{tg}\frac{\pi}{4}\cdot\sin\frac{\pi}{3}\cdot\operatorname{ctg}\frac{\pi}{6}=1\cdot\frac{\sqrt{3}}{2}\sqrt{3}=\frac{3}{2}$;

б) $2\sin\pi+3\cos\pi+\operatorname{ctg}\frac{\pi}{2}=0-3+0=-3$;

в) $\sin\frac{\pi}{3}=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$, $\operatorname{tg}\frac{\pi}{3}=\sqrt{3}\Rightarrow 0$

г) $\operatorname{tg}0=0$, $\cos\frac{3\pi}{2}=0$, $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}\Rightarrow -4,5$

6.10. а) $\operatorname{tg}\frac{\pi}{5}\cdot\operatorname{ctg}\frac{\pi}{5}=1$;

б) $3\operatorname{tg}2,3\cdot\operatorname{ctg}2,3=3$;

в) $\operatorname{tg}\frac{\pi}{7}\cdot\operatorname{ctg}\frac{\pi}{7}=1$;

г) $7\operatorname{ctg}\frac{\pi}{12}\cdot\operatorname{ctg}\frac{\pi}{12}=7$.

6.11. а) $\sin t\cdot\operatorname{ctg}t=\cos t$, $\sin t\cdot\frac{\cos t}{\sin t}=\cos t$;

б) $\frac{\sin t}{tg\ t}=\cos t$, $\sin t\cdot\frac{\cos t}{\sin t}=\cos t$;

в) $\cos t\cdot\operatorname{tg}t=\sin t$, $\cos t\frac{\sin t}{\cos t}=\sin t$;

г) $\frac{\cos t}{\operatorname{ctg}t}=\sin t$, $\cos t\cdot\frac{\sin t}{\cos t}=\sin t$.

6.12. а) $\sin t\cdot\cos t\cdot\operatorname{tg}t=\sin t\cdot\cos t\cdot\frac{\sin t}{\cos t}=\sin^2 t$;

б) $\sin t\cdot\cos t\cdot\operatorname{ctg}t-1=\sin t\cdot\cos t\cdot\frac{\cos t}{\sin t}-1=\cos^2 t-1=-\sin^2 t$;

в) $\sin^2 t - \text{tg}\, t \cdot \text{ctg}\, t = \sin^2 t - 1 = -\cos^2 t$;

г) $\dfrac{1-\cos^2 t}{1-\sin^2 t} = \dfrac{\sin^2 t}{\cos^2 t} = \text{tg}^2 t$.

6.13. а) $\cos 2t$, $t = \dfrac{\pi}{2}$, $\cos \pi = -1$; б) $\sin \dfrac{t}{2}$, $t = -\dfrac{\pi}{3}$, $\sin(-\dfrac{\pi}{6}) = -\dfrac{1}{2}$;

в) $\sin 2t$, $t = -\dfrac{\pi}{6}$, $\sin(-\dfrac{\pi}{3}) = -\dfrac{\sqrt{3}}{2}$; г) $\cos \dfrac{1}{2}$, $t = -\dfrac{\pi}{3}$, $\cos(-\dfrac{\pi}{6}) = \dfrac{\sqrt{3}}{2}$.

6.14. а) $\sin^2 t - \cos^2 t$, $t = \dfrac{\pi}{3}$, $\sin^2\dfrac{\pi}{3} - \cos^2\dfrac{\pi}{3} = \dfrac{3}{4} - \dfrac{1}{4} = \dfrac{1}{2}$;

б) $\sin^2 t + \cos^2 t$, $\forall t$, в том числе для $t = \dfrac{\pi}{4}$;

в) $\sin^2 t - \cos^2 t$, $t = \dfrac{\pi}{4}$, $\sin^2\dfrac{\pi}{4} - \cos^2\dfrac{\pi}{4} = 0$;

г) $\sin^2 t + \cos^2 t$, см. п. б).

6.15. а) $f(t) = 2\sin t$; $f_{\max} = 2$, $f_{\min} = -2$.

б) $f(t) = 3 + 4\cos t$; $f_{\max} = 7$, $f_{\min} = -1$.

в) $f(t) = -3\cos t$; $f_{\max} = 3$, $f_{\min} = -3$.

г) $f(t) = 3 - 5\sin t$; $f_{\max} = 8$, $f_{\min} = -2$.

6.16. а) $\cos t = \dfrac{\sqrt{2}}{2}$, $t = \pm\dfrac{\pi}{4} + 2\pi n$; б) $\sin t = -\dfrac{1}{2}$, $t = (-1)^{n+1}\dfrac{\pi}{6} + \pi n$;

в) $\cos t = -\dfrac{1}{2}$, $t = \pm\dfrac{2\pi}{3} + 2\pi n$; г) $\sin t = \dfrac{\sqrt{2}}{2}$, $t = (-1)^n\dfrac{\pi}{4} + \pi n$.

6.17. а) $\sin t = -\dfrac{\sqrt{3}}{2}$, $t = (-1)^{k+1}\dfrac{\pi}{3} + \pi k$;

б) $\cos t = \sqrt{3}$ решений нет $|\cos t| \le 1$;

в) $\cos t = -\dfrac{\sqrt{3}}{2}$, $t = \pm\dfrac{5\pi}{6} + 2\pi n$;

г) $\cos t = -\dfrac{\pi}{3}$ решений нет, т.к. $|\cos t| \le 1$.

6.18. а) $\sin t + 1 = 0$, $\sin t = -1$, $t = -\dfrac{\pi}{2} + 2\pi n$;

б) $\cos t - 1 = 0$, $\cos t = 1$, $t = 2\pi n$;

в) $1 - 2\sin t = 0$, $\sin t = \dfrac{1}{2}$, $t = (-1)^n\dfrac{\pi}{6} + \pi n$;

г) $2\cos t + 1 = 0$, $\cos t = -\dfrac{1}{2}$, $t = \pm\dfrac{2\pi}{3} + 2\pi n$.

6.19. а) $\dfrac{\sin t - 1}{\cos t}$, $\cos t \ne 0$, $t \ne \dfrac{\pi}{2} + \pi n$;

б) $2\sin t=\sqrt{3}\Rightarrow \sin t=\dfrac{\sqrt{3}}{2}\Rightarrow t=(-1)^n\dfrac{\pi}{3}+\pi n,\ n\in\mathbb{Z}$

в) $\sin t=1\Rightarrow t=\dfrac{\pi}{2}+2\pi n,\ n\in\mathbb{Z}$

г) $\cos t=\dfrac{1}{2}\Rightarrow t=\pm\dfrac{\pi}{4}+2\pi n,\ n\in\mathbb{Z}$

6.20. а) $\sin\dfrac{4\pi}{7}>0$; б) $\cos\left(-\dfrac{5\pi}{7}\right)<0$;

в) $\sin\dfrac{9\pi}{8}<0$; г) $\sin\left(-\dfrac{3\pi}{8}\right)<0$.

6.21. см. ответ

6.22. а) $\sin(-2)<0$; б) $\cos 3<0$; в) $\sin 5<0$; г) $\cos(-6)>0$.

6.23. а) $\sin 10<0$; б) $\cos(-12)>0$; в) $\sin(-15)<0$; г) $\cos 8<0$.

6.24. а) $\sin 1\cos 2<0$; б) $\sin\dfrac{\pi}{7}\cos\left(-\dfrac{7\pi}{5}\right)<0$;

в) $\cos 2\sin(-3)>0$; г) $\cos\left(-\dfrac{14\pi}{9}\right)\sin\left(-\dfrac{4\pi}{9}\right)<0$.

6.25. а) $\cos\dfrac{5\pi}{9}-\operatorname{tg}\dfrac{25\pi}{18}<0$; б) $\operatorname{tg}1-\cos 2>0$;

в) $\sin\dfrac{7\pi}{10}-\operatorname{ctg}\dfrac{3\pi}{5}>0$; г) $\sin 2-\operatorname{ctg}5{,}5>0$.

6.26. а) $\sin 1\cdot\cos 2\cdot\operatorname{tg}3\cdot\operatorname{ctg}4>0$;
б) $\sin(-5)\cos(-6)\ \operatorname{tg}(-7)\operatorname{ctg}(-8)<0$.

6.27. а) $\cos 1+\cos(1+\pi)+\sin\left(-\dfrac{\pi}{3}\right)+\cos\left(-\dfrac{\pi}{6}\right)=$

$=\cos 1-\cos 1-\dfrac{\sqrt{3}}{2}+\dfrac{\sqrt{3}}{2}=0$

б) $\sin 2+\sin(2+\pi)+\cos^2\left(-\dfrac{\pi}{12}\right)+\sin^2\dfrac{\pi}{12}=\sin 2-\sin 2+1=1.$

6.28. а) $\sin^2(1{,}5+2\pi k)+\cos^2 1{,}5+\cos\left(-\dfrac{\pi}{4}\right)+\sin\left(-\dfrac{\pi}{6}\right)=$

$=1+\dfrac{\sqrt{2}}{2}-\dfrac{1}{2}=\dfrac{\sqrt{2}+1}{2}$;

б) $\cos^2\left(\dfrac{\pi}{8}+4\pi\right)+\sin^2\left(\dfrac{\pi}{8}-44\pi\right)=\cos^2\dfrac{\pi}{8}+\sin^2\dfrac{\pi}{8}=1.$

6.29. а) $\operatorname{tg}2{,}5\cdot\operatorname{ctg}2{,}5+\cos^2\pi-\sin^2\dfrac{\pi}{8}-\cos^2\dfrac{\pi}{8}=1+1-1=1.$

б) $\sin^2\dfrac{3\pi}{7}-2\operatorname{tg}1\cdot\operatorname{ctg}1+\cos^2\left(-\dfrac{3\pi}{7}\right)+\sin^2\dfrac{5\pi}{2}=1-2+1=0.$

6.30. а) $10\sin t=\sqrt{75}$, $\sin t=\frac{5\sqrt{3}}{10}=\frac{\sqrt{3}}{2}$, $t=(-1)^n\ \frac{\pi}{3}+\pi n$;

б) $\sqrt{8}\ \sin t+2=0$, $\sin t=-\frac{\sqrt{2}}{2}$, $t=(-1)^{k+1}\ \frac{\pi}{4}+\pi k$

в) $8\cos t-\sqrt{32}=0$, $\cos t=\frac{\sqrt{2}}{2}$, $t=\pm\frac{\pi}{4}+2\pi k$;

г) $8\cos t=-\sqrt{48}$, $\cos t=-\frac{\sqrt{3}}{2}$, $t=\pm\frac{5\pi}{6}+2\pi k$

6.31. а) $\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}-\sin\sqrt{2}=0$, $\sin t=\frac{\sqrt{2}}{2}$, $t=(-1)^k\ \frac{\pi}{4}+\pi k$

б) $\sqrt{\frac{4}{3}}\ \cos t=\cos^2 1+\sin^2 1$, $\cos t=\frac{\sqrt{3}}{2}$, $t=\pm\frac{\pi}{6}+2\pi n.$

6.32. а) $|\sin t|=1$, $\sin t=\pm 1$, $t=\frac{\pi}{2}+\pi n$

б) $\sqrt{1-\sin^2 t}=\frac{1}{2}$, $|\cos t|=\frac{1}{2}$, $\cos t=\pm\frac{1}{2}$, $t=\pm\frac{\pi}{3}+\pi k$

в) $|\cos t|=1$, $\cos t=\pm 1$, $t=\pi n$

г) $\sqrt{1-\cos^2 t}=\frac{\sqrt{2}}{2}$, $|\sin t|=\frac{\sqrt{2}}{2}$ $\sin t=\pm\frac{\sqrt{2}}{2}$, $t=\frac{\pi}{4}+\frac{\pi n}{2}$.

6.33. а) $\sqrt{\sin 10,\ 2\pi}$ – Да; б) $\sqrt{\cos 1,3\pi}$ – Нет;

в) $\sqrt{\sin(-3,4)\pi}$ – Да; г) $\sqrt{\cos(-6,9\pi)}$ – Нет.

6.34. а) $\frac{\pi}{2}<\frac{7\pi}{10}<\frac{5\pi}{6}<\pi\Rightarrow a>b$ б) $a=\sin 2>b=\cos 2$;

в) $0<\frac{\pi}{8}<\frac{\pi}{3}<\frac{\pi}{2}\Rightarrow a>b$; г) $a=\sin 1>b=\cos 1$;

6.35. а) $\frac{10\pi}{9}=\pi+\frac{\pi}{9}\Rightarrow\sin\frac{2\pi}{9}-\sin\frac{10\pi}{9}=\sin\frac{2\pi}{9}+\sin\frac{\pi}{9}>0$

б) $\sin 1-\sin 1,1<0$, т.к. $0<1<1,1<\frac{\pi}{2}$

в) $\frac{15\pi}{8}=2\pi-\frac{\pi}{8}\Rightarrow\sin\frac{15\pi}{8}-\cos\frac{\pi}{4}=-\sin\frac{\pi}{8}-\cos\frac{\pi}{4}<0$

г) $\cos 1-\cos 0,9<0$, т.к. $0<0,9<1<\frac{\pi}{2}$

6.36. а) $\sin\frac{4\pi}{3}$, $\sin\frac{7\pi}{6}$, $\sin\frac{\pi}{7}$, $\sin\frac{\pi}{5}$, $\sin\frac{2\pi}{3}$;

б) $\cos\frac{5\pi}{6}$, $\cos\frac{5\pi}{4}$, $\cos\frac{\pi}{3}$, $\cos\frac{7\pi}{4}$, $\cos\frac{\pi}{8}$.

6.37. а) cos 4, sin 3, cos 5, sin 2; б) cos 3, cos 4, cos 7, cos 6;
в) sin 4, sin 6, sin 3, sin 7; г) cos 3, sin 5, sin 4, cos 2.

6.38. а) cos 1, sin 1, 1, tg 1. б) ctg 2, cos 2, sin 2, 2.

6.39. а) $\sin t > 0,\ t \in (2\pi k;\ \pi + 2\pi k)$;

б) $\sin < \frac{\sqrt{3}}{2},\ t \in (-\frac{4\pi}{3} + 2\pi k;\ \frac{\pi}{3} + 2\pi k)$;

в) $\sin t < 0,\ t \in (-\pi + 2\pi k;\ 2\pi k)$;

г) $\sin t > \frac{\sqrt{3}}{2},\ t \in (\frac{\pi}{3} + 2\pi k;\ \frac{2\pi}{3} + 2\pi k)$.

6.40. а) $\cos t > 0,\ t \in (-\frac{\pi}{2} + 2\pi k;\ \frac{\pi}{2} + 2\pi k)$;

б) $\cos t < \frac{\sqrt{2}}{2},\ t \in (\frac{\pi}{4} + 2\pi k;\ \frac{7\pi}{4} + 2\pi k)$;

в) $\cos t < 0,\ t \in (\frac{\pi}{2} + 2\pi k;\ \frac{3\pi}{2} + 2\pi k)$;

г) $\cos t > \frac{\sqrt{2}}{2},\ t \in (-\frac{\pi}{4} + 2\pi k;\ \frac{\pi}{4} + 2\pi k)$.

6.41. а) $\sin t > -\frac{\sqrt{2}}{2},\ t \in (-\frac{\pi}{4} + 2\pi k;\ \frac{5\pi}{4} + 2\pi k)$;

б) $\cos t > -\frac{\sqrt{3}}{2},\ t \in (-\frac{5\pi}{6} + 2\pi k;\ \frac{5\pi}{6} + 2\pi k)$;

в) $\sin t < -\frac{\sqrt{2}}{2},\ t \in (\frac{5\pi}{4} + 2\pi k;\ \frac{7\pi}{4} + 2\pi k)$;

г) $\cos t < -\frac{\sqrt{3}}{2},\ t \in (\frac{5\pi}{6} + 2\pi k;\ \frac{7\pi}{6} + 2\pi k)$.

§ 7. *Тригонометрические функции числового аргумента*

7.1. а) $1 - \sin^2 t = \cos^2 t$; б) $\cos^2 t - 1 = -\sin^2 t$;
в) $1 - \cos^2 t = \sin^2 t$; г) $\sin^2 t - 1 = -\cos^2 t$.

7.2. а) $(1 - \sin t)(1 + \sin t) = 1 - \sin^2 t = \cos^2 t$;
б) $\cos^2 t + 1 - \sin^2 t = \cos^2 t + \cos^2 t = 2\cos^2 t$;
в) $(1 - \cos t)(1 + \cos t) = 1 - \cos^2 t = \sin^2 t$;
г) $\sin^2 t + 2\cos^2 t - 1 = 1 + \cos^2 t - 1 = \cos^2 t$.

7.3. а) $\frac{1}{\cos^2 t} - 1 = \frac{1 - \cos^2 t}{\cos^2 t} = \mathrm{tg}^2 t$; б) $\frac{1 - \sin^2 t}{\cos^2 t} = 1$;

в) $1 - \frac{1}{\sin^2 t} = \frac{\sin^2 t - 1}{\sin^2 t} = -\mathrm{ctg}^2 t$; г) $\frac{1 - \cos^2 t}{1 - \sin^2 t} = \frac{\sin^2 t}{\cos^2 t} = \mathrm{tg}^2 t$.

7.4. а) $\frac{(\sin t+\cos t)^2}{1+2\sin t\cos t}=\frac{1+2\sin t\cos t}{1+2\sin t\cos t}=1;$

б) $\frac{1-2\sin t\cos t}{(\cos t-\sin t)^2}=\frac{1-2\sin t\cos t}{1-2\sin t\cos t}=1.$

7.5. а) $\frac{\cos^2 t}{1-\sin t}-\sin t=1,\quad \frac{1-\sin^2 t}{1-\sin t}-\sin t=\frac{1-\sin^2 t-\sin t+\sin^2 t}{1-\sin t}=1;$

б) $\frac{\sin^2 t}{1+\cos t}+\cos t=1,\quad \frac{1-\cos^2 t}{1+\cos t}+\cos t=\frac{\sin^2 t+\cos t+\cos^2 t}{1+\cos t}=1.$

7.6. а) $\sin^4 t-\cos^4 t+2\cos^2 t=1-2\cos^2 t+\cos^4 t-\cos^4 t+2\cos^2 t=1;$

б) $\frac{2-\sin^2 t-\cos^2 t}{3\sin^2 t+3\cos^2 t}=\frac{2-1}{3}=\frac{1}{3};$

в) $\sin^4 t+\cos^4 t+2\sin^2 t\cos^2 t=(\sin^2 t+\cos^2 t)^2=1;$

г) $\frac{\sin^4 t-\cos^4 t}{\sin^2 t-\cos^2 t}=\frac{(\sin^2 t-\cos^2 t)(\sin^2 t+\cos^2 t)}{\sin^2 t-\cos^2 t}=\sin^2 t+\cos^2 t=1.$

7.7. а) $\sin t=\frac{4}{5},\ \frac{\pi}{2}<t<\pi,\quad \cos t=-\frac{3}{5},\quad \operatorname{tg} t=-\frac{4}{3},\quad \operatorname{ctg} t=-\frac{3}{4};$

б) $\sin t=\frac{5}{13},\ 0<t<\frac{\pi}{2},\quad \cos t=\sqrt{1-\frac{25}{169}}=\frac{12}{13},\quad \operatorname{tg} t=\frac{5}{12},\quad \operatorname{ctg} t=\frac{12}{5};$

в) $\sin t=-0{,}6,\ -\frac{\pi}{2}<t<0,\quad \cos t=\frac{4}{5},\quad \operatorname{tg} t=-\frac{3}{4},\quad \operatorname{ctg} t=-\frac{4}{3};$

г) $\sin t=-0{,}28,\ \pi<t<\frac{3\pi}{2},$

$\cos t=-\sqrt{1-\frac{49}{625}}=-\frac{24}{25},\quad \operatorname{tg} t=\frac{7}{24},\quad \operatorname{ctg} t=\frac{24}{7}.$

7.8. а) $\cos t=0{,}8,\ 0<t<\frac{\pi}{2},\quad \sin t=\frac{3}{5},\quad \operatorname{tg} t=\frac{3}{4},\quad \operatorname{ctg} t=\frac{4}{3};$

б) $\cos t=-\frac{5}{13},\ \frac{\pi}{2}<t<\pi,\quad \sin t=\frac{12}{13},\quad \operatorname{tg} t=-\frac{12}{5},\quad \operatorname{ctg} t=-\frac{5}{12};$

в) $\cos t=0{,}6,\quad \frac{3\pi}{2}<t<2\pi,\quad \sin t=-\frac{4}{5},\quad \operatorname{tg} t=-\frac{4}{3},\quad \operatorname{ctg} t=-\frac{3}{4};$

г) $\cos t=-\frac{24}{25},\ \pi<t<\frac{3\pi}{2},\quad \sin t=-\frac{7}{25},\quad \operatorname{tg} t=\frac{7}{24},\quad \operatorname{ctg} t=\frac{24}{7}.$

7.9. а) $\operatorname{tg} t=\frac{3}{4},\quad 0<t<\frac{\pi}{2},\quad \operatorname{ctg} t=\frac{4}{3},\quad \sin t=\frac{3}{5},\quad \cos t=\frac{4}{5};$

б) $\operatorname{tg} t=2{,}4,\quad \pi<t<\frac{3\pi}{2},\quad \operatorname{ctg} t=\frac{5}{12},\quad \cos t=-\frac{5}{13},\quad \sin t=-\frac{12}{13};$

в) $\operatorname{tg} t=-\frac{3}{4}$, $\frac{\pi}{2}<t<\pi$, $\operatorname{ctg} t=-\frac{4}{3}$, $\sin t=\frac{3}{5}$, $\cos t=-\frac{4}{5}$;

г) $\operatorname{tg} t=-\frac{5}{12}$, $\frac{3\pi}{2}<t<2\pi$, $\operatorname{ctg} t=-\frac{12}{5}$, $\sin t=-\frac{5}{13}$; $\cos t=\frac{12}{13}$.

Во всех пунктах применяются формулы $1+\operatorname{tg}^2 t=\frac{1}{\cos^2 t}$ и

$1+\operatorname{ctg}^2 t=\frac{1}{\sin^2 t}$.

7.10. а) $\operatorname{ctg} t=\frac{12}{5}, \pi<t<\frac{3\pi}{2}$, $\operatorname{tg} t=\frac{5}{12}$, $\sin t=-\frac{5}{13}$, $\cos t=-\frac{12}{13}$;

б) $\operatorname{ctg} t=\frac{7}{24}$, $0<t<\frac{\pi}{2}$, $\operatorname{tg} t=\frac{24}{7}$, $\sin t=\frac{24}{25}$, $\cos t=\frac{7}{25}$;

в) $\operatorname{ctg} t=-\frac{5}{12}$, $\frac{3\pi}{2}<t<2\pi$, $\operatorname{tg} t=-\frac{12}{5}$, $\cos t=\frac{5}{13}$, $\sin t=-\frac{12}{13}$;

г) $\operatorname{ctg} t=-\frac{8}{15}, \frac{\pi}{2}<t<\pi, \operatorname{tg} t=-\frac{15}{8}, \frac{\cos t}{\sqrt{1-\cos^2 t}}=-\frac{8}{15}$,

$\cos^2 t=\frac{64}{225}-\frac{64}{225}\cos^2 t, \cos t=-\frac{8}{17}, \sin t=\sqrt{1-\frac{64}{289}}=\frac{15}{17}$.

7.11. а) $f(t)=1-(\cos^2 t-\sin^2 t)=2\sin^2 t$, $f_{\max}=2, f_{\min}=0$;

б) $f(t)=1-\sin t\cdot\cos t\cdot\operatorname{tg} t=1-\sin t\cdot\cos t\cdot\frac{\sin t}{\cos t}=\cos^2 t$, $f_{\max}=1$, $f_{\min}=0$.

в) $f(t)=\cos^2 t\cdot\operatorname{tg}^2 t+5\cos^2 t-1=\cos^2 t\cdot\frac{\sin^2 t}{\cos^2 t}+5\cos^2 t-1=\sin^2 t+5\cos^2 t-1=$

$=4\cos^2 t, f_{\max}=4$, $f_{\min}=0$.

г) $f(t)=\sin t+3\sin^2 t+3\cos^2 t=\sin t+3$, $f_{\max}=4$, $f_{\min}=2$.

7.12. а) $\operatorname{ctg} t-\frac{\cos t-1}{\sin t}=\frac{\cos t-\cos t+1}{\sin t}=\frac{1}{\sin t}$;

б) $\operatorname{ctg}^2 t-(\frac{1}{\sin^2 t}-1)=\frac{\cos^2 t-1+\sin^2 t}{\sin^2 t}=0$;

в) $\cos^2 t-(\operatorname{ctg}^2 t+1)\sin^2 t=\cos^2 t-\cos^2 t-\sin^2 t=-\sin^2 t$;

г) $\frac{\sin^2 t-1}{\cos^2 t-1}+\operatorname{tg} t\cdot\operatorname{ctg} t=\operatorname{ctg}^2 t+1=\frac{\sin^2 t+\cos^2 t}{\sin^2 t}=\frac{1}{\sin^2 t}$.

7.13. а) $\frac{\sin t}{1+\cos t}+\frac{\sin t}{1-\cos t}=\frac{\sin t-\sin t\ \cos t+\sin t+\cos t\ \sin t}{1-\cos^2 t}=$

$=\frac{2\sin t}{\sin^2 t}=\frac{2}{\sin t}$;

б) $\operatorname{ctg}^2 t(\cos^2 t-1)+1=-\cos^2 t+1=\sin^2 t$;

в) $\dfrac{\cos t}{1+\sin t}+\dfrac{\cos t}{1-\sin t}=\dfrac{\cos t-\sin t\ \cos t+\cos t+\sin t\ \cos t}{1-\sin^2 t}=$

$=\dfrac{2\cos t}{\cos^2 t}=\dfrac{2}{\cos t}$;

г) $\dfrac{\operatorname{tg} t+1}{1+\operatorname{ctg} t}=\dfrac{\dfrac{\sin t+\cos t}{\cos t}}{\dfrac{\sin t+\cos t}{\sin t}}=\operatorname{tg} t.$

7.14. а) $\dfrac{1-\sin^2 t}{1-\cos^2 t}+\operatorname{tg} t\cdot\operatorname{ctg} t=\dfrac{\cos^2 t}{\sin^2 t}+1=\operatorname{ctg}^2 t+1=\dfrac{1}{\sin^2 t}$;

б) $\dfrac{\cos^2 t-\operatorname{ctg}^2 t}{\sin^2 t-\operatorname{tg}^2 t}=\dfrac{\dfrac{\cos^2 t\cdot\sin^2 t-\cos^2 t}{\sin^2 t}}{\dfrac{\sin^2 t\cdot\cos^2 t-\sin^2 t}{\cos^2 t}}=\dfrac{\cos^2 t\ (\sin^2 t-1)\ \cos^2 t}{\sin^2 t\ \ \sin^2 t\ (\cos^2 t-1)}=$

$=\dfrac{-\cos^6 t}{-\sin^6 t}=\operatorname{ctg}^6 t$

7.15. а) $1+\sin t=\dfrac{\cos t+\operatorname{ctg} t}{\operatorname{ctg} t}$,

$\dfrac{\cos t+\operatorname{ctg} t}{\operatorname{ctg} t}=\cos t-\operatorname{tg} t+1=\sin t+1;$

б) $\dfrac{\sin t+\operatorname{tg} t}{\operatorname{tg} t}=1+\cos t,\ \sin t\cdot\dfrac{\cos t}{\sin t}+1=\cos t+1;$

в) $\dfrac{1-\sin t}{\cos t}=\dfrac{\cos t}{1+\sin t},\ \dfrac{1-\sin^2 t}{\cos t\ (1+\sin t)}=\dfrac{\cos t}{1+\sin t}$;

г) $\dfrac{\sin t}{1-\cos t}=\dfrac{1+\cos t}{\sin t},\ \dfrac{\sin t(1+\cos t)}{1-\cos^2 t}=\dfrac{1+\cos t}{\sin t}.$

7.16. а) $\dfrac{(\sin t+\cos t)^2-1}{\operatorname{ctg} t-\sin t\ \cos t}=2\operatorname{tg}^2 t,$

$\dfrac{2\sin t\cos t}{\cos t-\sin^2 t\cos t}\sin t=\dfrac{2\sin^2 t\cos t}{\cos t(1-\sin^2 t)}=2\operatorname{tg}^2 t.$

б) $\sin^3 t\ (1+\operatorname{ctg} t)+\cos^3 t\ (1+\operatorname{tg} t)=\sin t+\cos t,$

$\sin^3 t\cdot\dfrac{\sin t+\cos t}{\sin t}+\cos^3 t\ \dfrac{\sin t+\cos t}{\cos t}=$

$=(\sin t+\cos t)\ (\sin^2 t+\cos^2 t)=\ \sin t+\cos t.$

7.17. а) $\sin(4\pi+t)=\dfrac{3}{5},\ 0<t<\dfrac{\pi}{2},$

$\cos t=\dfrac{4}{5},\ \operatorname{tg} t=\dfrac{3}{4},\ \operatorname{tg}(-t)=-\dfrac{3}{4},\ \operatorname{tg}(\pi-t)=-\dfrac{3}{4}$;

б) $\cos(2\pi + t) = \frac{12}{13}$, $\frac{3\pi}{2} < t < 2\pi$,

$\sin t = -\frac{5}{13}$, $\operatorname{ctg} t = -\frac{12}{5}$, $\operatorname{ctg}(-t) = \frac{12}{5}$, $\operatorname{ctg}(\pi - t) = \frac{12}{5}$.

7.18. а) $\cos t = -\frac{5}{13}$, $8{,}5\pi < t < 9\pi$, $\sin t = \frac{12}{13}$, $\sin(-t) = -\frac{12}{13}$;

б) $\sin t = \frac{4}{5}$, $\frac{9\pi}{2} < t < 5\pi$,

$\cos t = -\frac{3}{5}$, $\cos(-t) = -\frac{3}{5}$,

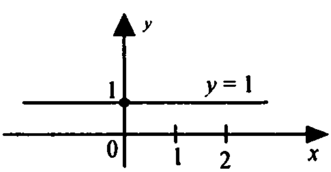
$\sin(-t) = -\frac{4}{5}$, $\cos(-t) + \sin(-t) = -\frac{7}{5}$.

7.19. $\operatorname{tg} t - \frac{1}{\operatorname{tg} t} = -\frac{7}{12}$, $0 < t < \frac{\pi}{2}$, $12\operatorname{tg}^2 t + 7\operatorname{tg} t - 12 = 0$,

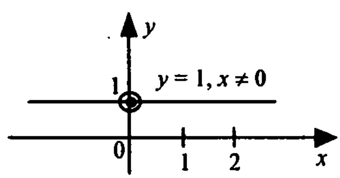
$\operatorname{tg} t = \frac{-7 \pm \sqrt{49 - 4\cdot 12(-12)}}{24} = \frac{-7 \pm 25}{24}$, $\operatorname{tg} t = -\frac{4}{3}$ не подходит, т.к. $0 < t < \frac{\pi}{2}$,

$\operatorname{tg} t = \frac{3}{4} \Rightarrow \cos t = \frac{4}{5}$, $\sin t = \frac{3}{5} \Rightarrow \sin t + \cos t = \frac{7}{5}$.

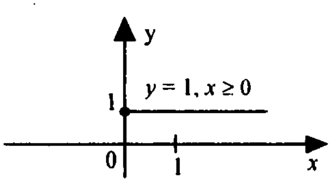
7.20. а) $y = \cos^2 t + \sin^2 t = 1$;



б) $y = \cos^2 \frac{1}{x} + \sin^2 \frac{1}{x} = 1$ $(x \neq 0)$

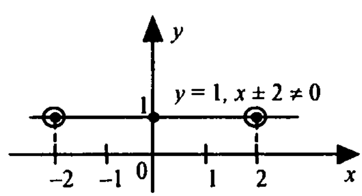


в) $y = \sin^2 \sqrt{x} + \cos^2 \sqrt{x} = 1$ $(x \geq 0)$



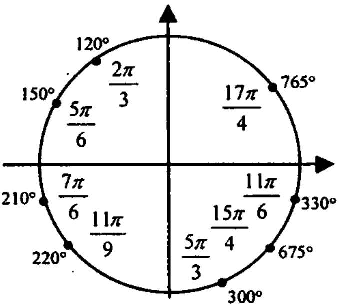
г) $y = \sin^2 \frac{1}{x^2 - 4} + \cos^2 \frac{1}{x^2 - 4} =$

$= 1$ $(x \neq \pm 2)$



§ 8. *Тригонометрические функции углового аргумента*

8.1 – 8.4 см. рис.



8.1. а) $120° = \frac{2\pi}{3}$; б) $220° = \frac{11\pi}{9}$;

в) $300° = \frac{5\pi}{3}$; г) $765° = \frac{17\pi}{4}$.

8.2. а) $210° = \frac{7\pi}{6}$; б) $150° = \frac{5\pi}{6}$;

в) $330° = \frac{11\pi}{6}$; г) $675° = \frac{15\pi}{4}$.

8.3. а) $\frac{3\pi}{4} = 135°$; б) $\frac{11\pi}{3} = 660°$;

в) $\frac{6\pi}{5} = 216°$; г) $\frac{46\pi}{9} = 920°$.

8.4. а) $\frac{5\pi}{8} = 112{,}5°$; б) $\frac{7\pi}{12} = 105°$; в) $\frac{11\pi}{12} = 165°$; г) $\frac{47\pi}{9} = 940°$.

8.5. а) $\sin 90° = 1$, $\cos 90° = 0$, $\mathrm{tg}\, 90°$ = не сущ., $\mathrm{ctg}\, 90° = 0$
б) $\sin 180° = 0$, $\cos 180° = -1$, $\mathrm{tg}\, 180° = 0$, $\mathrm{ctg}\, 180°$ = не сущ.
в) $\sin 270° = -1$, $\cos 270° = 0$, $\mathrm{tg}\, 270°$ = не сущ., $\mathrm{ctg}\, 270° = 0$
г) $\sin 360° = 0$, $\cos 360° = 1$, $\mathrm{tg}\, 360° = 0$, $\mathrm{ctg}\, 360°$ = не сущ.

8.6. а) $\sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\mathrm{tg}\, 30° = \frac{\sqrt{3}}{3}$, $\mathrm{ctg}\, 30° = \sqrt{3}$;

б) $\sin 150° = \frac{1}{2}$, $\cos 150° = -\frac{\sqrt{3}}{2}$, $\mathrm{tg}\, 150° = -\frac{\sqrt{3}}{3}$, $\mathrm{ctg}\, 150° = -\sqrt{3}$;

в) $\sin 210° = -\frac{1}{2}$, $\cos 210° = -\frac{\sqrt{3}}{2}$, $\mathrm{tg}\, 210° = \frac{\sqrt{3}}{3}$, $\mathrm{ctg}\, 210° \quad \sqrt{3}$;

г) $\sin 240° = -\frac{\sqrt{3}}{2}$, $\cos 240° = -\frac{1}{2}$, $\mathrm{tg}\, 240° = \sqrt{3}$, $\mathrm{ctg}\, 240° = \frac{\sqrt{3}}{3}$.

8.7. $\sin 160°$, $\sin 40°$, $\sin 120°$, $\sin 80°$

8.8. $\cos 160°$, $\cos 120°$, $\cos 80°$, $\cos 40°$

8.9. $\sin 210°$, $\sin 20°$, $\sin 400°$, $\sin 110°$

8.10. а) $\mathrm{tg}\,\alpha = \frac{x}{2}$, $x = 2\,\mathrm{tg}\,\alpha$; б) $\cos\alpha = \frac{x}{4}$, $x = 4\cos\alpha$;

в) $\cos\alpha = \frac{3}{x}$, $x = \frac{3}{\cos\alpha}$; г) $\mathrm{ctg}\,\alpha = x$.

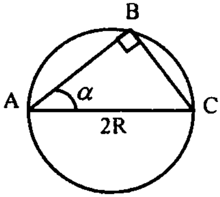
8.11. а) $\sin 30° = \frac{2}{x}$, $x = \frac{2}{\frac{1}{2}} = 4$; б) $x = \frac{\sqrt{2}}{2}$;

в) $x = \frac{2}{\sin 60°} = \frac{4}{\sqrt{3}}$; г) $\frac{x}{2} = \cos 60° = \frac{1}{2}$, $x=1$.

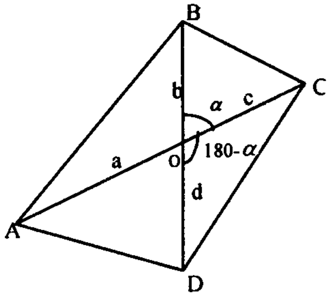
8.12. Во всех пунктах $a = c\cos\alpha$, $b = c\sin\alpha$, $S = \frac{1}{2}ab$ $R = \frac{c}{2}$

а) $c = 12$, $\alpha = 60°$, $a = 6$, $b = 6\sqrt{3}$, $S = 18\sqrt{3}$, $R = 6$;
б) $c = 6$, $\alpha = 45°$, $a = b = 3\cdot\sqrt{2}$, $S = 9$, $R = 3$;
в) $c = 4$, $\alpha = 30°$, $a = 2\sqrt{3}$, $b = 2$, $S = 2\cdot\sqrt{3}$, $R = 2$;
г) $c = 60$, $\alpha = 60°$, $a = 30$, $b = 30\sqrt{3}$, $S = 450\cdot\sqrt{3}$, $R = 30$.

8.13. $\cos\alpha = \frac{AB}{AC}$,
$AB = AC\cdot\cos\alpha = 2R\cos\alpha$.



8.14. $S_{ABCD} = \frac{1}{2}ab\sin\alpha + \frac{1}{2}bc\sin\alpha +$
$+\frac{1}{2}cd\sin\alpha + \frac{1}{2}da\sin\alpha =$
$= \frac{1}{2}\sin\alpha\,(ab + bc + cd + da) =$
$= \frac{1}{2}\sin\alpha\,(b(a+c) + d(c+a)) =$
$= \frac{1}{2}\sin\alpha\,(b+d)(a+c)$, ч.т.д.

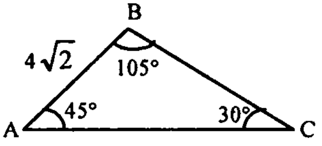


8.15. $\frac{AB}{\sin C} = \frac{BC}{\sin A}$, $\frac{4\sqrt{2}}{1/2} = \frac{BC}{\sqrt{2}/2} \Rightarrow BC = 8$,

$\angle B = 180° - 45° - 30° = 105°$,
$AC = 4\sqrt{2}\cdot\cos 45° + 8\cos 30° = 4 + 4\sqrt{3}$.

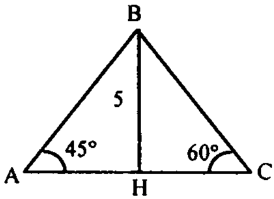
$S = \frac{1}{2}\cdot AC\cdot AB\cdot\sin\angle C$;

$S = \frac{1}{2}\cdot\frac{1}{2}(4\sqrt{3} + 4)\,8 = 8(\sqrt{3} + 1)$.



Ответ: $BC = 8$, $AC = 4\sqrt{3} + 4$, $S = 8(\sqrt{3} + 1)$ (см^2).

8.16. $AH = BH = 5$ (т.к. $\angle A = \angle ABH = 45^\circ$)



$\text{tg } 60^\circ = \dfrac{5}{HC}$, $\quad HC = \dfrac{5}{\text{tg}60^\circ} = \dfrac{5}{\sqrt{3}}$;

$S = \dfrac{1}{2} \cdot 5 \cdot \left(5 + \dfrac{5}{\sqrt{3}}\right) = \dfrac{25(3+\sqrt{3})}{6}$

Ответ: $\dfrac{25(3+\sqrt{3})}{6}$ (см^2).

§ 9. Формулы приведения

9.1. а) $\sin\left(\dfrac{\pi}{2} - t\right) = \cos t$; б) $\cos(2\pi - t) = \cos t$;

в) $\cos\left(\dfrac{3\pi}{2} + t\right) = -\sin t$; г) $\sin(\pi + t) = -\sin t$.

9.2. а) $\sin(\pi - t) = \sin t$; б) $\cos\left(\dfrac{\pi}{2} + t\right) = -\sin t$;

в) $\cos(2\pi + t) = \cos t$; г) $\sin\left(\dfrac{3\pi}{2} - t\right) = -\cos t$.

9.3. а) $\cos(90^\circ - \alpha) = \sin\alpha$; б) $\sin(360^\circ - \alpha) = -\sin\alpha$;
в) $\sin(270^\circ + \alpha) = -\cos\alpha$; г) $\cos(180^\circ + \alpha) = -\cos\alpha$.

9.4. а) $\text{tg}(90^\circ - \alpha) = \text{ctg}\,\alpha$; б) $\text{ctg}(180^\circ - \alpha) = -\text{ctg}\,\alpha$;
в) $\text{tg}(270^\circ + \alpha) = -\text{ctg}\,\alpha$; г) $\text{ctg}(360^\circ + \alpha) = \text{ctg}\,\alpha$.

9.5. а) $\sin 240^\circ = -\sin 60^\circ = -\dfrac{\sqrt{3}}{2}$; б) $\text{tg } 300^\circ = \text{tg } 120^\circ = -\text{tg } 60^\circ = -\sqrt{3}$;

в) $\cos 330^\circ = \cos(-30^\circ) = \dfrac{\sqrt{3}}{2}$; г) $\text{ctg } 315^\circ = -\text{ctg } 45^\circ = -1$.

9.6. а) $\cos\dfrac{5\pi}{3} = \dfrac{1}{2}$; б) $\sin\left(-\dfrac{11\pi}{6}\right) = \dfrac{1}{2}$;

в) $\sin\dfrac{7\pi}{6} = -\dfrac{1}{2}$; г) $\cos\left(-\dfrac{7\pi}{3}\right) = \dfrac{1}{2}$.

9.7. а) $\cos 630^\circ - \sin 1470^\circ - \text{ctg } 1125^\circ = -\dfrac{1}{2} - 1 = -\dfrac{3}{2}$;

б) $\sin(-7\pi) + 2\cos\dfrac{31\pi}{3} - \text{tg}\dfrac{7\pi}{4} = 1 + 1 = 2$;

в) $\text{tg } 1800^\circ - \sin 495^\circ + \cos 945^\circ = -\dfrac{\sqrt{2}}{2} - \dfrac{\sqrt{2}}{2} = -\sqrt{2}$;

г) $\cos(-9\pi) + 2\sin\left(-\dfrac{49\pi}{6}\right) - \text{ctg}\left(-\dfrac{21\pi}{4}\right) = -1 - 1 + 1 = -1$.

9.8. а) $\sin(90^\circ-\alpha)+\cos(180^\circ+\alpha)+\operatorname{tg}(270^\circ+\alpha)+\operatorname{ctg}(360^\circ+\alpha)=$
$=\cos\alpha-\cos\alpha-\operatorname{ctg}\alpha+\operatorname{ctg}\alpha=0;$

б) $\sin(\frac{\pi}{2}+t)-\cos(\pi-t)+\operatorname{tg}(\pi-t)+\operatorname{ctg}(\frac{5\pi}{2}-t)=$
$=\cos t+\cos t-\operatorname{tg} t+\operatorname{tg} t=2\cos t.$

9.9. а) $\dfrac{\cos(180+\alpha)\cdot\cos(-\alpha)}{\sin(-\alpha)\cdot\sin(90+\alpha)}=\dfrac{-\cos\alpha\cos\alpha}{-\sin\alpha\cos\alpha}=\operatorname{ctg}\alpha;$

б) $\dfrac{\sin(\pi-t)\cos(2\pi-t)}{\operatorname{tg}(\pi-t)\cos(\pi-t)}=\dfrac{\sin t\cos t}{-\operatorname{tg} t\cdot(-\cos t)}=\cos t;$

в) $\dfrac{\sin(-\alpha)\operatorname{ctg}(-\alpha)}{\cos(360-\alpha)\operatorname{tg}(180+\alpha)}=\dfrac{\cos\alpha}{\cos\alpha\ \operatorname{tg}\alpha}=\operatorname{ctg}\alpha;$

г) $\dfrac{\sin(\pi+t)\ \sin(2\pi+t)}{\operatorname{tg}(\pi+t)\ \cos(\frac{3\pi}{2}+t)}=\dfrac{-\sin t\ \sin t}{\frac{\sin t}{\cos t}\cdot\sin t}=-\cos t.$

9.10. а) $\dfrac{\cos(\pi-t)+\cos(\frac{\pi}{2}-t)}{\sin(2\pi-t)-\sin(\frac{3\pi}{2}-t)}=\dfrac{-\cos t+\sin t}{-\sin t+\cos t}=-1;$

б) $\dfrac{\sin^2(\pi-t)+\sin^2\left(\frac{\pi}{2}-t\right)}{\sin(\pi-t)}\operatorname{tg}(\pi-t)=\dfrac{\sin^2 t+\cos^2 t}{\sin t}\cdot\left(-\dfrac{\sin t}{\cos t}\right)=-\dfrac{1}{\cos t};$

9.11. а) $\dfrac{\operatorname{tg}(\pi-t)}{\cos(\pi+t)}\cdot\dfrac{\sin(\frac{3\pi}{2}+t)}{\operatorname{tg}(\frac{3\pi}{2}+t)}=\operatorname{tg}^2 t,\ \dfrac{-\operatorname{tg} t}{-\cos t}\cos(\frac{3\pi}{2}+t)=\operatorname{tg}^2 t;$

б) $\dfrac{\sin(\pi-t)}{\operatorname{tg}(\pi+t)}\cdot\dfrac{\operatorname{ctg}(\frac{\pi}{2}-t)}{\operatorname{tg}(\frac{\pi}{2}+t)}\cdot\dfrac{\cos(2\pi-t)}{\sin(-t)}=\sin t.$

$\dfrac{\sin t}{\operatorname{tg} t}\cdot\dfrac{\operatorname{tg} t}{-\operatorname{ctg} t}\cdot\dfrac{\cos t}{-\sin t}=\operatorname{tg} t\ \cos t=\sin t.$

9.12. а) $2\cos(2\pi+t)+\sin(\frac{\pi}{2}+t)=3,$

$2\cos t+\cos t=3,\ \cos t=1,\ t=2\pi n;$

б) $\sin(\pi+t)+2\cos(\frac{\pi}{2}+t)=3,$

$-\sin t-2\sin t=3,\ \sin t=-1,\ t=-\frac{\pi}{2}+2\pi n;$

в) $2\sin(\pi+t)+\cos(\frac{\pi}{2}-t)=-\frac{1}{2}$, $-2\sin t+\sin t=-\frac{1}{2}$, $\sin t=\frac{1}{2}$, $t=(-1)^k\ \frac{\pi}{6}+\pi k$;

г) $3\sin(\frac{\pi}{2}+t)-\cos(2\pi+t)=1$, $3\cos t-\cos t=1$, $\cos t=\frac{1}{2}$, $t=\pm\frac{\pi}{3}+2\pi n$.

9.13. а) $5\sin(\frac{\pi}{2}+t)-\sin(\frac{3\pi}{2}+t)-8\cos(2\pi-t)=1$, $5\cos t+\cos t-8\cos t=$

$=1$, $\cos t=-\frac{1}{2}$, $t=\pm\frac{2\pi}{3}+2\pi n$;

б) $\sin(2\pi+t)-\cos(\frac{\pi}{2}-t)+\sin(\pi-t)=1$, $\sin t-\sin t+\sin t=1$, $\sin t=1$,

$t=\frac{\pi}{2}+2\pi n$.

9.14. а) $\sin^2(\pi+t)+\cos^2(2\pi-t)=0$, $\sin^2 t+\cos^2 t=0$ корней нет;

б) $\sin^2(\pi+t)+\cos^2(2\pi-t)=1$, $\sin^2 t+\cos^2(2\pi-t)=1$, $\sin^2 t+\cos^2 t=1$, $t\in R$.

§ 10. *Функция* $y=\sin x$*, её свойства и график*

10.1. а) $\sin\pi=0$; б) $\sin(-\frac{\pi}{2})=-1$;

в) $\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}$; г) $\sin(-\frac{\pi}{3})=-\frac{\sqrt{3}}{2}$.

10.2. а) $f(x)=\sin x$, $f(-x)=-\sin x$; б) $f(x)=\sin x$, $f(2x)=\sin 2x$;
в) $f(x)=\sin x$, $f(x+1)=\sin(x+1)$; г) $f(x)=\sin x$, $f(x)-5=\sin x-5$.

10.3. а) $y=2\sin(x-\frac{\pi}{6})+1$, $x=\frac{4\pi}{3}$, $y=2\sin\frac{7\pi}{6}+1=-1+1=0$;

б) $y=-\sin(x+\frac{\pi}{4})$, $x=-\frac{\pi}{2}$, $y=-\sin(-\frac{\pi}{4})=\frac{\sqrt{2}}{2}$;

в) $y=2\sin(x-\frac{\pi}{6})+1$, $x=\frac{7\pi}{6}$, $y=2\sin\pi+1=1$;

г) $y=-\sin(x+\frac{\pi}{4})$, $x=-\frac{15\pi}{4}$, $y=-\sin(-\frac{14\pi}{4})=\sin\frac{7\pi}{2}=-1$.

10.4. а) $y=\sin x$, $\sin(-\frac{\pi}{2})=-1$, $(-\frac{\pi}{2};-1)$ принадлежит;

б) $y=\sin x$, $\frac{1}{2}\neq\sin\frac{\pi}{2}$, $(\frac{\pi}{2};\frac{1}{2})$ не принадлежит;

в) $y=\sin x$, $1\neq\sin\pi$, $(\pi;1)$ не принадлежит.

г) $y=\sin x$, $-1=\sin\frac{3\pi}{2}$, $(\frac{3\pi}{2};-1)$ принадлежит.

10.5. а) $y = \sin\left(x + \frac{\pi}{6}\right) + 2 = -\sin\frac{\pi}{6} + 2 = \frac{3}{2}$, $\left(0; \frac{3}{2}\right)$ принадлежит;

б) $y = -\sin\left(x + \frac{\pi}{6}\right) + 2 = -\sin\frac{\pi}{3} + 2 = -\frac{\sqrt{3}}{2} + 2$,

$\left(\frac{\pi}{6}; -\frac{\sqrt{3}}{2} + 2\right)$ принадлежит;

в) $y = -\sin\left(x + \frac{\pi}{6}\right) + 2$, $\frac{3}{2} = -\sin\frac{5\pi}{6} + 2 = -\frac{1}{2} + 2$,

$\left(\frac{2\pi}{3}; \frac{3}{2}\right)$ принадлежит;

г) $y = -\sin\left(x + \frac{\pi}{6}\right) + 2$, $-\sin\left(4\pi + \frac{\pi}{6}\right) + 2 = -\frac{1}{2} + 2 \neq 2{,}5$,

$(4\pi; 2{,}5)$ не принадлежит.

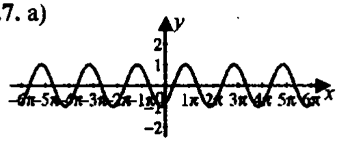
10.6. а) $y = \sin x$, $x \in \left[\frac{\pi}{4}; \frac{2\pi}{3}\right]$, $f_{max} = 1$, $f_{min} = \frac{\sqrt{2}}{2}$;

б) $y = \sin x$, $x \in \left[\frac{\pi}{4}; +\infty\right]$, $f_{max} = 1$, $f_{min} = -1$;

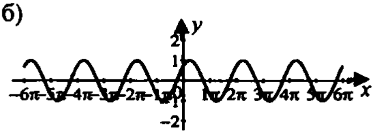
в) $y = \sin x$, $x \in \left(-\frac{3\pi}{2}; \frac{3\pi}{4}\right)$, $f_{max} = 1$, $f_{min} = -1$;

г) $y = \sin x$, $x \in \left(-\pi; \frac{\pi}{3}\right]$, $f_{max} = \frac{\sqrt{3}}{2}$, $f_{min} = -1$

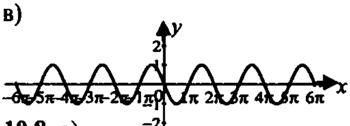
10.7. а)



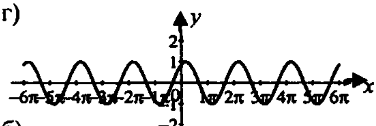
б)



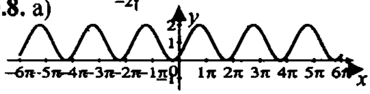
в)



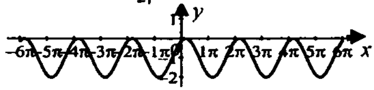
г)



10.8. а)

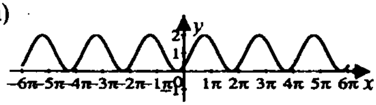


б)

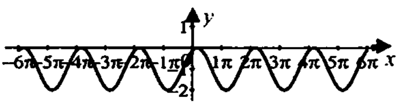


10.9.

а)

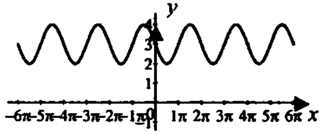


б)

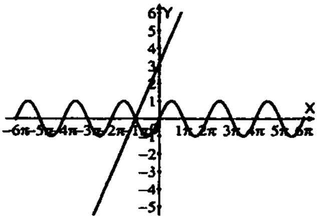


10.10. а)

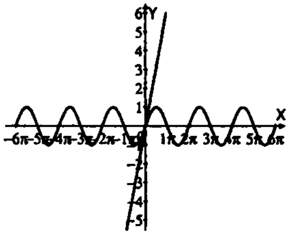
б)



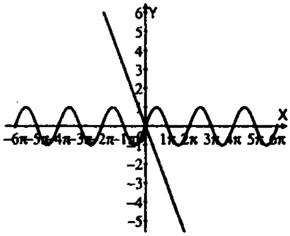
10.11. а) $\sin x = x + \pi$, $x = -\pi$;



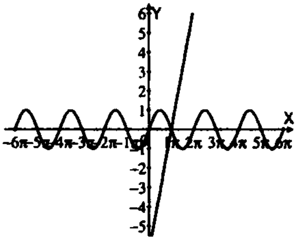
б) $\sin x = 2x$, $x = 0$;



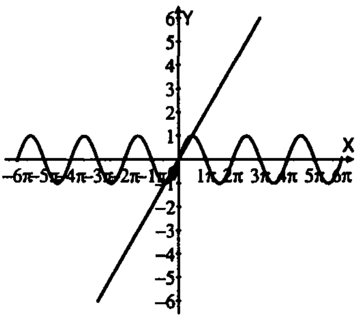
в) $\sin x = -x$, $x = 0$;



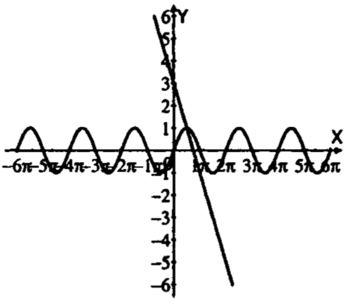
г) $\sin x = 2x - 2\pi$, $x = \pi$



10.12. а) $\sin x = \frac{2}{\pi}x$, $x = 0$, $x = \pm\frac{\pi}{2}$;

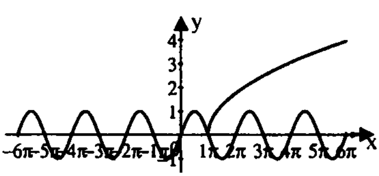


б) $\sin x = -\frac{4}{\pi}x + 3$, $x = \frac{\pi}{2}$.

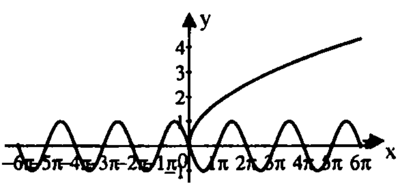


10.13.

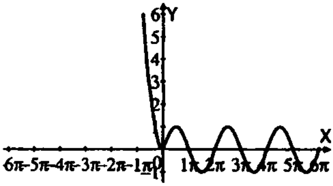
а) $\sin x = \sqrt{x - \pi}$, $x = \pi$.



б) $-\sin x = \sqrt{x}$, $x = 0$.



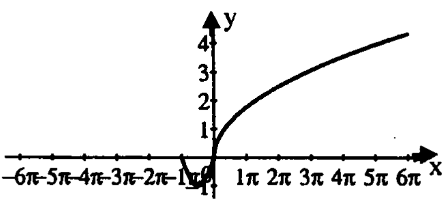
10.14. а) $f(x) = x + \sin x$,
$f(-x) = (-x) + \sin(-x) = -(x + \sin x) = -f(x)$;
б) $f(x) = x^3 \sin x^2$,
$f(-x) = -x^3 \cdot \sin(-x)^2 = -x^3 \sin x^2 = -f(x)$;
в) $f(x) = \dfrac{x^2 \sin x}{x^2 - 9}$, $f(-x) = -\dfrac{x^2 \sin x}{x^2 - 9} = -f(x)$;
г) $f(x) = x^3 - \sin x$, $f(-x) = -x^3 + \sin x = -f(x)$.
10.15. $f(x) = 2x^2 - x + 1$.
$f(\sin x) = 2\sin^2 x - \sin x + 1 = 2 - 2\cos^2 x - \sin x + 1 = 3 - 2\cos^2 x - \sin x$.
10.16.



10.17. $f(x) = \begin{cases} \sin x, & -\pi \le x \le 0 \\ \sqrt{x}, & x > 0 \end{cases}$

а) $f(-\dfrac{\pi}{2}) = \sin(-\dfrac{\pi}{2}) = -1$, $f(0) = 0$, $f(1) = 1$, $f(\pi^2) = \pi$

б)

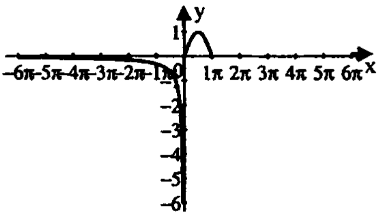


в)
1) $D(f) = [-\pi; +\infty)$;
2) $E(f) = [-1; +\infty)$;
3) непериодичная;
4) ни четная, ни нечетная;
5) $f(x) = 0$ при $x = 0$, $x = -\pi$;
6) $f(x) > 0$ при $x > 0$, $f(x) < 0$ при $x \in (-\pi; 0)$;
7) $f_{min} = -1$, $f_{max} = +\infty$;
8) убывает при $x \in \left[-\pi; -\dfrac{\pi}{2}\right]$, возрастает при $x \ge -\dfrac{\pi}{2}$.

10.18. $f(x) = \begin{cases} \dfrac{1}{x}, & x < 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$

а) $f(-2) = -\dfrac{1}{2}$, $f(0) = 0$, $f(1) = \sin 1$;

б)



в)

1) $D(f) = (-\infty; \pi]$; 2) $E(f) = (-\infty; +1]$;

3) непериодичная;

4) ни четная, ни нечетная;

5) $f(x) = 0$ при $x = 0$, $x = \pi$;

6) $f(x) < 0$ при $x < 0$, $f(x) > 0$ при $x \in (0; \pi)$;

7) $f_{\min} = -\infty$, $f_{\max} = 1$;

8) убывает при $x < 0$, $x \in \left[\frac{\pi}{2}; \pi\right]$, возрастает при $x \in \left[0; \frac{\pi}{2}\right]$.

§ 11. *Функция* $y = \cos x$, *ее свойства и график*

11.1. а) $\cos\frac{\pi}{2} = 0$; б) $\cos(-\pi) = -1$;

в) $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$; г) $\cos\left(-\frac{2\pi}{3}\right) = -\frac{1}{2}$.

11.2. а) $f(x) = \cos x$, $f(-x) = \cos x$; б) $f(x) = \cos x$, $f(3x) = \cos 3x$;

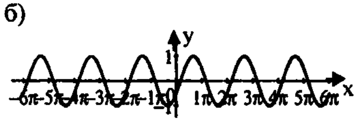
в) $f(x) = \cos x$, $f(x+2) = \cos(x+2)$; г) $f(x) = \cos x$, $f(x) - 6 = \cos x - 6$.

11.3. а) $y = 2\sin\left(-\frac{\pi}{2}\right) + \cos\left(-\frac{\pi}{2}\right) = -2$; б) $y = 2\sin\frac{\pi}{6} + \cos\frac{\pi}{6} = 1 + \frac{\sqrt{3}}{2}$.

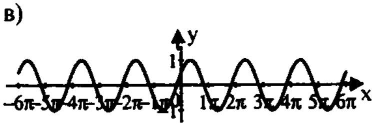
11.4. а) $y = 2\cos\left(-\frac{\pi}{2} - \frac{\pi}{4}\right) - 1 = -\sqrt{2} - 1$; б) $y = 2\cos\left(\frac{\pi}{4} - \frac{\pi}{4}\right) - 1 = 1$.

11.5. а)

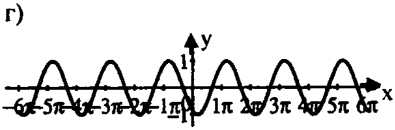
б)



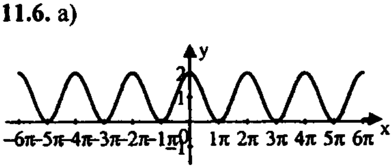
в)



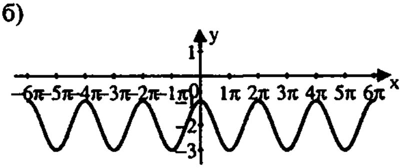
г)



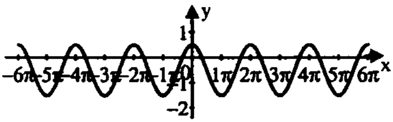
11.6. а)



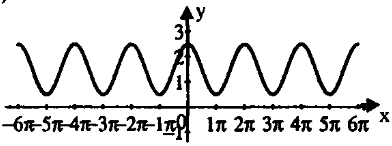
б)



в)



г)



11.7. $y = \cos x$.

а) $x \in [\frac{\pi}{6}; \frac{2\pi}{3}]$, $y_{min} = -\frac{1}{2}$, $y_{max} = \frac{\sqrt{3}}{2}$.

б) $x \in (-\pi; \frac{\pi}{4})$ y_{min} = не существует, $y_{max} = 1$.

в) $x \in [-\frac{\pi}{3}; +\infty)$, $y_{min} = -1$, $y_{max} = 1$.

г) $x \in [-\frac{\pi}{3}; \frac{3\pi}{2})$, $y_{min} = -1$, $y_{max} = 1$.

11.8. а) $f(x) = \begin{cases} x+2, & x<0 \\ \cos x, & x \geq 0 \end{cases}$

1) $D(f) = R$;

2) $E(f) = (-\infty; 2)$;

3) при $x \geq 0$ $T = 2\pi$;

4) ни четная, ни нечетная;

5) $f(x) = 0$ при $x = -2$, $x = \frac{\pi}{2} + \pi n$, $n \geq 0$;

6) $f(x) < 0$ при $x \in (-\infty; -2) \cup \left(\frac{\pi}{2} + 2\pi n; \frac{3}{2}\pi + 2\pi n\right)$, $n \geq 0$, $f(x) > 0$ при

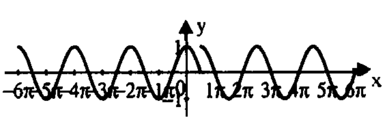
$x \in \left(-2; \frac{\pi}{2}\right) \cup \left(-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n\right)$, $n \geq 1$;

7) $f_{min} = -\infty$, f_{max} = не сущ.;

8) $f(x)$ возрастает при $x \in (-\infty; 0) \cup [-\pi + 2\pi n; 2\pi n]$, $n \geq 1$,

убывает при $x \in [2\pi n; 2\pi n + \pi]$, $n \geq 0$.

б) $f(x) = \begin{cases} \cos x, & x \leq \frac{\pi}{2} \\ \sin x, & x > \frac{\pi}{2} \end{cases}$



1) $D(f) = R$;

2) $E(f) = [-1; 1]$;

3) на промежутках $\left(-\infty; \frac{\pi}{2}\right]$ и $\left(\frac{\pi}{2}; +\infty\right)$ $T = 2\pi$;

4) ни четная, ни нечетная;

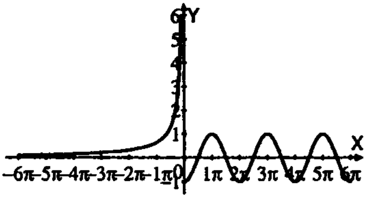
5) $f(x) = 0$ при $x = \dfrac{\pi}{2} - \pi n$, $n \geq 0$, $x = \pi(1+k)$, $k \geq 0$;

6) $f(x)>0$ при $x \in \left(-\dfrac{\pi}{2} + 2\pi n; \dfrac{\pi}{2} + 2\pi n\right)$, $n \leq 0$, $x \in \left(\dfrac{\pi}{2}, \pi\right)$, $x \in (2\pi n; 2\pi n+\pi)$, $n \geq 1$;

7) $f_{\min} = -1$, $f_{\max} = 1$;

8) $f(x)$ возрастает при $x \in [-\pi+2\pi n; 2\pi n]$, $n \leq 0$, $x \in \left[-\dfrac{\pi}{2} + 2\pi k; \dfrac{\pi}{2} + 2\pi k\right]$, $k \geq 1$, убывает при $x \in [2\pi n; \pi+2\pi n)$, $n \leq -1$, $x \in \left[0; \dfrac{\pi}{2}\right]$;

$x \in \left[\dfrac{\pi}{2} + 2\pi k; \dfrac{3}{2}\pi + 2\pi k\right]$, $k \geq 0$;



в) $f(x) = \begin{cases} -\dfrac{2}{x}, x < 0 \\ -\cos x, x \geq 0 \end{cases}$

1) $D(f) = R$;
2) $E(f) = [-1; +\infty)$;
3) при $x \geq 0$ $T = 2\pi$;

4) ни четная, ни нечетная;

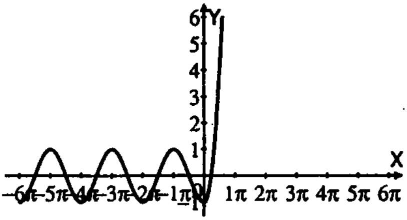
5) $f(x) = 0$ при $x = \dfrac{\pi}{2} + \pi n$, $n \geq 0$;

6) $f(x) < 0$ при $x \in \left[0; \dfrac{\pi}{2}\right) \cup \left(-\dfrac{\pi}{2} + 2\pi n; \dfrac{\pi}{2} + 2\pi n\right)$, $n \geq 1$,

$f(x) > 0$ при $x \in (-\infty; 0) \cup \left(\dfrac{\pi}{2} + 2\pi n; \dfrac{3}{2}\pi + 2\pi n\right)$, $n \geq 0$;

7) $f_{\min} = -1$, $f_{\max} = +\infty$;
8) $f(x)$ возрастает при $x \in (-\infty; 0) \cup [2\pi n; \pi+2\pi n)$, $n \geq 0$, убывает при $x \in [2\pi k-\pi; 2\pi k]$, $k \geq 1$.

г) $f(x) = \begin{cases} -\cos x, x < 0 \\ 2x^2 - 1, x \geq 0 \end{cases}$



1) $D(f) = R$; 2) $E(f) = [-1; +\infty)$;
3) при $x < 0$ $T = 2\pi$;

4) ни четная, ни нечетная;

5) $f(x) = 0$ при $x = \dfrac{\pi}{2} - \pi n$, $n \geq 1$, $x = \dfrac{\sqrt{2}}{2}$;

6) $f(x) < 0$ при $x \in \left(-\frac{\pi}{2} - 2\pi n; \frac{\pi}{2} - 2\pi n\right) \cup \left(-\frac{\pi}{2}; \frac{\sqrt{2}}{2}\right)$, $n \geq 1$,

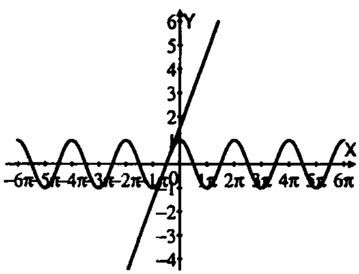
$f(x) > 0$ при $x \in \left(\frac{\pi}{2} - 2\pi k; \frac{3}{2}\pi - 2\pi k\right) \cup \left(\frac{\sqrt{2}}{2}; +\infty\right)$, $k \geq 1$;

7) $f_{\min} = -1$, $f_{\max} = +\infty$;

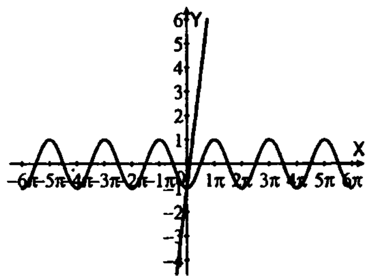
8) $f(x)$ возрастает при $x \in [-2\pi n; -2\pi n + \pi] \cup [0; +\infty)$, $n \geq 1$,
убывает при $x \in [-2\pi n - \pi; -2\pi n]$, $n \geq 0$.

11.9.

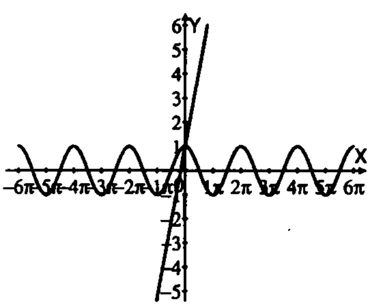
а) $\cos x = x + \frac{\pi}{2}$; $x = -\frac{\pi}{2}$.



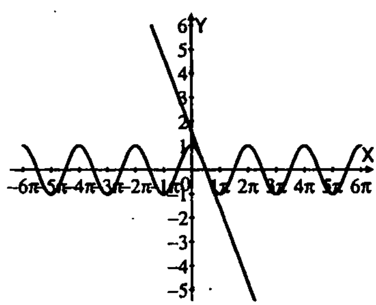
б) $-\cos x = 3x - 1$; $x = 0$.



в) $\cos x = 2x + 1$; $x = 0$.

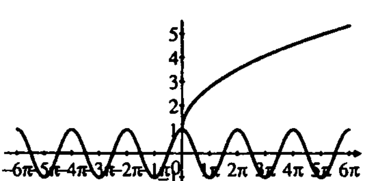


г) $\cos x = -x + \frac{\pi}{2}$; $x = \frac{\pi}{2}$.

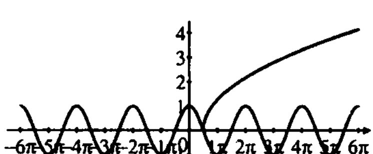


11.10.

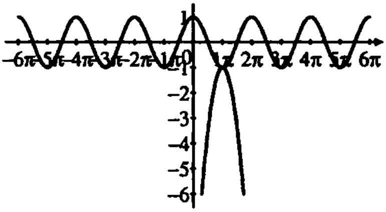
а) $\cos x = \sqrt{x} + 1$, $x = 0$;



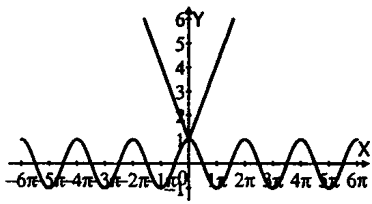
б) $\cos x = \sqrt{x - \frac{\pi}{2}}$, $x = \frac{\pi}{2}$;



в) $\cos x + 1 = -(x-\pi)^2$; $x=\pi$



г) $\cos x = |x| + 1$, $x = 0$.



11.11. а) $f(x) = x^2\cos x$, $f(-x) = (-x)^2\cos(-x) = x^2\cos x = f(x)$;

б) $f(x) = \dfrac{\cos x^3}{4-x^2}$, $f(-x) = \dfrac{\cos(-x)^3}{4-(-x)^2} = \dfrac{\cos x^3}{4-x^2} = f(x)$;

в) $f(x) = \dfrac{\cos 5x + 1}{|x|}$, $f(-x) = \dfrac{\cos(-5x)+1}{|-x|} = \dfrac{\cos 5x + 1}{|x|} = f(x)$;

г) $f(x) = (4+\cos x)(\sin^6 x - 1)$,
$f(-x) = (4 + \cos(-x))(\sin^6(-x) - 1) = (4+\cos x)(\sin^6 x - 1) = f(x)$.

11.12. а) $f(x) = \sin x\cos x$, $f(-x) = \sin(-x)\cos(-x) = -\sin x\cos x = -f(x)$;

б) $f(x) = x^5\cos 3x$, $f(-x) = (-x)^5\cos(-3x) = -x^5\cos 3x = -f(x)$;

в) $f(x) = \dfrac{\cos x^3}{x(25-x^2)}$, $f(-x) = \dfrac{\cos(-x)^3}{-x(25-(-x)^2)} = -\dfrac{\cos x^3}{x(25-x^2)} = -f(x)$;

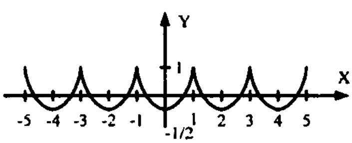
г) $f(x) = x^{11}\cdot\cos x + \sin x$,
$f(-x) = (-x)^{11}\cdot\cos(-x) + \sin(-x) = (-x)^{11}\cdot\cos x - \sin x = f(-x)$.

11.13. а) $f(x) = 2x^2 - 3x - 2$,
$-f(\cos x) = -2\cos^2 x + 3\cos x + 2 = 2(1-\cos^2 x) + 3\cos x = 2\sin^2 x + 3\cos x$

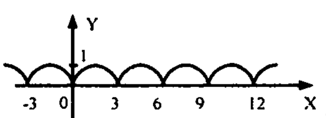
б) $f(x) = 5x^2 + x + 4$,
$f(\cos x) = 5\cos^2 x + \cos x + 4 = 5 - 5\sin^2 x + \cos x + 4 = -5\sin^2 x + \cos x + 9$.

§ 12. Периодичность функций $y = \sin x$, $y = \cos x$

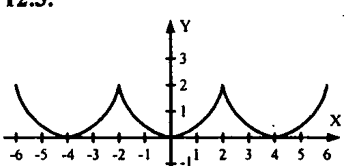
12.1.



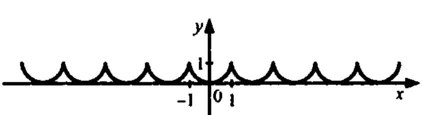
12.2.



12.3.



12.4.



12.5. 32π является периодом функций $y = \sin x$, $y = \cos x$, но не основным.

12.6. а) $\sin 50,5\pi = \sin\frac{\pi}{2} = 1$; б) $\cos 51,75\pi = \cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$;

в) $\sin 25,25\pi = \sin\frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$;

г) $\sin 29,5\pi = \sin\left(30\pi - \frac{\pi}{2}\right) = -\sin\frac{\pi}{2} = -1$

12.7. а) $\sin 390° = \sin 30° = \frac{1}{2}$; б) $\cos 750° = \cos 30° = \frac{\sqrt{3}}{2}$;

в) $\sin 540° = \sin 180° = 0$; г) $\cos 930° = \cos 210° = -\frac{\sqrt{3}}{2}$.

12.8. а) $\sin^2(x - 8\pi) = 1 - \cos^2(16\pi - x)$, $\sin^2(x - 8\pi) = \sin^2 x$;
$1 - \cos^2(16\pi - x) = 1 - \cos^2 x = \sin^2 x$;
б) $\cos^2(4\pi + x) = 1 - \sin^2 x\,(22\pi - x)$; $\cos^2(4\pi + x) = \cos^2 x$,
$1 - \sin^2 x(22\pi - x) = 1 - \sin^2 x = \cos^2 x$.

12.9. а) $\sin(t + 2\pi) + \sin(t - 4\pi) = 1$, $\sin t + \sin t = 1$,

$\sin t = \frac{1}{2}$; $t = (-1)^k\frac{\pi}{6} + \pi k$;

б) $3\cos(2\pi + t) + \cos(t - 2\pi) + 2 = 0$,

$4\cos t = -2$, $\cos t = -\frac{1}{2}$, $t = \pm\frac{2\pi}{3} + 2\pi n$;

в) $\sin(t + 4\pi) + \sin(t - 6\pi) = \sqrt{3}$,

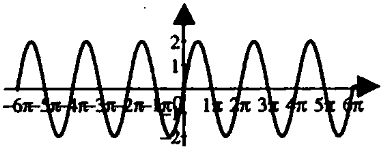
$2\sin t = \sqrt{3}$, $\sin t = \frac{\sqrt{3}}{2}$, $t = (-1)^k\frac{\pi}{3} + \pi k$;

г) $\cos(t + 2\pi) + \cos(t - 8\pi) = \sqrt{2}$,

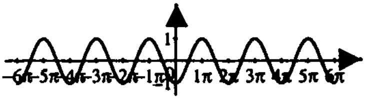
$2\cos t = \sqrt{2}$, $\cos t = \frac{\sqrt{2}}{2}$, $t = \pm\frac{\pi}{4} + 2\pi k$.

§ 13. Преобразования графиков тригонометрических функций

13.1. а)

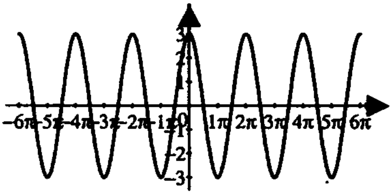


б)

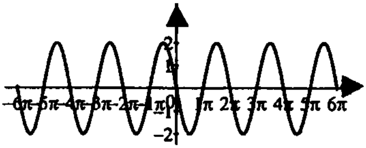


в)

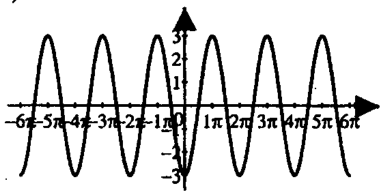
г)



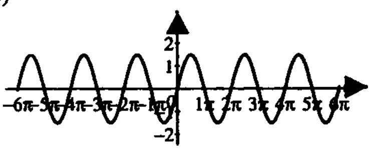
13.2. а)



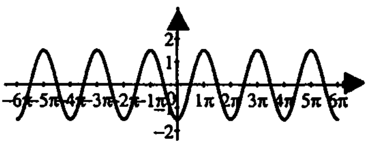
б)



в)



г)



13.3. $y = 2\cos x$

а) $x \in [-\frac{\pi}{2}; \frac{\pi}{2}]$, $y_{\max} = 2$, $y_{\min} = 0$;

б) $x \in (0; \frac{3\pi}{2})$, $y_{\min} = -2$, $y_{\max}$ не существует.

в) $x \in \left[\frac{\pi}{3}; \frac{3\pi}{2}\right)$, $y_{\max} = 1$, $y_{\min} = -2$;

г) $x \in [-\frac{3\pi}{2}; -\frac{\pi}{4}]$, $y_{\max} = \sqrt{2}$, $y_{\min} = -2$.

13.4. . $y = -3\sin x$.;

а) $x \in [0; +\infty)$, $y_{\max} = 3$, $y_{\min} = -3$;

б) $x \in (-\infty; \frac{\pi}{2})$, $y_{\max} = 3$, $y_{\min} = -3$;

в) $x \in [\frac{\pi}{4}; +\infty)$, $y_{\max} = 3$, $y_{\min} = -3$;

г) $x \in (-\infty; 0)$, $y_{\max} = 3$, $y_{\min} = -3$.

13.5. $f(x) = 3\sin x$;

а) $f(-x) = -3\sin x$;

б) $2f(x) = 6\sin x$;

в) $2f(x) + 1 = 6\sin x + 1$;

г) $f(-x) + f(x) = -3\sin x + 3\sin x = 0$.

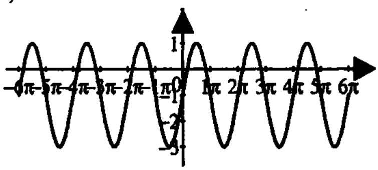
13.6. $f(x) = -\frac{1}{2}\cos x;$

а) $f(-x) = -\frac{1}{2}\cos x;$ б) $2f(x) = -\cos x;$

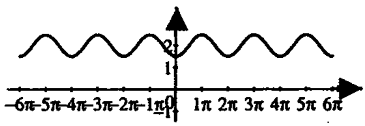
в) $f(x+2\pi) = -\frac{1}{2}\cos x;$ г) $f(-x) - f(x) = -\frac{1}{2}\cos x + \frac{1}{2}\cos x = 0.$

13.7.

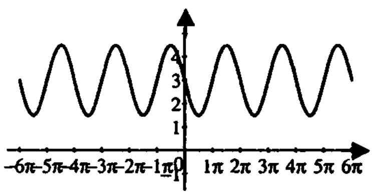
а)



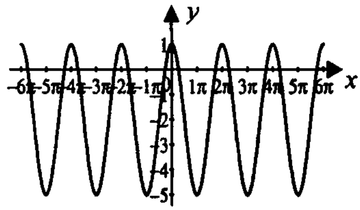
б)



в)

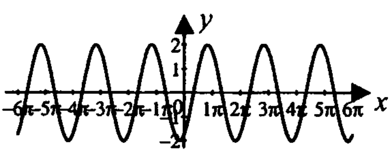


г)

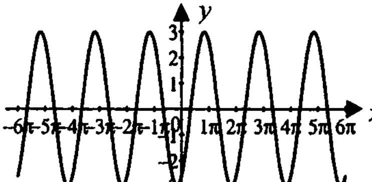


13.8.

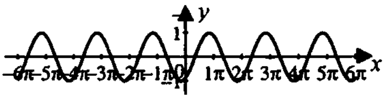
а)



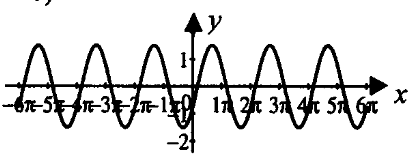
б)



в)



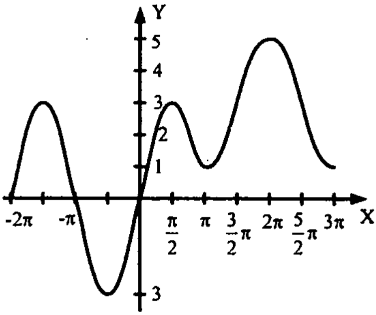
г)



13.9. а) $\begin{cases} x^2, & x < 0 \\ \frac{1}{2}\sin x, & 0 \le x \le \pi \end{cases}$

б) $\begin{cases} 1{,}5\cos x, & x \in [-\frac{\pi}{2}; \frac{\pi}{2}] \\ x - \frac{\pi}{2}, & x > \frac{\pi}{2} \end{cases}$

13.10. а) $f(x)=\begin{cases}3\sin x,\ x<\dfrac{\pi}{2}\\ 2\cos x+3,\ x\geq\dfrac{\pi}{2}\end{cases}$



1) $D(f) = R$;
2) $E(f) = [-3; 5]$;
3) при $x<\dfrac{\pi}{2}$ $T=2\pi$,

при $x\geq\dfrac{\pi}{2}$ $T=2\pi$;

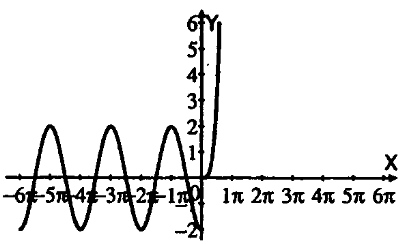
4) ни четная, ни нечетная;
5) $f(x) = 0$ при $x = -\pi n$, $n \geq 0$
6) $f_{\min} = -3$ $f_{\max} = 5$
7) $f(x) < 0$ при $-\pi+2\pi n < x < 2\pi n$, $n \leq 0$
$f(x) > 0$ при $2\pi n < x < \pi + 2\pi n$, $n \leq -1$ и $x > 0$;
8) Убывает на промежутках

$\left[\dfrac{\pi}{2}+2\pi n, \dfrac{3\pi}{2}+2\pi n\right]$, $\left[\dfrac{\pi}{2};\pi\right]$, $[2\pi k; \pi+2\pi k]$ $n\leq -1$, $k\geq 1$.

Возрастает на промежутках

$\left[-\dfrac{\pi}{2}+2\pi n, \dfrac{\pi}{2}+2\pi n\right]$ $n \leq 0$, $[-\pi+2\pi k, 2\pi k]$ $k \geq 1$.

б) $f(x)=\begin{cases}-2\cos x,\ \ x<0\\ \dfrac{1}{2}x^4,\ \ x\geq 0\end{cases}$



1) D(f) = R;
2) E(f) = [–2; +∞];
3) при $x < 0$ T = 2 π;
4) ни четная, ни нечетная;
5) $f(x) = 0$ при $x = -\dfrac{\pi}{2} - \pi n$, $n \geq 0$,

$x = 0$;
6) $f_{\min} = -2$, $f_{\max} = +\infty$;

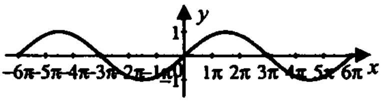
7) $f(x) < 0$ при $x \in \left(-\dfrac{\pi}{2} - 2\pi n; \dfrac{\pi}{2} - 2\pi n\right) \cup \left(-\dfrac{\pi}{2};0\right)$, $n \geq 1$,

$f(x) > 0$ при $x \in \left(\dfrac{\pi}{2} - 2\pi n; \dfrac{3}{2}\pi - 2\pi n\right) \cup (0; +\infty)$, $n \geq 1$;

8) $f(x)$ возрастает при $x \in [-2\pi n; -2\pi n + \pi]$, $n \geq 1$, $x \geq 0$,
$f(x)$ убывает при $x \in [-2\pi n - \pi; -2\pi n]$, $n \geq 0$.

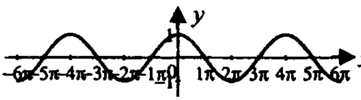
13.11.

а)



б)

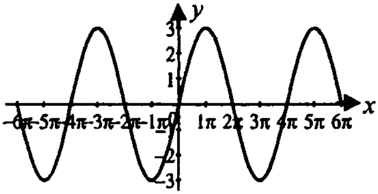
в)



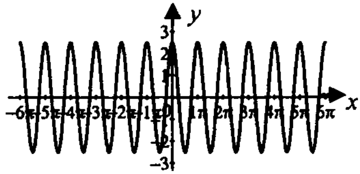
г)

13.12.

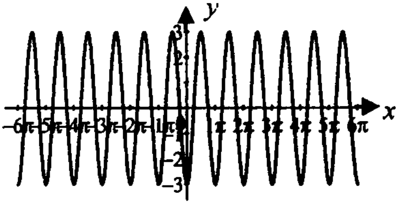
а)



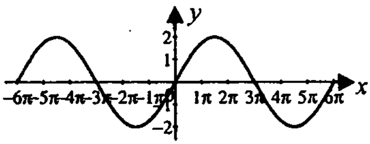
б)



в)

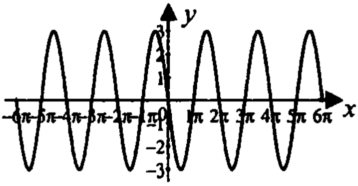


г)

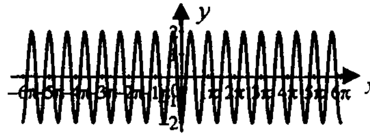


13.13.

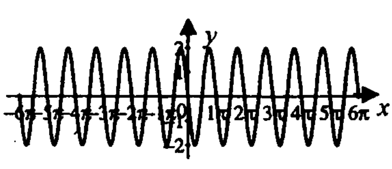
а)



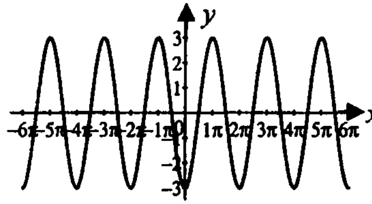
б)



в)



г)



13.14. $y = \sin 2x$

а) $x \in [-\frac{\pi}{2}; 0]$, $y_{max} = 0$, $y_{min} = -1$; б) $x \in (-\frac{\pi}{4}; \frac{\pi}{2})$, $y_{max} = 1$;

в) $x \in [-\frac{\pi}{4}; \frac{\pi}{4}]$, $y_{min} = -1$, $y_{max} = 1$; г) $x \in (0; \pi]$, $y_{min} = -1$, $y_{max} = 1$.

13.15. $y = \cos \frac{x}{3}$

а) $x \in [0; +\infty)$, $y_{max} = 1$, $y_{min} = -1$;
б) $x \in (-\infty; \pi)$, $y_{max} = 1$, $y_{min} = -1$;

в) $x \in [-\infty; \frac{\pi}{2}]$, $y_{max} = 1$, $y_{min} = -1$;

г) $x \in (\frac{\pi}{3}, +\infty)$, $y_{max} = 1$, $y_{min} = -1$.

13.16. $f(x) = \cos \frac{x}{3}$

а) $f(-x) = \cos \frac{x}{3}$; б) $3f(x) = 3\cos \frac{x}{3}$;

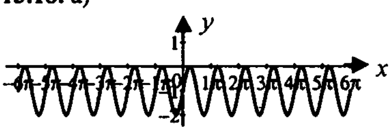
в) $f(-3x) = \cos x$; г) $f(-x) - f(x) = 0$.

13.17. $f(x) = \sin 2x$

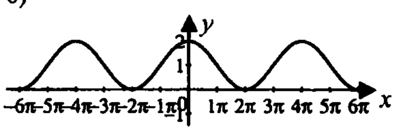
а) $f(-x) = -\sin 2x$; б) $2f(x) = 2\sin 2x$;

в) $f(-3x) = -\sin 6x$; г) $f(-x) + f(x) = 0$.

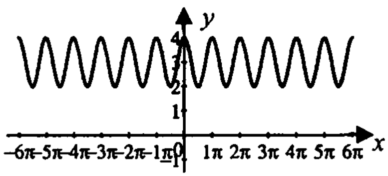
13.18. а)



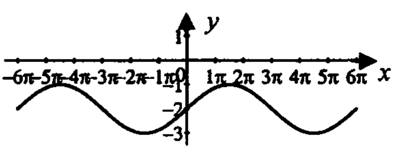
б)



в)



г)



13.19. а) $f(x) = \begin{cases} \cos 2x, x \le \pi \\ -\frac{1}{2}, x > \pi \end{cases}$

1) $D(f) = R$; 2) $E(f) = [-1;1]$;
3) при $x \le \pi$ $T = \pi$; 4) ни четная, ни нечетная;

5) $f(x) = 0$ при $x = -\frac{\pi}{4} + \frac{\pi n}{2}\pi$, $n \le -2$;

6) $f_{min}=-1, f_{max}=1$;

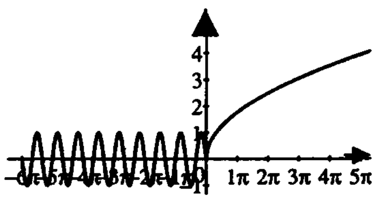
7) $f(x)<0$ при $x\in\left(\frac{\pi}{4}-\pi n;\ \frac{3}{4}\pi-\pi n\right)\cup(\pi;+\infty)$, $n\geq 0$,

$f(x)>0$ при $x\in\left(-\frac{\pi}{4}-\pi n;\ \frac{\pi}{4}-\pi n\right)\cup(\frac{3}{4}\pi;\pi]$, $n\geq 0$;

8) $f(x)$ возрастает при $x\in\left[\frac{\pi}{2}-\pi n;\pi-\pi n\right]$, $n\geq 0$,

$f(x)$ убывает при $x\in\left[-\pi n;\frac{\pi}{2}-\pi n\right]$, $n\geq 0$.

б) $f(x)=\begin{cases}-\sin 3x, x<0\\ \sqrt{x}, x\geq 0\end{cases}$



1) $D(f)=R$;
2) $E(f)=[-1;+\infty)$;
3) при $x\leq 0$ $T=\frac{2}{3}\pi$;
4) ни четная, ни нечетная;
5) $f(x)=0$ при $x=-\frac{\pi n}{3}$, $n\geq 0$;
6) $f_{min}=-1$, $f_{max}=+\infty$;
7) $f(x)<0$ при $x\in\left(-\frac{2}{3}\pi;\frac{\pi}{3}-\frac{2}{3}\pi n\right)$, $n\geq 1$, $f(x)>0$ при

$x\in\left(-\frac{\pi}{3}-\frac{2}{3}\pi n;\ -\frac{2}{3}\pi n\right)$, $n\geq 0$, $x\geq 0$;

8) $f(x)$ возрастает при $x\in\left[\frac{\pi}{6}-\frac{2}{3}\pi n;\frac{\pi}{2}-\frac{2}{3}\pi n\right]$, $n\geq 1$, $x\in[0;+\infty)$,

$f(x)$ убывает при $x\in\left[-\frac{\pi}{6}-\frac{3}{2}\pi n;\frac{\pi}{6}-\frac{3}{2}\pi n\right]$, $n\geq 1$, $x\in\left[-\frac{\pi}{3};0\right]$.

13.20.

а) $y=\begin{cases}-x, & x<0\\ \sin 2x, & x\geq 0\end{cases}$;

б) $y=\begin{cases}\cos 3x, & x\in[-\frac{\pi}{6};\ \frac{\pi}{3}]\\ -1, & x>\frac{\pi}{3}\end{cases}$

в) $y=\begin{cases}\sin 2x, & x\leq 0\\ 2\cos x, & x>0\end{cases}$;

г) $y=\begin{cases}-2\sin x, & x\in[-2\pi;\ 0]\\ \cos\frac{x}{2}, & x\in(0;3\pi]\end{cases}$

В ответах учебника для пункта в) ошибка.

§ 14. Функции $y = \operatorname{tg} x$, $y = \operatorname{ctg} x$, их свойства и графики

14.1. $y = \operatorname{tg} x$

а) $\operatorname{tg} \frac{\pi}{4} = 1$; б) $\operatorname{tg} \frac{2\pi}{3} = -\sqrt{3}$;

в) $\operatorname{tg} \frac{3\pi}{4} = -1$; г) $\operatorname{tg} \pi = 0$.

14.2. $y = \operatorname{tg} x$.

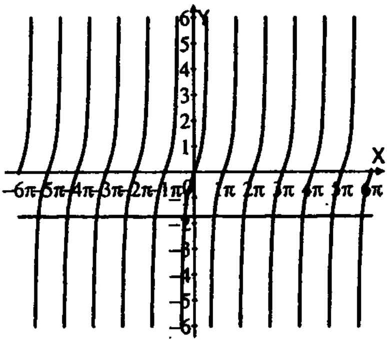
а) $x \in (\frac{\pi}{2}; \frac{3\pi}{2})$, $y_{\min} = -\infty$, $y_{\max} = +\infty$;

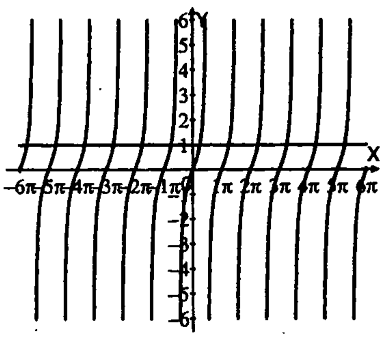
б) $x \in (\frac{3\pi}{4}; \pi]$, $y_{\min}$ не существует, $y_{\max} = 0$;

в) $x \in [-\frac{\pi}{4}; \frac{\pi}{6}]$, $y_{\min} = -1$, $y_{\max} = \frac{\sqrt{3}}{3}$

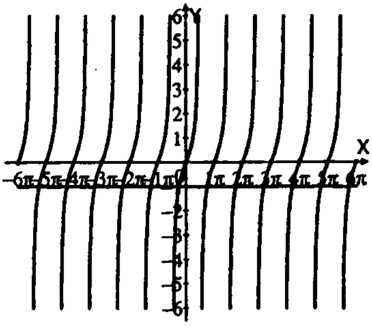
г) $x \in [\pi; \frac{3\pi}{2})$, $y_{\min} = 0$, $y_{\max}$ не существует.

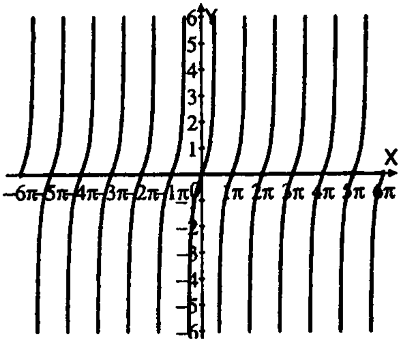
14.3. а) $x = -\frac{\pi}{3} + \pi n$; б) $x = \frac{\pi}{4} + \pi n$;





в) $x = -\frac{\pi}{4} + \pi n$; г) $x = \pi n$.





14.4. а) $\operatorname{ctg}\frac{\pi}{4}=1$; б) $\operatorname{ctg}\frac{\pi}{3}=\frac{\sqrt{3}}{3}$;

в) $\operatorname{ctg}2\pi$ не существует; г) $\operatorname{ctg}\frac{\pi}{2}=0$.

14.5. $y=\operatorname{ctg}x$.

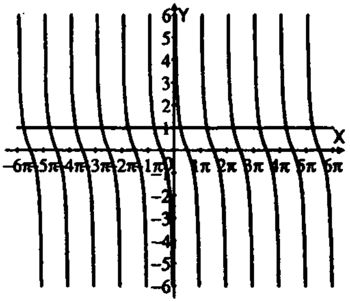
а) $x\in[\frac{\pi}{4};\frac{\pi}{2}]$, $y_{max}=1$, $y_{min}=0$;

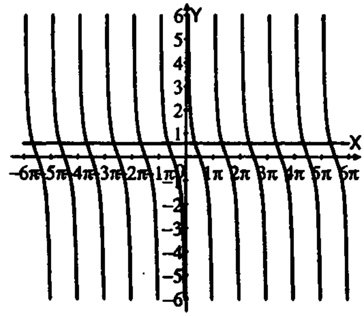
б) $x\in[\frac{\pi}{2};\pi)$, $y_{max}=0$, y_{min} не существует;

в) $x\in(-\pi;0)$, y_{max}, y_{min} не существуют;

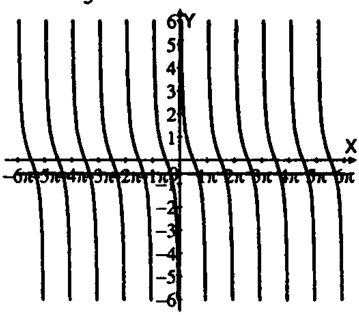
г) $x\in[\frac{\pi}{6};\frac{3\pi}{4}]$, $y_{max}=\sqrt{3}$, $y_{min}=-1$.

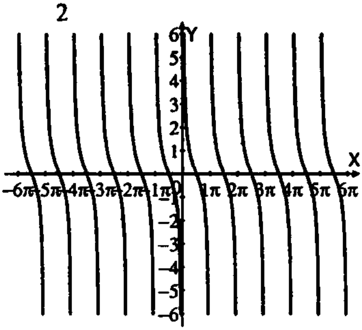
14.6. а) $x=\frac{\pi}{4}+\pi k$; б) $x=\frac{\pi}{3}+\pi k$;





в) $x=-\frac{\pi}{3}+\pi k$; г) $x=\frac{\pi}{2}+\pi k$.





14.7. а) $f(x)=\operatorname{tg}x-\cos x$, $f(-x)=-\operatorname{tg}x-\cos x$, ни четная, ни нечетная;
б) $f(x)=\operatorname{tg}x+x$, $f(-x)=-\operatorname{tg}x-x=-f(x)$, нечетная;
в) $f(x)=\operatorname{ctg}^2x-x^4$, $f(-x)=\operatorname{ctg}^2x-x^4$, четная;
г) $f(x)=x^3-\operatorname{ctg}x$, $f(-x)=-x^3+\operatorname{ctg}x=-f(x)$, нечетная.

14.8. $\operatorname{tg}(9\pi-x)=-\frac{3}{4}$; $\operatorname{tg}(9\pi-x)=-\operatorname{tg}x$; $\operatorname{tg}x=\frac{3}{4}$, $\operatorname{ctg}x=\frac{4}{3}$.

14.9. $\operatorname{ctg}(7\pi - x) = \frac{5}{7}$; $\operatorname{ctg} x = -\frac{5}{7}$, $\operatorname{tg} x = -\frac{7}{5}$

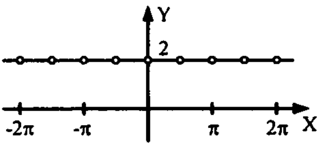
14.10. а) $\operatorname{tg} 200^\circ - \operatorname{tg} 201^\circ < 0$; б) $\operatorname{tg} 1 - \operatorname{tg} 1{,}01 < 0$;

в) $\operatorname{tg} 2{,}2 - \operatorname{tg} 2{,}1 > 0$; г) $\operatorname{tg}\frac{3\pi}{5} - \operatorname{tg}\frac{6\pi}{5} < 0$.

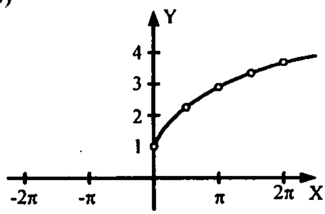
14.11. $f(x) = \operatorname{tg} x$, $f(2x + 2\pi) + f(7\pi - 2x) = \operatorname{tg}(2x + 2\pi) + \operatorname{tg}(7\pi - 2x) =$
$= \operatorname{tg} 2x - \operatorname{tg} 2x = 0$.

14.12. $f(x) = x^2 + 1$, $f(\operatorname{tg} x) = \operatorname{tg}^2 x + 1 = \frac{1}{\cos^2 x}$.

14.13. а)

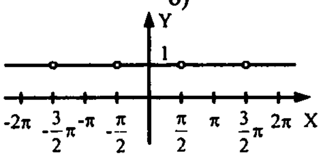


б)

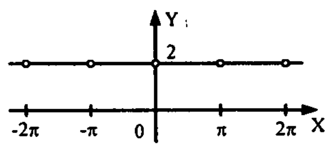


14.14.

а)

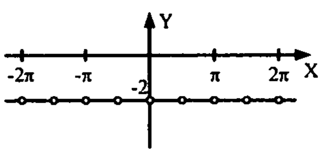


б)



14.15. а) см. график 276 (а)

б)



Глава 3. Тригонометрические уравнения

§ 15. Арккосинус. Решение уравнения $\cos t = a$.

15.1. а) $\arccos 0 = \frac{\pi}{2}$; б) $\arccos 1 = 0$;

в) $\arccos \frac{\sqrt{3}}{2} = \frac{\pi}{6}$; г) $\arccos \frac{1}{2} = \frac{\pi}{3}$.

15.2. а) $\arccos (-\frac{\sqrt{2}}{2}) = \frac{3\pi}{4}$; б) $\arccos (-\frac{\sqrt{3}}{2}) = \frac{5\pi}{6}$;

в) $\arccos (-1) = \pi$; г) $\arccos (-\frac{1}{2}) = \frac{2\pi}{3}$.

15.3. а) $\arccos (-1) + \arccos 0 = \pi + \frac{\pi}{2} = \frac{3\pi}{2}$;

б) $\arccos \frac{1}{2} - \arccos \frac{\sqrt{3}}{2} = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6}$;

в) $\arccos (-\frac{\sqrt{2}}{2}) + \arccos \frac{\sqrt{2}}{2} = \frac{3\pi}{4} + \frac{\pi}{4} = \pi$;

г) $\arccos (-\frac{1}{2}) - \arccos \frac{1}{2} = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3}$.

15.4. а) $\sin (\arccos (-\frac{1}{2} \frac{1}{2})) = \sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$;

б) $\operatorname{tg} (\arccos \frac{\sqrt{3}}{2})) = \operatorname{tg} \frac{\pi}{6} = \frac{\sqrt{3}}{3}$;

в) $\operatorname{ctg} (\arccos 0) = \operatorname{ctg} \frac{\pi}{2} = 0$;

г) $\sin (\arccos \frac{\sqrt{2}}{2})) = \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

15.5. а) $\cos (2 \arccos \frac{1}{2} - 3 \arccos 0 - \arccos (-\frac{1}{2})) = \cos (\frac{2\pi}{3} \frac{3\pi}{2} - \frac{2\pi}{3}) =$

$= \cos \frac{3\pi}{2} = 0$;

б) $\cos x = \frac{\sqrt{2}}{2}$, $x = \pm \frac{\pi}{4} + 2\pi n$.

в) $\cos t = 1$, $t = 2\pi n, n \in \mathbb{Z}$

г) $\cos t = \frac{\sqrt{3}}{2}$, $t = \pm \frac{\pi}{6} + 2\pi n, n \in \mathbb{Z}$

15.6. а) $\cos t = -1$, $t = \pi + 2\pi n$, $n \in \mathbb{Z}$

б) $\cos x = -\frac{\sqrt{3}}{2}$, $x = \pm\frac{5\pi}{6} + 2\pi n$;

в) $\cos t = -\frac{1}{2} \Rightarrow t = \pm\frac{2\pi}{3} + 2\pi n$, $n \in \mathbb{Z}$

г) $\cos x = -\frac{\sqrt{2}}{2}$, $t = \pm\frac{3\pi}{4} + 2\pi n$;

15.7. а) $\cos x = \frac{1}{3}$, $x = \pm \arccos\frac{1}{3} + 2\pi n$;

б) $\cos x = -1{,}1$, решений нет;

в) $\cos t = -\frac{3}{7} \Rightarrow t = \pm\arccos\left(-\frac{3}{7}\right) + 2\pi n$, $n \in \mathbb{Z}$

г) $\cos t = 2{,}04 \Rightarrow$ решений нет.

15.8. а) $\cos\left(2\arccos\frac{1}{2} - 3\arccos 0 - \arccos\left(-\frac{1}{2}\right)\right) = \cos\left(\frac{2\pi}{3} - \frac{3\pi}{2} - \frac{2\pi}{3}\right) =$

$= \cos\frac{3\pi}{2} = 0$;

б) $\frac{1}{3}\left(\arccos\frac{1}{3} + \arccos\left(-\frac{1}{3}\right)\right) = \frac{1}{3}\pi = \frac{\pi}{3}$.

15.9. а) $[-1; 1]$; б) $\left[-\frac{1}{2}; \frac{1}{2}\right]$; в) $[0; 2]$; г) $[1; 2]$.

15.10. а) $\arccos\sqrt{5}$, – нет; б) $\arccos\sqrt{\frac{2}{3}}$, – да;

в) $\arccos\frac{\pi}{5}$, – да; г) $\arccos(-\sqrt{3})$, – нет.

15.11. $\operatorname{tg}(\arccos 0{,}1 + \arccos(-0{,}1) + x) = \operatorname{tg} x$, $\operatorname{tg}(\pi + x) = \operatorname{tg} x$.

15.12. а) $\frac{8\cos x - 3}{3\cos x + 2} = 1$, $\frac{8\cos x - 3 - 3\cos x - 2}{3\cos x + 2} = 0$,

$\begin{cases} 5\cos x - 5 = 0 \\ \cos x \neq -\frac{2}{3} \end{cases}$, $\cos x = 1$, $x = 2\pi n$;

б) $\frac{3\cos x + 1}{2} + \frac{5\cos x - 1}{3} = 1\frac{3}{4}$,

$9\cos x + 3 + 10\cos x - 2 = \frac{7}{4} \cdot 6 = \frac{21}{2}$,

$19\cos x = \frac{19}{2}$, $\cos x = \frac{1}{2}$, $x = \pm\frac{\pi}{3} + 2\pi n$.

15.13. а) $6\cos^2 x + 5\cos x + 1 = 0,$

$\cos x = \dfrac{-5+1}{12} = -\dfrac{1}{3},$

$x = \pm \arccos(-\dfrac{1}{3}) + 2\pi n.$

$\cos x = \dfrac{-5-1}{12} = -\dfrac{1}{2}, \ t = \pm\ \frac{2}{3}\pi + 2\pi n.$

б) $3 + 9\cos x = 5\sin x, \quad 5\cos^2 x + 9\cos x - 2 = 0;$

$\cos x = \dfrac{-9 \pm 11}{10}$ $\cos x = -2$ не существует.

$\cos x = \dfrac{1}{5}$ $x = \pm\arccos\dfrac{1}{5} + 2\pi n$.

15.14. а) $\cos x = \dfrac{\sqrt{3}}{2}, x \in [0, 2\pi]$

$x = \pm\dfrac{\pi}{6} + 2\pi n, n \in Z$

Ответ: $\dfrac{\pi}{6}, \dfrac{11}{6}\pi$;

б) $\cos x = -\dfrac{1}{2}, x \in [2\pi, 4\pi]$

$x = \pm\dfrac{2}{3}\pi + 2\pi n, n \in Z$

Ответ: $\dfrac{8}{3}\pi, \dfrac{10}{3}\pi$;

в) $\cos x = \dfrac{\sqrt{2}}{2}, x \in [-\pi, 3\pi]$

$x = \pm\dfrac{\pi}{4} + 2\pi n, n \in Z$

Ответ: $-\dfrac{\pi}{4}, \dfrac{\pi}{4}, \dfrac{7\pi}{4}, \dfrac{9\pi}{4}$;

г) $\cos x = -1, x \in \left[-\dfrac{3\pi}{2}, 2\pi\right]$

$x=\pi+2\pi n,\ n \in Z$

Ответ: $-\pi, \pi$.

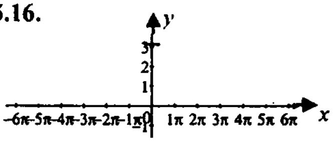
15.15. а) $\cos x = \dfrac{1}{2}, x \in (1, 6)$ $x = \pm\dfrac{\pi}{3} + 2\pi n$. Ответ: $\dfrac{\pi}{3}, \dfrac{5\pi}{3}$;

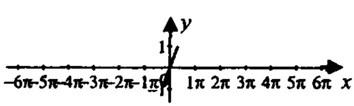
б) $\cos x = -\dfrac{1}{2}, x \in (2, 10)$ $x = \pm\dfrac{2\pi}{3} + 2\pi n$. Ответ: $\dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \dfrac{8\pi}{3}$;

в) $\cos x = \dfrac{\sqrt{2}}{2}, \ x \in \left(-\dfrac{\pi}{4}, 12\right), \ x = \pm\dfrac{\pi}{4} + 2\pi n$. Ответ: $\dfrac{\pi}{4}, \dfrac{7\pi}{4}, \dfrac{9\pi}{4}, \dfrac{15\pi}{4}$;

г) $\cos x = -\dfrac{\sqrt{2}}{2}, \ x \in \left(-4, \dfrac{5\pi}{4}\right), \ x = \pm\dfrac{3\pi}{4} + 2\pi n$. Ответ: $-\dfrac{3\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}$.

15.16.
а)





15.17. а) $\cos t > \frac{1}{2}$, $t \in (-\frac{\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k)$;

б) $\cos t \le -\frac{\sqrt{2}}{2}$, $t \in [\frac{3\pi}{4} + 2\pi k; \frac{5\pi}{4} + 2\pi k]$;

в) $\cos t \ge -\frac{\sqrt{2}}{2}$, $t \in [-\frac{3\pi}{4} + 2\pi k; \frac{3\pi}{4} + 2\pi k]$;

г) $\cos t < \frac{1}{2}$. $t \in (\frac{\pi}{3} + 2\pi k; \frac{5\pi}{3} + 2\pi k)$.

15.18. а) $\cos t < \frac{2}{3}$, $t \in (\arccos\frac{2}{3} + 2\pi k; 2\pi - \arccos\frac{2}{3} + 2\pi k)$;

б) $\cos t > -\frac{1}{7}$, $t \in (-\arccos(-\frac{1}{7}) + 2\pi k; \arccos(-\frac{1}{7}) + 2\pi k)$;

в) $\cos t > \frac{2}{3}$, $t \in (-\arccos\frac{2}{3} + 2\pi k; \arccos\frac{2}{3} + 2\pi k)$;

г) $\cos t < -\frac{1}{7}$, $t \in (\arccos(-\frac{1}{7}) + 2\pi k; 2\pi - \arccos(-\frac{1}{7}) + 2\pi k)$.

15.19. а) $3\cos^2 t - 4\cos t \ge 4$, $3\cos^2 t - 4\cos t - 4 = 0$.

Найдем корни квадратного уравнения:

$$\cos t = \frac{4 \pm \sqrt{16 + 4 \cdot 3 \cdot 4}}{6} = \frac{4 \pm 8}{6},$$

$\cos t = -\frac{2}{3}$, $\cos t = 2$ не подходит,

$$\cos t \le -\frac{2}{3}, \quad t \in \left[\arccos\left(-\frac{2}{3}\right) + 2\pi k, 2\pi - \arccos\left(-\frac{2}{3}\right) + 2\pi k\right];$$

б) $6\cos^2 t + 1 > 5\cos t$.

Найдем корни квадратного уравнения:

$6\cos^2 t - 5\cos t + 1 = 0$,

$\cos t = \frac{5+1}{12} = \frac{1}{2}$, $\cos t = \frac{1}{3}$,

$t \in (-\frac{\pi}{3} + 2\pi k; \frac{\pi}{3} + 2\pi k) \cup (\arccos\frac{1}{3} + 2\pi k; 2\pi - \arccos\frac{1}{3} + 2\pi k)$;

в) $3\cos^2 t - 4\cos t < 4$,

$3\cos^2 t - 4\cos t - 4 < 0$.

Найдем корни квадратного уравнения:

$3\cos^2 t - 4\cos t - 4 = 0$,

$$\cos t = \frac{4 \pm \sqrt{16 + 4 \cdot 3 \cdot 4}}{6} = \frac{2 \pm 4}{3},$$

$\cos t = 2$ – не подходит,

$\cos t = -\dfrac{2}{3} \to \cos t > -\dfrac{2}{3}$,

$t \in (-\arccos\left(-\dfrac{2}{3}\right) + 2\pi k; \arccos\left(-\dfrac{2}{3}\right) + 2\pi k)$;

г) $6\cos^2 t + 1 \le 5\cos t,\ 6\cos^2 t - 5\cos t + 1 \le 0$.
Найдем корни квадратного уравнения:
$6\cos^2 t - 5\cos t + 1 = 0$,

$\cos t = \dfrac{5 \pm \sqrt{25 - 4\cdot 6\cdot 1}}{12} = \dfrac{5 \pm 1}{12}$,

$\cos t = \dfrac{1}{2},\ \cos t = \dfrac{1}{3}$,

$t \in \left[-\arccos\dfrac{1}{3} + 2\pi k; -\dfrac{\pi}{3} + 2\pi k\right] \cup \left[\dfrac{\pi}{3} + 2\pi k; \arccos\dfrac{1}{3} + 2\pi k\right]$.

15.20. а) $4\cos^2 t < 1,\ \cos^2 t < \dfrac{1}{4}$,

$\cos t \in (-\dfrac{1}{2}; \dfrac{1}{2}),\ t \in (\dfrac{\pi}{3} + \pi k; \dfrac{2}{3}\pi + \pi k)$;

б) $3\cos^2 t < \cos t,\ \cos t(3\cos t - 1) < 0,\ \cos t \in (0; \dfrac{1}{3})$,

$t \in (-\dfrac{\pi}{2} + 2\pi n; -\arccos\dfrac{1}{3} + 2\pi n) \cup (\arccos\dfrac{1}{3} + 2\pi n; \dfrac{\pi}{2} + 2\pi n)$;

в) $9\cos^2 t > 1,\quad \cos^2 t - \dfrac{1}{9} > 0$

$$\left[\begin{array}{l}\cos t > \dfrac{1}{3}\\ \cos t < -\dfrac{1}{3}\end{array}\right.\qquad t \in \left(-\arccos\dfrac{1}{3} + 2\pi n, \arccos\dfrac{1}{3} + 2\pi n\right) \cup$$
$$\cup \left(\arccos\left(-\dfrac{1}{3}\right) + 2\pi n, 2\pi - \arccos\left(-\dfrac{1}{3}\right) + 2\pi n\right);$$

г) $3\cos^2 t - \cos t > 0$

$\cos t\left(\cos t - \dfrac{1}{3}\right) > 0$

$$\left[\begin{array}{l}\cos t < 0\\ \cos t > \dfrac{1}{3}\end{array}\right.\qquad t \in \left(-\arccos\dfrac{1}{3} + 2\pi n, \arccos\dfrac{1}{3} + 2\pi n\right) \cup$$
$$\cup \left(\dfrac{\pi}{2} + 2\pi n, \dfrac{3\pi}{2} + 2\pi n\right).$$

15.21. а) $\sin\left(\arccos\dfrac{3}{5}\right) = \sqrt{1 - \cos^2\left(\arccos\dfrac{3}{5}\right)} = \dfrac{4}{5}$

б) $\sin(\arccos(-0{,}8)) = \sqrt{1 - \cos^2(\arccos(-0{,}8))} = 0{,}6$

15.22. а) $\operatorname{tg}\left(\arccos\left(-\frac{5}{13}\right)\right)=\frac{\sqrt{1-\cos^2\left(\arccos\left(-\frac{5}{13}\right)\right)}}{\cos\left(\arccos\left(-\frac{5}{13}\right)\right)}=-\frac{12}{5}$

б) $\operatorname{ctg}\left(\arccos\frac{4}{5}\right)=\frac{\cos\left(\arccos\left(\frac{4}{5}\right)\right)}{\sqrt{1-\cos^2\left(\arccos\left(\frac{4}{5}\right)\right)}}=\frac{4}{3}$

§ 16. Арксинус. Решение уравнения sin t = a

16.1. а) $\arcsin\frac{\sqrt{3}}{2}=\frac{\pi}{3}$; б) $\arcsin 1=\frac{\pi}{2}$;

в) $\arcsin\frac{\sqrt{2}}{2}=\frac{\pi}{4}$; г) $\arcsin 0=0$.

16.2. а) $\arcsin\left(-\frac{\sqrt{3}}{2}\right)=-\frac{\pi}{3}$; б) $\arcsin\left(-\frac{1}{2}\right)=-\frac{\pi}{6}$;

в) $\arcsin(-1)=-\frac{\pi}{2}$; г) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)=-\frac{\pi}{4}$.

16.3. а) $\arcsin 0+\arccos 0=\frac{\pi}{2}$; б) $\arcsin\frac{\sqrt{2}}{2}+\arccos\frac{\sqrt{3}}{2}=\frac{\pi}{2}$;

в) $\arcsin\left(-\frac{\sqrt{2}}{2}\right)+\arccos\frac{1}{2}=\frac{\pi}{12}$; г) $\arcsin(-1)+\arccos\frac{\sqrt{3}}{2}=-\frac{\pi}{3}$.

16.4. а) $\arccos\left(-\frac{1}{2}\right)+\arcsin\left(-\frac{1}{2}\right)=\frac{\pi}{2}$;

б) $\arccos\left(-\frac{\sqrt{2}}{2}\right)-\arcsin(-1)=\frac{5\pi}{4}$;

в) $\arccos\left(-\frac{\sqrt{3}}{2}\right)+\arcsin\left(-\frac{\sqrt{3}}{2}\right)=\frac{\pi}{2}$;

г) $\arccos\frac{\sqrt{2}}{2}-\arcsin\left(-\frac{\sqrt{3}}{2}\right)=\frac{7\pi}{12}$.

16.5. а) $\sin x=\frac{\sqrt{3}}{2}$, $x=(-1)^k\frac{\pi}{3}+\pi k$;

б) $\sin x=\frac{\sqrt{2}}{2}$, $x=(-1)^k\frac{\pi}{4}+\pi k$;

в) $\sin x = 1$, $x = \frac{\pi}{2} + 2\pi n$;

г) $\sin x = \frac{1}{2}$, $x = (-1)^k \frac{\pi}{6} + \pi k$.

16.6. а) $\sin x = -1$, $x = -\frac{\pi}{2} + 2\pi n$;

б) $\sin x = -\frac{\sqrt{2}}{2}$, $x = (-1)^{k+1} \frac{\pi}{4} + \pi k$;

в) $\sin x = -\frac{1}{2}$, $x = (-1)^{k+1} \frac{\pi}{6} + \pi k$;

г) $\sin x = -\frac{\sqrt{3}}{2}$, $x = (-1)^{k+1} \frac{\pi}{3} + \pi k$.

16.7.

а) $\sin x = \frac{1}{4}$, $x = (-1)^k \arcsin \frac{1}{4} + \pi k$; б) $\sin x = 1{,}02$, решений нет;

в) $\sin x = -\frac{1}{7}$, $x = (-1)^k \arcsin\left(-\frac{1}{7}\right) + \pi k$; г) $\sin x = \frac{\pi}{3}$, решений нет.

16.8. а) $\sin(\arccos x + \arccos(-x)) = 0$, $\sin \pi = 0$;
б) $\cos(\arcsin x + \arcsin(-x)) = 1$, $\cos 0 = 1$.

16.9. а) $\sin x = \frac{1}{2}, x \in [0, 2\pi]$

$x = (-1)^n \frac{\pi}{6} + \pi n$.

Ответ: $\frac{\pi}{6}, \frac{5\pi}{6}$.

б) $\cos x = -\frac{1}{2}, x \in [-\pi, \pi]$

$x = \pm\frac{2\pi}{3} + 2\pi n$.

Ответ: $\pm\frac{2\pi}{3}$

в) $\sin x = -\frac{\sqrt{2}}{2}, x \in [-\pi, 2\pi]$

$x = (-1)^{n+1} \frac{\pi}{4} + \pi n$.

Ответ: $-\frac{\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$;

г) $\cos x = \frac{\sqrt{3}}{2}, x \in [-2\pi, \pi]$

$x = \pm\frac{\pi}{6} + 2\pi n$.

Ответ: $-\frac{11}{6}\pi, -\frac{\pi}{6}, \frac{\pi}{6}$.

16.10. а) $\sin x = \frac{1}{2}, x \in \left(\frac{1}{2}, \frac{11\pi}{4}\right)$

$x = (-1)^n \frac{\pi}{6} + \pi n$.

Ответ: $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$.

б) $\sin x = -\frac{1}{2}, x \in \left(-\frac{5\pi}{6}, 6\right)$

$x = (-1)^{n+1} \frac{\pi}{6} + \pi n$.

Ответ: $-\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

в) $\sin x = \frac{\sqrt{2}}{2}$, $x \in (-4, 3)$

$x = (-1)^n \frac{\pi}{4} + \pi n$.

Ответ: $-\frac{5\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4}$.

г) $\sin x = \frac{\sqrt{2}}{2}$, $x \in (-3, 6)$

Ответ: $\frac{\pi}{4}, \frac{3\pi}{4}$.

16.11. а) $\arcsin x$, $x \in [-1; 1]$;

б) $\arcsin(5 - 2x)$, $x \in [2; 3]$;

в) $\arcsin \frac{x}{2}$, $x \in [-2; 2]$;

г) $\arcsin(x^2 - 3)$, $x \in [-2; -\sqrt{2}\,] \cup [\sqrt{2}; 2]$.

16.12. а) $\arcsin(-\frac{2}{3})$. Да; б) $\arcsin 1{,}5$. Нет;

в) $\arcsin(3 - \sqrt{20})$. Нет; г) $\arcsin(4 - \sqrt{20})$. Да.

16.13. а) $(2\cos x + 1)(2\sin x - \sqrt{3}) = 0$,

$\left[\begin{array}{l} \cos x = -\frac{1}{2} \\ \sin x = \frac{\sqrt{3}}{2} \end{array}\right.$, $x = \pm\frac{2\pi}{3} + 2\pi k$, $x = (-1)^n \frac{\pi}{3} + \pi n$; $\pm\frac{2\pi}{3} + 2\pi k$, $\frac{\pi}{3} + 2\pi k$;

б) $2\cos x - 3\sin x \cos x = 0$, $\cos x(2 - 3\sin x) = 0$,

$\left[\begin{array}{l} \cos x = 0 \\ \sin x = \frac{2}{3} \end{array}\right.$, $x = \frac{\pi}{2} + \pi n$, $x = (-1)^k \arcsin\frac{2}{3} + \pi k$;

в) $4\sin^2 x - 3\sin x = 0$, $\sin x(4\sin x - 3) = 0$, $\sin x = 0$, $\sin x = \frac{3}{4}$,

$x = \pi n$, $x = (-1)^k \arcsin\frac{3}{4} + \pi k$;

г) $2\sin^2 x - 1 = 0$, $\sin x = \pm\frac{\sqrt{2}}{2}$, $x = \frac{\pi}{4} + \frac{\pi n}{2}$.

16.14. а) $6\sin^2 x + \sin x - 2 = 0$, $\sin x = \frac{-1+7}{12} = \frac{1}{2}$, $(-1)^n \frac{\pi}{6} + \pi n$,

$\sin x = -\frac{2}{3}$, $(-1)^{k+1} \arcsin\frac{2}{3} + \pi k$;

б) $3\cos^2 x = 7(\sin x + 1)$, $3 - 3\sin^2 x = 7\sin x + 7$, $3\sin^2 x + 7\sin x + 4 = 0$,

$\sin x = \frac{-7 + \sqrt{49 - 4\cdot 3\cdot 4}}{6} = \frac{-7 \pm 1}{6}$, $\sin x = \frac{-8}{6}$ — не существует,

$\sin x = -1$, $x = -\frac{\pi}{2} + 2\pi n$.

16.15. а) $\sin t > \frac{\sqrt{3}}{2}$, $t \in (\frac{\pi}{3} + 2\pi k;\ \frac{2\pi}{3} + 2\pi k)$;

б) $\sin t > -\frac{1}{2}$, $t \in (-\frac{\pi}{6} + 2\pi k;\ \frac{7\pi}{6} + 2\pi k)$;

в) $\sin t < \frac{\sqrt{3}}{2}$, $t \in (-\frac{4\pi}{3} + 2\pi k;\ \frac{\pi}{3} + 2\pi k)$;

г) $\sin t \le -\frac{1}{2}$, $t \in \left[\frac{7\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k\right]$.

16.16. а) $\sin t < \frac{1}{3}$, $t \in (-\pi - \arcsin\frac{1}{3} + 2\pi k;\ \arcsin\frac{1}{3} + 2\pi k)$;

б) $\sin t \ge -\frac{3}{5}$, $t \in \left[-\arcsin\frac{3}{5} + 2\pi k; \pi + \arcsin\frac{3}{5} + 2\pi k\right]$;

в) $\sin t \ge \frac{1}{3}$, $t \in \left[\arcsin\frac{1}{3} + 2\pi k; \pi - \arcsin\frac{1}{3} + 2\pi k\right]$;

г) $\sin t < -\frac{3}{5}$, $t \in (\pi + \arcsin\frac{3}{5} + 2\pi k;\ 2\pi - \arcsin\frac{3}{5} + 2\pi k)$.

16.17.

а) $5\sin^2 t > 11\sin t + 12$, $5\sin^2 t - 11\sin t - 12 = 0$,

$\sin t = \frac{11+19}{10}$, не существует. $\sin t = -\frac{8}{10}$,

$t \in (\pi + \text{arccsin}\ \frac{4}{5} + 2\pi k;\ 2\pi - \arcsin\frac{4}{5} + 2\pi k)$

б) $5\sin^2 t \le 11\sin t + 12$, $5\sin^2 t - 11\sin t - 12 = 0$,

$\sin t = -\frac{4}{5}$, $t \in \left[-\arcsin\frac{4}{5} + 2\pi k; \pi + \arcsin\frac{4}{5} + 2\pi k\right]$.

16.18. а) $6\cos^2 t + \sin t > 4$,

$6 - 6\sin^2 t + \sin t - 4 > 0$,

$6\sin^2 t - \sin t - 2 < 0$,

$\sin t = \frac{1+7}{12} = \frac{3}{4}$, $\sin t = -\frac{1}{2}$,

$t \in (-\frac{\pi}{6} + 2\pi k;\ \arcsin\frac{2}{3} + 2\pi k) \cup (\pi - \arcsin\frac{2}{3} + 2\pi k; \frac{7\pi}{6} + 2\pi k)$;

б) $6\cos^2 t + \sin t \le 4$,

$6\sin^2 t - \sin t - 2 = 0$,

$\sin t = \frac{3}{4}$, $\sin t = -\frac{1}{2}$,

$t \in [\arcsin\frac{2}{3} + 2\pi k;\ \pi - \arcsin\frac{2}{3} + 2\pi k]$, $t \in [\frac{7\pi}{6} + 2\pi k;\ \frac{11\pi}{6} + 2\pi k]$.

16.19. а) $\cos\left(\arcsin\left(-\frac{5}{13}\right)\right)=\sqrt{1-\sin^2\left(\arcsin\left(-\frac{5}{13}\right)\right)}=\frac{12}{13}$

б) $\operatorname{tg}(\arcsin(0,6))=\frac{\sin(\arcsin(0,6))}{\sqrt{1-\sin^2(\arcsin(0,6))}}=\frac{3}{4}$

в) $\cos\left(\arcsin\frac{8}{17}\right)=\sqrt{1-\sin^2\left(\arcsin\left(\frac{8}{17}\right)\right)}=\frac{15}{17}$

г) $\operatorname{ctg}(\arcsin(-0,8))=\frac{\sqrt{1-\sin^2(\arcsin(-0,8))}}{\sin(\arcsin(-0,8))}=-\frac{3}{4}$

§ 17. Арктангенс и арккотангенс. Решение уравнений $\operatorname{tg}x=a$, $\operatorname{ctg}x=a$.

17.1. а) $\operatorname{arctg}\frac{\sqrt{3}}{3}=\frac{\pi}{6}$; б) $\operatorname{arctg}1=\frac{\pi}{4}$;

в) $\operatorname{arctg}\sqrt{3}=\frac{\pi}{3}$; г) $\operatorname{arctg}0=0$.

17.2. а) $\operatorname{arctg}(-1)=-\frac{\pi}{4}$; б) $\operatorname{arctg}(-\sqrt{3})=-\frac{\pi}{3}$;

в) $\operatorname{arctg}(-\frac{\sqrt{3}}{3})=-\frac{\pi}{6}$; г) $\operatorname{arctg}(-\frac{1}{\sqrt{3}})=-\frac{\pi}{6}$.

17.3. а) $\operatorname{arcctg}\frac{\sqrt{3}}{3}=\frac{\pi}{3}$; б) $\operatorname{arcctg}1=\frac{\pi}{4}$;

в) $\operatorname{arcctg}(-\frac{\sqrt{3}}{3})=\frac{2\pi}{3}$; г) $\operatorname{arcctg}0=\frac{\pi}{2}$.

17.4. а) $\operatorname{arcctg}(-1)+\operatorname{arctg}(-1)=\frac{\pi}{2}$;

б) $\arcsin(-\frac{\sqrt{2}}{2})+\operatorname{arcctg}(-\sqrt{3})=\frac{7\pi}{12}$;

в) $\operatorname{arcctg}(-\frac{\sqrt{3}}{3})-\operatorname{arctg}(\frac{\sqrt{3}}{3})=\frac{\pi}{2}$;

г) $\arccos(-\frac{1}{2})-\operatorname{arcctg}(-\sqrt{3})=-\frac{\pi}{6}$.

17.5. а) $\operatorname{tg}x=1$, $x=\frac{\pi}{4}+\pi n$; б) $\operatorname{tg}x=-\frac{\sqrt{3}}{3}$, $x=-\frac{\pi}{6}+\pi n$;

в) $\operatorname{tg}x=-1$, $x=-\frac{\pi}{4}+\pi n$; г) $\operatorname{tg}x=-\frac{\sqrt{3}}{3}$, $x=-\frac{\pi}{6}+\pi n$.

17.6. а) $\text{tg}\, x = 0$, $x = \pi n$; б) $\text{tg}\, x = -2$, $x = -\text{arctg}\, 2 + \pi n$;

в) $\text{tg}\, x = -3$, $x = -\text{arctg}\, 3 + \pi n$; г) $\text{tg}\, x = \frac{1}{2}$, $x = \text{arctg}\, \frac{1}{2} + \pi n$.

17.7. а) $\text{ctg}\, x = 1$, $x = \frac{\pi}{4} + \pi n$; б) $\text{ctg}\, x = -\sqrt{3}$, $x = -\frac{\pi}{6} + \pi n$;

в) $\text{ctg}\, x = 0$, $x = \frac{\pi}{2} + \pi n$; г) $\text{ctg}\, x = -\frac{\sqrt{3}}{3}$, $x = -\frac{\pi}{3} + \pi n$;

17.8. а) $\text{tg}^2 x - 6\text{tg}\, x + 5 = 0$, $\text{tg}\, x = 5$, $\text{tg}\, x = 1$,

$x = \text{arctg}\, 5 + \pi n$, $x = \frac{\pi}{4} + \pi n$;

б) $\text{tg}^2 x - 2\, \text{tg}\, x - 3 = 0$, $\text{tg}\, x = 3$, $\text{tg}\, x = -1$,

$x = \text{arctg}\, 3 + \pi n$, $x = -\frac{\pi}{4} + \pi n$.

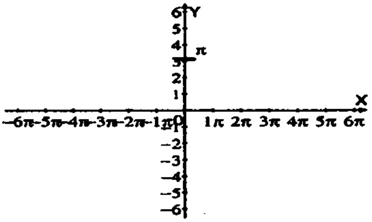
17.9. а) $\text{tg}\, (\pi + x) = \sqrt{3}$, $\text{tg}\, x = \sqrt{3}$, $x = \frac{\pi}{3} + \pi n$;

б) $2\, \text{ctg}\, (2\pi + x) - \text{tg}(\frac{\pi}{2} + x) = \sqrt{3}$, $2\text{ctg}\, x + \text{ctg}\, x = \sqrt{3}$, $\text{ctg}\, x = \frac{\sqrt{3}}{3}$, $x = \frac{\pi}{3} + \pi n$;

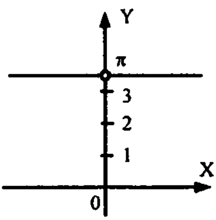
в) $-\sqrt{3}\ \text{tg}\, (\pi - x) = 1$, $\text{tg}\, x = \frac{\sqrt{3}}{3}$, $x = \frac{\pi}{6} + \pi n$;

г) $\text{ctg}\, (2\pi - x) + \text{ctg}\, (\pi - x) = 2$, $\text{ctg}\, x = -1$, $x = \frac{3\pi}{4} + \pi n$.

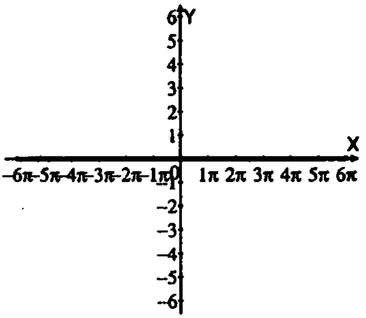
17.10. а)



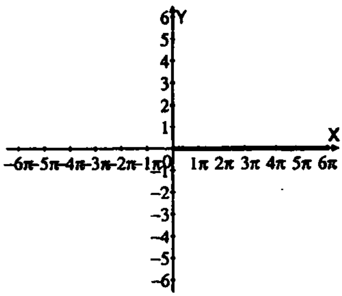
б)



в)



г)



§ 18. Тригонометрические уравнения

18.1. а) $\sin 2x = \frac{\sqrt{2}}{2}$, $2x = (-1)^k \frac{\pi}{4} + \pi k$, $x = (-1)^k \frac{\pi}{8} + \frac{\pi k}{2}$;

б) $\cos \frac{x}{3} = -\frac{1}{2}$, $\frac{x}{3} = \pm \frac{2\pi}{3} + 2\pi n$, $x = \pm 2\pi + 6\pi n$;

в) $\sin \frac{x}{4} = \frac{1}{2}$, $\frac{x}{4} = (-1)^k \frac{\pi}{6} + \pi k$, $x = (-1)^k \frac{2\pi}{3} + 4\pi k$;

г) $\cos 4x = 0$, $4x = \frac{\pi}{3} + \pi n$, $x = \frac{\pi}{8} + \frac{\pi n}{4}$.

18.2. а) $\sin(-\frac{x}{3}) = \frac{\sqrt{2}}{2}$, $\sin \frac{x}{3} = -\frac{\sqrt{2}}{2}$, $\frac{x}{3} = (-1)^{k+1} \frac{\pi}{4} + \pi k$,

$x = (-1)^{k+1} \frac{3\pi}{4} + 3\pi k$;

б) $\cos(-2x) = -\frac{\sqrt{3}}{2}$, $2x = \pm \frac{5\pi}{6} + 2\pi n$, $x = \pm \frac{5\pi}{12} + \pi n$;

в) $\mathrm{tg}(-4x) = \frac{\sqrt{3}}{3}$, $\mathrm{tg}\, 4x = -\frac{\sqrt{3}}{3}$, $4x = -\frac{\pi}{6} + \pi n$, $x = -\frac{\pi}{24} + \frac{\pi n}{4}$;

г) $\mathrm{ctg}(-\frac{x}{2}) = 1$, $\mathrm{ctg}\, \frac{x}{2} = -1$, $\frac{x}{2} = -\frac{\pi}{4} + \pi n$, $x = -\frac{\pi}{2} + 2\pi n$.

18.3. а) $2\cos(\frac{\pi}{3} - \frac{\pi}{6}) = \sqrt{3}$, $\frac{x}{2} - \frac{\pi}{6} = \pm \frac{\pi}{6} + 2\pi n$, $x = \pm \frac{\pi}{3} + \frac{\pi}{3} + 4\pi n$;

б) $\sqrt{3}\, \mathrm{tg}(\frac{x}{3} + \frac{\pi}{6}) = 3$, $\mathrm{tg}(\frac{x}{3} + \frac{\pi}{6}) = \sqrt{3}$, $\frac{x}{3} + \frac{\pi}{6} = \frac{\pi}{3} + \pi n$, $x = \frac{\pi}{2} + 3\pi n$;

в) $2\sin(3x - \frac{\pi}{4}) = -\sqrt{2}$, $3x - \frac{\pi}{4} = (-1)^{k+1} \frac{\pi}{4} + \pi k$,

$x = (-1)^{k+1} \frac{\pi}{12} + \frac{\pi}{12} + \frac{\pi k}{3}$;

г) $\sin(\frac{x}{2} - \frac{\pi}{6}) + 1 = 0$, $\frac{x}{2} - \frac{\pi}{6} = -\frac{\pi}{2} + 2\pi n$, $x = -\frac{2\pi}{3} + 4\pi n$.

18.4. а) $\cos(\frac{\pi}{6} - 2x) = -1$, $2x - \frac{\pi}{6} = \pi + 2\pi n$, $x = \frac{7\pi}{12} + \pi n$;

б) $\mathrm{tg}(\frac{\pi}{4} - \frac{x}{2}) = -1$, $\frac{x}{2} - \frac{\pi}{4} = \frac{\pi}{4} + \pi n$, $x = \pi + 2\pi n$;

в) $2\sin(\frac{\pi}{3} - \frac{x}{4}) = \sqrt{3}$, $\frac{x}{4} - \frac{\pi}{3} = (-1)^{n+1} \frac{\pi}{3} + \pi n$, $x = (-1)^{n+1} \frac{4\pi}{3} + \frac{4\pi}{3} + 4\pi n$;

г) $2\cos(\frac{\pi}{4} - 3x) = \sqrt{2}$, $3x - \frac{\pi}{4} = \pm \frac{\pi}{4} + 2\pi n$, $x = \pm \frac{\pi}{12} + \frac{\pi}{12} + \frac{2\pi n}{3}$.

18.5. а) $\sin\left(\frac{\pi}{2}+t\right)-\cos(\pi+t)=1$, $\cos t+\cos t=1$,

$\cos t=\frac{1}{2}\Rightarrow t=\pm\frac{\pi}{3}+2\pi n, n\in\mathbb{Z}$.

б) $\sin(\pi+t)+\sin(2\pi-t)-\cos\left(\frac{3\pi}{2}+t\right)+1.5=0$, $-\sin t-\sin t-\sin t=-1,5$

$\sin t=0,5\Rightarrow t=(-1)^n\frac{\pi}{6}+\pi n, n\in\mathbb{Z}$.

в) $\cos\left(\frac{\pi}{2}-t\right)-\sin(\pi+t)=\sqrt{2}$, $\sin t+\sin t=\sqrt{2}$,

$\sin t=\frac{\sqrt{2}}{2}\Rightarrow t=(-1)^n\frac{\pi}{4}+\pi n$, $n\in\mathbb{Z}$

г) $\sin(\pi+t)+\cos\left(\frac{\pi}{2}+t\right)=\sqrt{3}$, $-\sin t-\sin t=\sqrt{3}$,

$\sin t=-\frac{\sqrt{3}}{2}\Rightarrow t=(-1)^{n+1}\frac{\pi}{3}+\pi n$, $n\in\mathbb{Z}$

18.6. а) $3\sin^2x-5\sin x-2=0$, $\sin x=\frac{5+7}{6}$ не существует.

$\sin x=-\frac{1}{3}$, $x=(-1)^{k+1}\arcsin\frac{1}{3}+\pi k$;

б) $3\sin^2 2x+10\sin 2x+3=0$,

$\sin 2x=\frac{-10\pm\sqrt{100-4\cdot3\cdot3}}{6}=\frac{-5\pm4}{3}$,

$\sin 2x=\frac{-5-7}{3}$ не существует;

$\sin 2x=-\frac{1}{3}$, $x=(-1)^{k+1}\frac{\arcsin\frac{1}{2}}{2}+\frac{\pi k}{2}$;

в) $4\sin^2x+11\sin x-3=0$, $\sin x=\frac{-10\pm\sqrt{100-4\cdot3\cdot3}}{6}=\frac{-5\pm4}{3}$,

$\sin x=\frac{-11-13}{8}$ не существует; $\sin x=\frac{1}{4}$, $x=(-1)^n\arcsin\frac{1}{4}+\pi n$;

г) $2\sin^2\frac{x}{2}-3\sin\frac{x}{2}+1=0$, $\sin\frac{x}{2}=\frac{3\pm\sqrt{9-4\cdot2\cdot1}}{4}=\frac{3\pm1}{4}$,

$\sin\frac{x}{2}=\frac{3+1}{4}=1$, $\frac{x}{2}=\frac{\pi}{2}+2\pi n$, $x=\pi+4\pi n$; $\sin\frac{x}{2}=\frac{1}{2}$,

$\frac{x}{2}=(-1)^k\frac{\pi}{6}+\pi k$, $x=(-1)^k\frac{\pi}{3}+2\pi k$.

18.7. а) $6\cos^2 x + \cos x - 1 = 0$,

$\cos x = \dfrac{-10 \pm \sqrt{100 - 4 \cdot 3 \cdot 3}}{6} = \dfrac{-5 \pm 4}{3}$,

$\cos x = -\dfrac{1}{2}$; $x = \pm\dfrac{2}{3}\pi + 2\pi n$,

$\cos x = \dfrac{1}{3}$, $x = \pm\arccos\dfrac{1}{3} + 2\pi n$;

б) $2\cos^2 3x - 5\cos 3x - 3 = 0$,

$\cos 3x = \dfrac{5+7}{4}$ не существует,

$\cos 3x = -\dfrac{1}{2}$, $3x = \pm\dfrac{2\pi}{3} + 2\pi n$, $x = \pm\dfrac{2\pi}{9} + \dfrac{2\pi n}{3}$.

в) $2\cos^2 x - \cos x - 3 = 0$,

$\cos x = \dfrac{1+5}{4}$ не существует,

$\cos x = -1$, $x = \pi + 2\pi n$;

г) $2\cos^2\dfrac{x}{3} + 3\cos\dfrac{x}{3} - 2 = 0$,

$\cos\dfrac{x}{3} = \dfrac{-3-5}{4}$ не существует,

$\cos\dfrac{x}{3} = \dfrac{1}{2}$, $\dfrac{x}{3} = \pm\dfrac{\pi}{3} + 2\pi n$, $x = \pm\pi + 6\pi n$.

18.8. а) $2\sin^2 x + 3\cos x = 0$,

$2 - 2\cos^2 x - 3\cos x - 2 = 0$,

$\cos x = \dfrac{3+5}{4}$ не существует,

$\cos x = -\dfrac{1}{2}$, $x = \pm\dfrac{2\pi}{3} + 2\pi n$;

б) $8\sin^2 2x + \cos 2x + 1 = 0$,

$8 - 8\cos^2 x + \cos 2x + 1 = 0$, $8\cos^2 x - \cos 2x - 9 = 0$;

$\cos 2x = \dfrac{1+17}{16}$ не существует, $\cos 2x = -1$, $2x = \pi + 2\pi n$, $x = \dfrac{\pi}{2} + \pi n$;

в) $5\cos^2 x + 6\sin x - 6 = 0$,

$5 - 5\sin^2 x + 6\sin x - 6 = 0$, $5\sin^2 x - 6\sin x + 1 = 0$,

$\sin x \dfrac{6 \pm \sqrt{36 - 4 \cdot 5 \cdot 1}}{10} = \dfrac{3 \pm 2}{5}$,

$\sin x = 1$, $x = \dfrac{\pi}{2} + 2\pi n$, $\sin x = \dfrac{1}{5}$, $x = (-1)^n \arcsin\dfrac{1}{5} + \pi n$;

г) $4\sin 3x + \cos^2 3x = 4$,

$\sin^2 3x - 4\sin 3x + 3 = 0$, $\sin 3x = 3$ не существует.

$\sin 3x = 1$, $3x = \dfrac{\pi}{2} + 2\pi n$, $x = \dfrac{\pi}{6} + \dfrac{2\pi n}{3}$.

18.9. а) $3\,\mathrm{tg}^2 x + 2\,\mathrm{tg}\, x - 1 = 0$,

$\mathrm{tg}\, x = \dfrac{-1+2}{3} = \dfrac{1}{3}$, $x = \mathrm{arctg}\,\dfrac{1}{3} + \pi n$,

$\mathrm{tg}\, x = -1$, $x = -\dfrac{\pi}{4} + \pi n$;

б) $\mathrm{ctg}^2\, 2x - 6\,\mathrm{ctg}\, 2x + 5 = 0$,
$\mathrm{ctg}\, 2x = 5$, $2x = \mathrm{arcctg}\, 5 + \pi n$,

$x = \dfrac{\mathrm{arcctg}\, 5}{2} + \dfrac{\pi n}{2}$, $\mathrm{ctg}\, 2x = 1$, $2x = \dfrac{\pi}{4} + \pi n$, $x = \dfrac{\pi}{8} + \dfrac{\pi n}{2}$;

в) $2\,\mathrm{tg}^2 x + 3\,\mathrm{tg}\, x - 2 = 0$,

$\mathrm{tg}\, x = \dfrac{-3+5}{4} = \dfrac{1}{2}$, $x = \mathrm{arctg}\,\dfrac{1}{2} + \pi n$, $\mathrm{tg}\, x = -2$,

$x = -\mathrm{arctg}\, 2 + \pi n$;

г) $7\,\mathrm{ctg}^2 \dfrac{x}{2} + 2\,\mathrm{ctg}\,\dfrac{x}{2} = 5$,

$7\,\mathrm{ctg}^2 \dfrac{x}{2} + 2\,\mathrm{ctg}\,\dfrac{x}{2} - 5 = 0$, $\mathrm{ctg}\,\dfrac{x}{2} = \dfrac{-1-6}{7} = -1$,

$\dfrac{x}{2} = -\dfrac{\pi}{4} + \pi n$, $x = -\dfrac{\pi}{2} + 2\pi n$,

$\mathrm{ctg}\,\dfrac{x}{2} = \dfrac{-1+6}{7} = \dfrac{5}{7}$, $\dfrac{x}{2} = \mathrm{arcctg}\,\dfrac{5}{7} + \pi n$,

$x = 2\,\mathrm{arcctg}\,\dfrac{5}{7} + 2\pi n$.

18.10. а) $\sin x + \sqrt{3}\cos x = 0$,

$\mathrm{tg}\, x = -\sqrt{3}$, $\cos x \neq 0$, $x = -\dfrac{\pi}{3} + \pi n$;

б) $\sin x + \cos x = 0$,

$\mathrm{tg}\, x = -1$; $\cos x \neq 0$, $x = -\dfrac{\pi}{4} + \pi n$;

в) $\sin x - 3\cos x = 0$,
$\mathrm{tg}\, x = 3$, $\cos x \neq 0$, $x = \mathrm{arctg}\, 3 + \pi n$;
г) $\sqrt{3}\sin x + \cos x = 0$,

$\mathrm{tg}\, x = -\dfrac{\sqrt{3}}{3}$, $\cos x \neq 0$, $x = -\dfrac{\pi}{6} + \pi n$.

18.11. а) $\sin^2 x + \sin x \cos x = 0$,
$\sin x(\sin x + \cos x) = 0$, $\sin x = 0$, $x = \pi n$,

$\sin x + \cos x = 0$, $x = -\dfrac{\pi}{4} + \pi n$;

б) $\sqrt{3}\ \sin x \cos x + \cos^2 x = 0$,

$\cos x\,(\sqrt{3}\ \sin x + \cos x) = 0$, $\cos x = 0$,

$x = \frac{\pi}{2} + \pi n$, $\sqrt{3}\ \sin x + \cos x = 0$, $x = -\frac{\pi}{6} + \pi n$;

в) $\sin^2 x = 3 \sin x \cos x$,
$\sin x\,(\sin x - 3 \cos x) = 0$, $\sin x = 0$, $x = \pi n$,
$\sin x - 3 \cos x = 0$, $x = \operatorname{arctg} 3 + \pi n$;

г) $\sqrt{3}\ \cos^2 x = \sin x \cos x$,

$\cos x\,(\sqrt{3}\ \cos x - \sin x) = 0$, $\cos x = 0$,

$x = \frac{\pi}{2} + \pi n$, $\sqrt{3}\ \cos x - \sin x = 0$, $\operatorname{tg} x = \sqrt{3}$, $x = \frac{\pi}{3} + \pi n$.

18.12. а) $\sin^2 x + 2 \sin x \cos x - 3 \cos^2 x = 0$,
$\operatorname{tg}^2 x + 2 \operatorname{tg} x - 3 = 0, \cos x \neq 0, \operatorname{tg} x = -3$,

$x = -\operatorname{arctg} 3 + \pi n$, $\operatorname{tg} x = 1$, $x = \frac{\pi}{4} + \pi n$;

б) $\sin^2 x - 4 \sin x \cos x + 3 \cos^2 x = 0$,
$\operatorname{tg}^2 x - 4 \operatorname{tg} x + 3 = 0$, $\cos x \neq 0$; $\operatorname{tg} x = 3$,

$x = \operatorname{arctg} 3 + \pi n$, $\operatorname{tg} x = 1$, $x = \frac{\pi}{4} + \pi n$;

в) $\sin^2 x + \sin x \cos x - 2 \cos^2 x = 0$,
$\cos x \neq 0$, $\operatorname{tg}^2 x + \operatorname{tg} x - 2 = 0$,

$\operatorname{tg} x = \frac{-1 \pm 3}{2}$, $\operatorname{tg} x = 1$; $x = \frac{\pi}{4} + \pi n$, $\operatorname{tg} x = -2$, $x = -\operatorname{arctg} 2 + \pi n$;

г) $3\sin^2 x + \sin x \cos x - 2\cos^2 x = 0$,
$3\operatorname{tg}^2 x + \operatorname{tg} x - 2 = 0$, $\cos x \neq 0$,

$\operatorname{tg} x = \frac{-1 \pm \sqrt{1 + 4 \cdot 3 \cdot 2}}{6} = \frac{-1 \pm 5}{6}$,

$\operatorname{tg} x = -1$, $x = -\frac{\pi}{4} + \pi n$, $\operatorname{tg} x = \frac{2}{3}$, $x = \operatorname{arctg} \frac{2}{3} + \pi n$.

18.13. а) $(\sin x - \frac{1}{2})(\sin x + 1) = 0$,

$\sin x = \frac{1}{2}$, $x = (-1)^k \frac{\pi}{6} + \pi k$,

$\sin x = -1$, $x = -\frac{\pi}{2} + 2\pi n$, т.к. $x \in [0, 2\pi] \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$;

б) $(\cos x + \frac{1}{2})(\cos x - 1) = 0$, $\cos x = -\frac{1}{2}$, $x = \pm\frac{2\pi}{3} + 2\pi n$, $\cos x = 1$,

$x = 2\pi n$, т.к. $x \in [0, 2\pi] \Rightarrow x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$;

в) $(\cos x - \frac{\sqrt{2}}{2})(\sin x + \frac{\sqrt{2}}{2}) = 0,$

$\cos x = \frac{\sqrt{2}}{2}, \quad x = \pm\frac{\pi}{4} + 2\pi n.$

$\sin x = -\frac{\sqrt{2}}{2}, \quad x = (-1)^{n+1}\frac{\pi}{4} + \pi n,$ т.к. $x \in [0, 2\pi] \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4};$

г) $(1 + \cos x)\cdot(\sqrt{2}\sin x - 1) = 0,$

$\cos x = -1, \quad x = \pi + 2\pi n.$

$\sin x = \frac{\sqrt{2}}{2}, \quad x = (-1)^n\frac{\pi}{4} + \pi n,$ т.к. $x \in [0, 2\pi] \Rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \pi.$

18.14. а) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

б) $-\frac{4\pi}{3}, -\frac{2\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}$

18.15. а) $\sin 3x = \frac{\sqrt{2}}{2}, \quad x \in [0; 2\pi];$

$3x = (-1)^k\frac{\pi}{4} + \pi k; \; x = (-1)^k\frac{\pi}{12} + \frac{\pi k}{3}; x = \frac{\pi}{3}, \frac{\pi}{4}, \frac{1}{2}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12};$

б) $\cos 3x = \frac{\sqrt{3}}{2}, \; x \in [-\pi; \pi];$

$3x = \pm\frac{\sqrt{2}}{2} + 2\pi n; \; x = \pm\frac{\pi}{18} + \frac{2\pi n}{3}, x = -\frac{13\pi}{18};$

$-\frac{11\pi}{18}; -\frac{5\pi}{6}; \frac{\pi}{18}; \frac{5\pi}{6}; \frac{13\pi}{18};$

в) $\operatorname{tg}\frac{x}{2} = \frac{\sqrt{3}}{3}, \quad x \in [-3\pi; 3\pi];$

$\frac{x}{2} = \frac{\pi}{6} + \pi n; x = \frac{\pi}{3} + 2\pi n, \; x = -\frac{5\pi}{3}; \frac{\pi}{3}; \frac{7\pi}{3};$

г) $\operatorname{ctg} 4x = -1, \; x \in [0; \pi];$

$4x = -\frac{\pi}{4} + \pi n, \; x = -\frac{\pi}{16} + \frac{\pi n}{4}; x = \frac{3\pi}{16}; \frac{7\pi}{16}; \frac{11\pi}{16}; \frac{15\pi}{16}.$

18.16. а) $\sin 3x = -\frac{1}{2}, \; x \in [-4; 4];$

$3x = (-1)^{k+1}\frac{\pi}{6} + \pi k; x = (-1)^{k+1}\frac{\pi}{18} + \frac{\pi k}{3}; \quad x = -\frac{\pi}{6}; -\frac{5\pi}{6}; \frac{7\pi}{6}.$

б) $\cos x = 1, \; x \in [-6; 16]; \quad x = 2\pi n, \quad x = 0; 2\pi; 4\pi.$

18.17. а) $\sin\frac{x}{2}=0,\ x\in[-12;18]$,

$\frac{x}{2}=\pi n,\quad x=-2\pi;\ 0;\ 2\pi;\ 4\pi;$

б) $\cos 3x=\frac{-\sqrt{2}}{2},\ x\in[1;7]$,

$3x=\pm\frac{3\pi}{4}+2\pi n,\quad x=\pm\frac{\pi}{4}+\frac{2\pi n}{3},$

$x=\frac{11\pi}{12};\ \frac{19\pi}{12};\ \frac{5}{12}\pi;\ \frac{13}{12}\pi;\ \frac{7}{4}\pi.$

18.18. $\sin(2x-\frac{\pi}{4})=-1,\quad 2x-\frac{\pi}{4}=-\frac{\pi}{2}+2\pi n,\quad x=-\frac{\pi}{8}+\pi n.$

а) $x=\frac{7\pi}{8}$; б) $-\frac{\pi}{8};\ \frac{7\pi}{8}$; в) $-\frac{\pi}{8}$; г) $-\frac{\pi}{8}$.

18.19. $\cos(\frac{\pi}{3}-2x)=\frac{1}{2}$,

$2x-\frac{\pi}{3}=\pm\frac{\pi}{3}+2\pi n,\quad x=\frac{\pi}{3}+\pi n,\quad x=\pi n.$

а) $\frac{\pi}{3}$; б) $0;\ \frac{\pi}{3};\ \pi;\ \frac{4\pi}{3}$; в) $-\frac{2\pi}{3}$; г) $-\frac{2\pi}{3};\ 0;\ \frac{\pi}{3}$.

18.20. а) $\sin^2\frac{3x}{4}-\frac{\sqrt{2}}{2}=\sin x-\cos^2\frac{3x}{4}+1$,

$1-\frac{\sqrt{2}}{2}-1=\sin x,\ \sin x=-\frac{\sqrt{2}}{2},\quad x=(-1)^{k+1}\frac{\pi}{4}+\pi k;$

б) $\cos^2 2x-1-\cos x=\frac{\sqrt{3}}{2}-\sin^2 2x$,

$\cos x=-\frac{\sqrt{3}}{2},\quad x=\pm\frac{5\pi}{6}+2\pi n.$

18.21. а) $\mathrm{tg}\,x-2\,\mathrm{ctg}\,x+1=0$,
$\mathrm{tg}^2x+\mathrm{tg}\,x-2=0,\ \mathrm{tg}\,x=-2$,

$x=-\mathrm{arctg}\,2+\pi n,\quad \mathrm{tg}\,x=1,\quad x=\frac{\pi}{4}+\pi n;$

б) $\frac{\mathrm{tg}\,x+5}{2}=\frac{1}{\cos^2 x}$,

$2\,\mathrm{tg}^2x-\mathrm{tg}\,x-3=0,\ \mathrm{tg}\,x=\frac{1+5}{4}=\frac{3}{2},\ x=\mathrm{arctg}\,\frac{3}{2}+\pi k,$

$\mathrm{tg}\,x=-1;\ x=-\frac{\pi}{4}+\pi k;$

в) $2\ \mathrm{ctg}\, x - 3\ \mathrm{tg}\, x + 5 = 0$,

$2\mathrm{ctg}^2 x + 5\ \mathrm{ctg}\, x - 3 = 0$, $\mathrm{ctg}\, x = \dfrac{-5+7}{4} = \dfrac{1}{2}$,

$x = \mathrm{arcctg}\, \dfrac{1}{2} + \pi n$, $\mathrm{ctg}\, x = -3$, $x = -\mathrm{arcctg}\, 3 + \pi n$;

г) $\dfrac{7 - \mathrm{ctg}\, x}{4} = \dfrac{1}{\sin^2 x}$,

$7 - \mathrm{ctg}\, x = 4\ \mathrm{ctg}^2 x + 4$, $4\ \mathrm{ctg}^2 x + \mathrm{ctg}\, x - 3 = 0$,

$\mathrm{ctg}\, x = \dfrac{-1+7}{8} = \dfrac{3}{4}$,

$x = \mathrm{arcctg}\, \dfrac{3}{4} + \pi n$, $\mathrm{ctg}\, x = -1$, $x = -\dfrac{\pi}{4} + \pi n$.

18.22. а) $2\cos^2 \dfrac{x}{2} + \sqrt{3} \cos \dfrac{x}{2} = 0$,

$\cos \dfrac{x}{2} (2 \cos \dfrac{x}{2} + \sqrt{3}) = 0$, $\cos \dfrac{x}{2} = 0$, $\dfrac{x}{2} = \dfrac{\pi}{2} + \pi k$, $x = \pi + 2\pi k$,

$\cos = \dfrac{\sqrt{3}}{2}$, $\dfrac{x}{2} = \pm \dfrac{5\pi}{6} + 2\pi n$, $x = \pm \dfrac{5}{3}\pi + 4\pi n$;

б) $4 \cos^2 (x - \dfrac{\pi}{6}) - 3 = 0$,

$\cos (x - \dfrac{\pi}{6}) = \pm \dfrac{\sqrt{3}}{2}$, $x - \dfrac{\pi}{6} = \pm \dfrac{\pi}{6} + 2\pi n$, $x = \dfrac{\pi}{3} + 2\pi n$, $x = 2\pi n$,

$x - \dfrac{\pi}{6} = \pm \dfrac{5\pi}{6} + 2\pi n$, $x = \pi + 2\pi n$, $x = -\dfrac{2}{3}\pi + 2\pi n$, => $x = \dfrac{\pi}{3} + \pi n$, $x = \pi n$;

в) $\sqrt{3}\ \mathrm{tg}^2 3x - 3\ \mathrm{tg}\, 3x = 0$,

$\mathrm{tg}\, 3x (\sqrt{3}\ \mathrm{tg}\, 3x - 3) = 0$, $\mathrm{tg}\, 3x = 0$,

$3x = \pi n$, $x = \dfrac{\pi n}{3}$, $\mathrm{tg}\, 3x = \sqrt{3}$, $3x = \dfrac{\pi}{3} + \pi n$, $x = \dfrac{\pi}{9} + \dfrac{\pi n}{3}$;

г) $4 \sin^2 (2x + \dfrac{\pi}{3}) - 1 = 0$,

$\sin (2x + \dfrac{\pi}{3}) = \pm \dfrac{1}{2}$, $2x + \dfrac{\pi}{3} = (-1)^n \dfrac{\pi}{6} + \pi n$,

$x = (-1)^n \dfrac{\pi}{12} - \dfrac{\pi}{6} + \dfrac{\pi n}{2}$,

$2x + \dfrac{\pi}{3} = (-1)^{n+1} \dfrac{\pi}{6} + \pi n$, $x = (-1)^{n+1} \dfrac{\pi}{12} - \dfrac{\pi}{6} + \dfrac{\pi n}{2}$ => $x = -\dfrac{\pi}{12} + \dfrac{\pi k}{2}$;

$x = -\dfrac{\pi}{4} + \dfrac{\pi n}{2}$.

18.23. а) $\sin^2 x - \frac{12-\sqrt{2}}{2} \cdot \sin x - 3\sqrt{2} = 0,$

$\sin x = 6$, не существует.

$\sin x = -\frac{\sqrt{2}}{2}, \quad x = (-1)^{k+1}\frac{\pi}{4} + \pi k;$

б) $\cos^2 x - \frac{8-\sqrt{3}}{2}\cos x - 2\sqrt{3} = 0,$

$\cos x = 4$, не существует.

$\cos x = -\frac{\sqrt{2}}{2}, \quad x = \pm\frac{5\pi}{6} + 2\pi n.$

18.24. а) $\sin 2x = \cos 2x, \quad \mathrm{tg}\, 2x = 1, \quad \cos 2x \neq 0, \quad 2x = \frac{\pi}{4} + \pi n, \; x = \frac{\pi}{8} + \frac{\pi n}{2};$

б) $\sqrt{3}\sin 3x = \cos 3x, \quad \mathrm{ctg}\, 3x = \sqrt{3}, \quad \sin 3x \neq 0, \quad 3x = \frac{\pi}{6} + \pi n, \quad x = \frac{\pi}{18} + \frac{\pi n}{3};$

в) $\sin\frac{x}{2} = \sqrt{3}\cos\frac{x}{2}, \quad \mathrm{tg}\frac{x}{2} = \sqrt{3}, \quad \cos\frac{x}{2} \neq 0, \quad \frac{x}{2} = \frac{\pi}{3} + \pi n, \quad x = \frac{2}{3}\pi + 2\pi n;$

г) $\sqrt{2}\sin 17x = \sqrt{6}\cos 17x, \; \mathrm{tg}\, 17x = \sqrt{3}, \; \cos 17x \neq 0, \; 17x = \frac{\pi}{3} + \pi n, \; x = \frac{\pi}{51} + \frac{\pi n}{17}.$

18.25. а) $2\sin^2 2x - 5\sin 2x\cos 2x + 2\cos^2 2x = 0,$
$2\,\mathrm{tg}^2\, 2x - 5\,\mathrm{tg}\, 2x + 2 = 0, \cos 2x \neq 0,$

$\mathrm{tg}\, 2x = \frac{5 \pm 3}{4}, \; \mathrm{tg}\, 2x = 2;$

$x = \frac{1}{2}\mathrm{arctg}\, 2 + \frac{\pi n}{2}, \quad \mathrm{tg}\, 2x = \frac{1}{2}, \quad x = \frac{1}{2}\mathrm{arctg}\frac{1}{2} + \frac{\pi n}{2};$

б) $3\sin^2 3x + 10\sin 3x\cos 3x + 3\cos^2 3x = 0,$

$3\,\mathrm{tg}^2\, 3x + 10\,\mathrm{tg}\, 3x + 3 = 0, \quad \mathrm{tg}\, 3x = \frac{-10 \pm \sqrt{100 - 4\cdot 3\cdot 3}}{6} = \frac{-10 \pm 8}{6},$

$\mathrm{tg}\, 3x = -3; \; x = \frac{1}{3}\mathrm{arctg}(-3) + \frac{1}{3}\pi n, \quad \mathrm{tg}\, 3x = -\frac{1}{3}; \; x = \frac{1}{3}\mathrm{arctg}\left(-\frac{1}{3}\right) + \frac{1}{3}\pi n.$

18.26. а) $\sin^2\frac{x}{2} = 3\cos^2\frac{x}{2},$

$\cos^2\frac{x}{2} = \frac{1}{4}, \quad \cos\frac{x}{2} = \pm\frac{1}{2}, \; \frac{x}{2} = \pm\frac{\pi}{3} + \pi k; \; x = \pm\frac{2\pi}{3} + 2\pi k;$

б) $\sin^2 4x = \cos^2 4x,$
$\mathrm{tg}^2\, 4x = 1, \cos 4x \neq 0, \quad \mathrm{tg}\, 4x = \pm 1,$

$4x = \frac{\pi}{4} + \pi n, \quad x = \frac{\pi}{16} + \frac{\pi n}{4}, \quad 4x = -\frac{\pi}{4} + \pi n, \quad x = -\frac{\pi}{16} + \frac{\pi n}{4} \Rightarrow$

$x = \pm\frac{\pi}{16} + \frac{\pi n}{4}.$

18.27. а) $5\sin^2 x - 14\sin x\cos x - 3\cos^2 x = 2$,
$3\sin^2 x - 14\sin x\cos x - 5\cos^2 x = 0$, $3\text{tg}^2 x - 14\text{tg}\, x - 5 = 0$, $\cos x \neq 0$,

$\text{tg}\, x = \dfrac{7+8}{3} = 5$, $x = \text{arctg}\, 5 + \pi k$, $\text{tg}\, x = -\dfrac{1}{3}$, $x = -\text{arctg}\,\dfrac{1}{3} + \pi k$;

б) $3\sin^2 x - \sin x\cos x = 2$,
$\sin^2 x - \sin x\cos x - 2\cos^2 x = 0$, $\text{tg}^2 x - \text{tg}\, x - 2 = 0$,

$\cos x \neq 0$, $\text{tg}\, x = 2$, $x = \text{arctg}\, 2 + \pi n$, $\text{tg}\, x = -1$; $x = -\dfrac{\pi}{4} + \pi n$.

в) $2\cos^2 x - \sin x\cos x + 5\sin^2 x = 3$,
$2\sin^2 x - \sin x\cos x - \cos^2 x = 0$,

$2\text{tg}^2 x - \text{tg}\, x - 1 = 0$, $\cos x \neq 0$, $\text{tg}\, x = 1$, $x = \dfrac{\pi}{4} + \pi k$,

$\text{tg}\, x = -\dfrac{1}{2}$, $x = -\text{arctg}\dfrac{1}{2} + \pi k$;

г) $4\sin^2 x - 2\sin x\cos x = 3$,
$\sin^2 x - 2\sin x\cos x - 3\cos^2 x = 0$,

$\text{tg}^2 x - 2\,\text{tg}\, x - 3 = 0$, $\cos x \neq 0$, $\text{tg}\, x = -1$, $x = -\dfrac{\pi}{4} + \pi k$,

$\text{tg}\, x = 3$, $x = \text{arctg}\, 3 + \pi k$.

18.28. а) $\sqrt{3}\sin x\cos x + \cos^2 x = 0$,
$\cos x(\sqrt{3}\sin x + \cos x) = 0$, $\cos x = 0$,

$x = \dfrac{\pi}{2} + \pi n$, $\sqrt{3}\sin x + \cos x = 0$, $\sqrt{3}\,\text{tg}\, x = -1$, $\text{tg}\, x = -\dfrac{\sqrt{3}}{3}$, $x = -\dfrac{\pi}{6} + \pi n$;

б) $2\sin^2 x - 3\sin x\cos x + 4\cos^2 x = 4$,
$-3\sin x\cos x = 2 - 2\cos^2 x$,
$-3\sin x\cos x = 2\sin^2 x$, $\sin x(3\cos x + 2\sin x) = 0$, $\sin x = 0$, $x = \pi n$,

$3\cos x + 2\sin x = 0$, $\text{tg}\, x = -\dfrac{3}{2}$, $\cos x \neq 0$, $x = -\text{arctg}\,\dfrac{3}{2} + \pi n$.

18.29. а) $3\sin^2 2x - 2 = \sin 2x\cos 2x$,
$\sin^2 2x - \sin 2x\cos 2x - 2\cos^2 2x = 0$,
$\text{tg}^2 2x - \text{tg}\, 2x - 2 = 0$, $\cos^2 2x \neq 0$, $\text{tg}\, 2x = 2$, $2x = \text{arctg}\, 2 + \pi n$,

$x = \dfrac{1}{2}\text{arctg}\, 2 + \dfrac{\pi n}{2}$, $\text{tg}\, 2x = -1$, $2x = -\dfrac{\pi}{4} + \pi n$, $x = -\dfrac{\pi}{8} + \dfrac{\pi n}{2}$;

б) $2\sin^2 4x - 4 = 3\sin 4x\cos 4x - 4\cos^2 4x$,
$2 + 2\cos^2 4x - 4 = 3\sin 4x\cos 4x$,
$2\sin^2 4x + 3\sin 4x\cos 4x = 0$, $\sin 4x(2\sin 4x + 3\cos 4x) = 0$, $\sin 4x = 0$,

$4x = \pi n$, $x = \dfrac{\pi n}{4}$, $2\sin 4x + 3\cos 4x = 0$, $2\text{tg}\, 4x = -3$, $\cos 4x \neq 0$,

$x = -\dfrac{1}{4}\text{arctg}\,\dfrac{3}{2} + \dfrac{\pi n}{4}$.

18.30. а) $4\sin^2\frac{x}{2}-3=2\sin\frac{x}{2}\cos\frac{x}{2}$,

$\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}-3\cos^2\frac{x}{2}=0$, $\operatorname{tg}^2\frac{x}{2}-2\operatorname{tg}\frac{x}{2}-3=0$; $\cos\frac{x}{2}\neq 0$.

$\operatorname{tg}\frac{x}{2}=-1$, $x=-\frac{\pi}{2}+2\pi n$, $\operatorname{tg}\frac{x}{2}=3$, $x=2\operatorname{arctg}3+2\pi n$;

б) $3\sin^2\frac{x}{3}+4\cos^2\frac{x}{3}=3+\sqrt{3}\sin\frac{x}{3}\cos\frac{x}{3}$,

$\cos^2\frac{x}{3}-\sqrt{3}\sin\frac{x}{3}\cos\frac{x}{3}=0$,

$\cos\frac{x}{3}(\cos\frac{x}{3}-\sqrt{3}\sin\frac{x}{3})=0$, $\cos\frac{x}{3}=0$, $\frac{x}{3}=\frac{\pi}{2}+\pi n$, $x=\frac{3\pi}{2}+3\pi n$,

$\cos\frac{x}{3}-\sqrt{3}\sin\frac{x}{3}=0$, $\operatorname{ctg}\frac{x}{3}=\sqrt{3}$, $\sin\frac{x}{3}\neq 0$,

$\frac{x}{3}=\frac{\pi}{6}+\pi n$, $x=\frac{\pi}{2}+3\pi n$.

18.31. а) $\sin(\frac{\pi}{2}+2x)+\cos(\frac{\pi}{2}-2x)=0$,

$\cos 2x+\sin 2x=0$,

$\operatorname{tg}2x=-1$, $\cos 2x\neq 0$, $2x=-\frac{\pi}{4}+\pi n$, $x=-\frac{\pi}{8}+\frac{\pi n}{2}$;

б) $2\sin(\pi-3x)+\cos(2\pi-3x)=0$,
$2\sin 3x+\cos 3x=0$,

$\operatorname{tg}3x=-\frac{1}{2}$, $\cos 3x\neq 0$, $3x=-\operatorname{arctg}\frac{1}{2}+\pi n$, $x=-\frac{1}{3}\operatorname{arctg}\frac{1}{2}+\frac{\pi n}{3}$.

18.32. а) $\cos(\frac{\pi}{2}-\frac{x}{2})-3\cos(\pi-\frac{x}{2})=0$,

$\sin\frac{x}{2}+3\cos\frac{x}{2}=0$,

$\operatorname{tg}\frac{x}{2}=-3$, $\cos\frac{x}{2}\neq 0$, $\frac{x}{2}=-\operatorname{arctg}3x+\pi n$, $x=-2\operatorname{arctg}3+2\pi n$;

б) $\sqrt{3}\sin(\pi-\frac{x}{2})+3\sin(\frac{\pi}{2}-\frac{x}{3})=0$,

$\sqrt{3}\sin\frac{x}{3}+3\cos\frac{x}{3}=0$, $\operatorname{tg}\frac{x}{3}=-\sqrt{3}$, $\cos\frac{x}{3}\neq 0$, $x=-\pi+3\pi n$.

18.33. а) $\sqrt{16-x^2}\sin x=0$,
$|x|\leq 4$, $x=4$, $x=-4$, $\sin x=0$, $x=\pi n$, $n=0,\pm 1$.
Ответ: $x=0, \pm 4; \pm\pi$.

б) $\sqrt{7x-x^2}(2\cos x-1)=0$,

$0 \le x \le 7$, $7x-x^2=0$, $x=0, x=7$,

$2\cos x-1=0$, $\cos x=\frac{1}{2}$, $x=\frac{\pi}{3}$, $x=\frac{5\pi}{3}$.

Ответ: $x=0;\frac{\pi}{3};\frac{5\pi}{3};7$.

18.34. а) $(\sqrt{2}\cos x-1)\cdot\sqrt{4x^2-7x+3}=0$, $4x^2-7x+3\ge 0$,

$x\ge\frac{7+1}{8}=1$, $x\le\frac{3}{4}$, $\sqrt{2}\cos x-1=0$, $x=\pm\frac{\pi}{4}+2\pi n$.

Ответ: $x=1, x=\frac{3}{4}, x=-\frac{\pi}{4}$; $x=\pm\frac{\pi}{4}+2\pi n$. $n=\pm 1;\pm 2;\pm 3\ldots$

б) $(2\sin x-\sqrt{3})\sqrt{3x^2-7x+4}=0$;

$3x^2-7x+4\ge 0$; $x\le 1; x\ge\frac{4}{3}$

$2\sin x-\sqrt{3}=0$, $\sin x=\frac{\sqrt{3}}{2}$, $x=\frac{2\pi}{3}+2\pi k; x=\frac{\pi}{3}+2\pi k; k=\pm1,\pm2,\ldots$;

$3x^2-7x+4=0$ $x=1; x=\frac{4}{3}$.

Ответ: $1; \frac{4}{3};\frac{2\pi}{3}+2\pi k;\frac{\pi}{3}+2\pi k; k=\pm1;\pm2;\ldots$

18.35. а) $y=\cos 3x+\sqrt{\cos^2 3x-1}=\cos 3x+\sqrt{-\sin^2 3x}$,

$\sin^2 3x\ge 0$, $\sin 3x=0, x=\frac{\pi n}{3}$.

Область значений функции: $\{-1;1\}$.

б) $y=\sin 2x+\sqrt{\sin^2 4x-1}=\sin 2x+\sqrt{-\cos^2 4x}$,

$\cos^2 4x\ge 0$, $\cos 4x=0$,

$x=\frac{\pi}{8}+\frac{\pi n}{4}$, $2x=\frac{\pi}{4}+\frac{\pi n}{2}$.

Область значений функции: $\{\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\}$.

Глава 4. Преобразование тригонометрических выражений

§ 19. Синус и косинус суммы и разности аргументов

19.1. а) $\sin 105° = \sin(60° + 45°) = \sin 60° \cos 45° + \sin 45° \cos 60° =$
$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$;

б) $\cos 105° = \cos(60° + 45°) = \frac{1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{2}-\sqrt{6}}{4}$.

19.2. а) $\sin(\alpha + \beta) - \sin\alpha\cos\beta = \sin\alpha\cos\beta + \sin\beta\cos\alpha - \sin\alpha\cos\beta =$
$= \sin\beta\cos\alpha$.

б) $\sin(\frac{\pi}{3} + \alpha) - \frac{1}{2}\sin\alpha = \sin\frac{\pi}{3}\cos\alpha + \cos\frac{\pi}{3}\sin\alpha - \frac{1}{2}\sin\alpha =$
$= \frac{\sqrt{3}}{2}\cos\alpha$.

в) $\sin\alpha\sin\beta + \cos(\alpha + \beta) = \sin\alpha\sin\beta + \cos\alpha\cos\beta - \sin\alpha\sin\beta = \cos\alpha\cos\beta$.

г) $\cos(\alpha + \frac{\pi}{4}) + \frac{\sqrt{2}}{2}\sin\alpha = \frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{2}}{2}\sin\alpha + \frac{\sqrt{2}}{2}\sin\alpha = \frac{\sqrt{2}}{2}\cos\alpha$.

19.3. а) $\sin(\frac{5\pi}{6} - \alpha) - \frac{1}{2}\cos\alpha = \sin\frac{5\pi}{6}\cos\alpha - \sin\alpha\cos\frac{5\pi}{6} - \frac{1}{2}\cos\alpha =$
$= \frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha - \frac{1}{2}\cos\alpha = \frac{\sqrt{3}}{2}\sin\alpha$.

б) $\sqrt{3}\cos\alpha - 2\cos(\alpha - \frac{\pi}{6}) = \sqrt{3}\cos\alpha - 2\cos\alpha\cos\frac{\pi}{6} - 2\sin\alpha\sin\frac{\pi}{6} = -\sin\alpha$.

в) $\frac{\sqrt{3}}{2}\sin\alpha + \cos(\alpha - \frac{5\pi}{3}) = \frac{\sqrt{3}}{2}\sin\alpha + \cos\alpha\cos\frac{5\pi}{3} + \sin\alpha\sin\frac{5\pi}{3} = \frac{1}{2}\cos\alpha$.

г) $\sqrt{2}\sin(\alpha - \frac{\pi}{4}) - \sin\alpha = \sqrt{2}\sin\alpha\cos\frac{\pi}{4} - \sqrt{2}\cos\alpha\sin\frac{\pi}{4} - \sin\alpha = -\cos\alpha$.

19.4. а) $\cos(\alpha - \beta) - \cos\alpha\cos\beta = \cos\alpha\cos\beta + \sin\alpha\sin\beta - \cos\alpha\cos\beta =$
$= \sin\alpha\sin\beta$

б) $\sin(\alpha + \beta) + \sin(\alpha - \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha + \sin\alpha\cos\beta - \sin\beta\cos\alpha =$
$= 2\sin\alpha\cos\beta$

в) $\sin\alpha\cos\beta - \sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\alpha\cos\beta + \sin\beta\cos\alpha =$
$= \sin\beta\cos\alpha$

г) $\cos(\alpha - \beta) - \cos(\alpha + \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta - \cos\alpha\cos\beta + \sin\alpha\sin\beta =$
$= 2\sin\alpha\sin\beta$.

19.5. а) $\sin(\alpha + \beta) + \sin(-\alpha)\cos(-\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha - \sin\alpha\cos\beta =$
$= \sin\beta\cos\alpha$, т.е. тождество верно.

б) $\cos(\alpha + \beta) + \sin(-\alpha)\sin(-\beta) = \cos\alpha\cos\beta$.

$\cos(\alpha+\beta)+\sin(-\alpha)\sin(-\beta)=\cos\alpha\cos\beta - \sin\alpha\sin\beta + \sin\alpha\sin\beta =$
$= \cos\alpha\cos\beta$

19.6.а) $\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x = \sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\sin x = \sin\left(\frac{\pi}{3} - x\right)$

б) $\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x = \cos\left(\frac{\pi}{3} - x\right)$

19.7. а) $\cos(\alpha - \beta) + \sin(-\alpha)\sin\beta = \cos\alpha\cos\beta$,
$\cos\alpha\cos\beta + \sin\alpha\sin\beta - \sin\alpha\sin\beta = \cos\alpha\cos\beta$;

б) $\sin(30° - \alpha) + \sin(30° + \alpha) = \cos\alpha$,

$\frac{1}{2}\cos\alpha - \frac{\sqrt{3}}{2}\sin\alpha + \frac{1}{2}\cos\alpha + \frac{\sqrt{3}}{2}\sin\alpha = \cos\alpha$.

19.8. а) $\sin 5x\cos 3x + \cos 5x\sin 3x = \sin 8x$, $\sin(5x + 3x) = \sin 8x$;

б) $\cos 5x\cos 3x - \sin 5x\sin 3x = \cos 8x$, $\cos(5x + 3x) = \cos 8x$;

19.9. а) $\sin 7x\cos 4x - \cos 7x\sin 4x = \sin(7x - 4x) = \sin 3x$

б) $\cos 2x\cos 12x + \sin 2x\sin 12x = \cos(2x - 12x) = \cos 10x$

19.10. а) $\cos 107°\cos 17° + \sin 107°\sin 17° = \cos(107° - 17°) = \cos 90° = 0$.

б) $\cos 36°\cos 24° - \sin 36°\sin 24° = \cos(36° + 24°) = \cos 60° = \frac{1}{2}$.

в) $\sin 63°\cos 27° + \cos 63°\sin 27° = \sin(63° + 27°) = \sin 90° = 1$.

г) $\sin 51°\cos 21° - \cos 51°\sin 21° = \sin(51° - 21°) = \frac{1}{2}$.

19.11. а) $\cos\frac{5\pi}{8}\cos\frac{3\pi}{8} + \sin\frac{5\pi}{8}\sin\frac{3\pi}{8} = \cos(\frac{5\pi}{8} - \frac{3\pi}{8}) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

б) $\sin\frac{2\pi}{15}\cos\frac{\pi}{5} + \cos\frac{2\pi}{15}\sin\frac{\pi}{5} = \sin(\frac{2\pi}{15} + \frac{\pi}{5}) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$.

в) $\cos\frac{\pi}{12}\cos\frac{\pi}{4} - \sin\frac{\pi}{12}\sin\frac{\pi}{4} = \cos(\frac{\pi}{12} + \frac{\pi}{4}) = \cos\frac{\pi}{3} = \frac{1}{2}$.

г) $\sin\frac{\pi}{12}\cos\frac{\pi}{4} - \cos\frac{\pi}{12}\sin\frac{\pi}{4} = \sin(\frac{\pi}{12} - \frac{\pi}{4}) = -\sin\frac{\pi}{6} = -\frac{1}{2}$.

19.12. а) $\sin 2x\cos x + \cos 2x\sin x = 1$, $\sin 3x = 1$,

$3x = \frac{\pi}{2} + 2\pi n$, $x = \frac{\pi}{6} + \frac{2\pi n}{3}$;

б) $\cos 3x\cos 5x = \sin 3x\sin 5x$, $\cos 3x\cos 5x - \sin 3x\sin 5x = 0$,

$\cos 8x = 0$, $8x = \frac{\pi}{2} + \pi n$, $x = \frac{\pi}{16} + \frac{\pi n}{8}$.

19.13. а) $\sin 6x\cos x + \cos 6x\sin x = \frac{1}{2}$, $\sin 7x = \frac{1}{2}$,

$7x = (-1)^k \frac{\pi}{6} + \pi k$, $x = (-1)^k \frac{\pi}{42} + \frac{\pi k}{7}$;

б) $\cos 5x\cos 7x - \sin 5x\sin 7x = -\frac{\sqrt{3}}{2}$, $\cos 12x = -\frac{\sqrt{3}}{2}$,

$12x = \pm\frac{5\pi}{6} + 2\pi n$, $x = \pm\frac{5\pi}{72} + \frac{\pi n}{6}$.

19.14. а) $\cos 6x\cos 5x + \sin 6x\sin 5x = -1$,
$\cos(6x - 5x) = -1$, $\cos x = -1$, $x = \pi + 2\pi n$;

б) $\sin 3x\cos 5x - \sin 5x\cos 3x = \frac{1}{2}$,

$\sin(3x - 5x) = \frac{1}{2}$, $\sin 2x = -\frac{1}{2}$,

$2x = (-1)^{k+1}\frac{\pi}{6} + \pi k$, $x = (-1)^{k+1}\frac{\pi}{12} + \frac{\pi k}{2}$.

19.15. а) $\sin x\cos 45° + \cos x\sin 45° = \cos 17°\cos 13° - \sin 17°\cos 13°$

$\sin(x + 45°) = \cos 30° = \frac{\sqrt{3}}{2}$

$x + 45° = (-1)^n 60° + 180\cdot n, n\in Z$; $x = 15°$

б) $\cos x\cos 60° - \sin x\sin 60° = \sin 200°\cos 25° + \cos 200°\sin 25°$;

$\cos(x + 60°) = \sin(225°) = -\frac{\sqrt{2}}{2}$; $x = 75°$

19.16. а) $\sin 0{,}2x\cos 0{,}8x + \cos 0{,}2x\sin 0{,}8x = \cos 3x\cos 2x + \sin 3x\sin 2x$, $x\in[0, 3\pi]$
$\sin(0{,}2x + 0{,}8x) = \cos(3x - 2x)$, $\sin x = \cos x$

$x = \frac{\pi}{4} + \pi n$, $x = \frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}$;

б) $\cos 0{,}7x\cos 1{,}3x - \sin 0{,}7x\sin 1{,}3x = \sin 7x\cos 9x - \sin 9x\cos 7x$, $x \in [-\pi\ \pi]$
$\cos(0{,}7x + 1{,}3x) = \sin(7x - 9x)$, $\cos 2x = -\sin 2x$; $\mathrm{tg}\, 2x = -1$, $\cos 2x \neq 0$,

$x = -\frac{\pi}{8} + \frac{\pi n}{2}$, $x = -\frac{5\pi}{8}, -\frac{\pi}{8}, \frac{3\pi}{8}, \frac{7\pi}{8}$.

19.17. $\sin t = \frac{3}{5}$, $0 < t < \frac{\pi}{2}$, $\cos t = \frac{4}{5}$.

а) $\sin\left(\frac{\pi}{3} + t\right) = \sin\frac{\pi}{3}\cos t + \sin t\cos\frac{\pi}{3} = \frac{\sqrt{3}}{2}\cdot\frac{4}{5} + \frac{1}{2}\cdot\frac{3}{5} =$

$\frac{2\sqrt{3}}{5} + \frac{3}{10} = \frac{4\sqrt{3} + 3}{10}$.

б) $\cos(\frac{\pi}{2}+t)=-\sin t=-\frac{3}{5}$;

в) $\sin(\frac{\pi}{2}+t)=\cos t=\frac{4}{5}$;

г) $\cos(\frac{\pi}{3}+t)=\cos\frac{\pi}{3}\cos t-\sin\frac{\pi}{3}\sin t=\frac{1}{2}\cdot\frac{4}{5}-\frac{\sqrt{3}}{2}\cdot\frac{3}{5}=\frac{4-3\sqrt{3}}{10}$.

19.18. а) $\sin\left(t-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}\sin t-\frac{1}{2}\cos t=\frac{\sqrt{3}}{2}\sqrt{1-\cos^2 t}+\frac{5}{26}=\frac{5+12\sqrt{3}}{26}$

б) $\cos\left(t-\frac{3\pi}{2}\right)=-\sin t=-\sqrt{1-\cos^2 t}=-\frac{12}{13}$

в) $\cos\left(t-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}\cos t+\frac{1}{2}\sin t=-\frac{5\sqrt{3}}{26}+\frac{1}{2}\sqrt{1-\cos^2 t}=\frac{12-5\sqrt{3}}{26}$

г) $\sin\left(t-\frac{3\pi}{2}\right)=\cos t=-\frac{5}{13}$

19.19. а) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta=$

$=\frac{4}{5}\sqrt{1-\cos^2\alpha}-\frac{15}{17}\sqrt{1-\cos^2\beta}=\frac{4}{5}\cdot\frac{8}{17}-\frac{15}{17}\cdot\frac{3}{5}=-\frac{13}{85}$

б) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta=$

$=\frac{15}{17}\cdot\frac{4}{5}+\sqrt{1-\cos^2\alpha}\sqrt{1-\cos^2\beta}=\frac{60}{85}+\frac{8}{17}\cdot\frac{3}{5}=\frac{84}{85}$

19.20. а) $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta=$

$=-\frac{4}{5}\cdot\frac{15}{17}+\sqrt{1-\sin^2\alpha}\sqrt{1-\cos^2\alpha}=\frac{3}{5}\cdot\frac{8}{17}-\frac{4\cdot 15}{5\cdot 17}=-\frac{36}{85}$

б) $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta=$

$=\frac{15}{17}\sqrt{1-\sin^2\alpha}+\frac{4}{5}\sqrt{1-\cos^2\beta}=\frac{5}{17}\cdot\frac{3}{5}+\frac{4}{5}\cdot\frac{8}{17}=\frac{77}{85}$

19.21. а) Опечатка в условии:

$\sin 77°\cos 17°-\sin 13°\cos 73°=\sin 77°\cos 17°-\cos 77\sin 17°=$

$\sin(77°-17°)=\sin 60°=\frac{\sqrt{3}}{2}$;

б) $\cos 125°\cos 5°+\sin 55°\cos 85°=-\cos 55°\cos 5°+\sin 55°\cos 5°=$

$=-\cos 60°=-\frac{1}{2}$.

19.22. а) $\frac{\cos 105°\cdot\cos 5°+\sin 105°\cdot\cos 85°}{\sin 95°\cdot\cos 5°-\sin 95°\cdot\sin 185°}=$

$=\frac{\cos 105°\cdot\cos 5°+\sin 105°\cdot\sin 5°}{\sin 95°\cdot\cos 5°+\cos 95°\cdot\sin 5°}=\frac{\cos 100°}{\sin 100°}=-\mathrm{tg}\,70°$

б) $\frac{\sin 75°\cdot\cos 5° - \cos 75°\cdot\cos 85°}{\cos 375°\cdot\cos 5° - \sin 15°\cdot\sin 365°} =$

$= \frac{\sin 75°\cdot\cos 5° - \cos 75°\cdot\sin 5°}{\cos 15°\cdot\cos 5° - \sin 15°\cdot\sin 5°} = \frac{\sin 70°}{\cos 20°} = \frac{\cos 20°}{\cos 20°} = 1$

19.23. а) $\sqrt{2}\ \cos\left(\frac{\pi}{4} - x\right) - \cos x = \frac{1}{2}$.

$\cos x + \sin x - \cos x = \frac{1}{2}$,

$\sin x = \frac{1}{2},\ x = (-1)^k \frac{\pi}{6} + \pi k;$

б) $\sqrt{2}\ \sin\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sin\frac{x}{2} = \frac{\sqrt{3}}{2}$,

$\cos\frac{x}{2} - \sin\frac{x}{2} + \sin\frac{x}{2} = \frac{\sqrt{3}}{2}$,

$\cos\frac{x}{2} = \frac{\sqrt{3}}{2},\quad \frac{x}{2} = \pm\frac{\pi}{6} + 2\pi n,\quad x = \pm\frac{\pi}{3} + 4\pi n.$

19.24. а) $\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x = 1$,

$\sin\left(x - \frac{\pi}{4}\right) = 1$,

$x - \frac{\pi}{4} = \frac{\pi}{2} + 2\pi n,\ x = \frac{3\pi}{4} + 2\pi n;$

б) $\sin x - \cos x = 1$,

$\sin\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2},\ x - \frac{\pi}{4} = (-1)^n\frac{\pi}{4} + \pi n,$

$x = (-1)^n\frac{\pi}{4} + \frac{\pi}{4} + \pi n;$

в) $\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x = 1$,

$\cos\left(x - \frac{\pi}{6}\right) = 1,\quad x - \frac{\pi}{6} = 2\pi n,\ x = \frac{\pi}{6} + 2\pi n;$

г) $\sqrt{3}\ \cos x + \sin x = 1$,

$\cos\left(x - \frac{\pi}{6}\right) = \frac{1}{2},\ x - \frac{\pi}{6} = \pm\frac{\pi}{3} + 2\pi n;\ x = \pm\frac{\pi}{3} + \frac{\pi}{6} + 2\pi n.$

19.25. а) $\frac{\sqrt{2}}{2}\sin x + \frac{\sqrt{2}}{2}\cos x = 1$,

$\sin\frac{\pi}{4}\sin x+\frac{\sqrt{2}}{2}\cos x=1,$

$\cos(x-\frac{\pi}{4})=1,\quad x-\frac{\pi}{4}=2\pi n, x=\frac{\pi}{4}+2\pi n;$

б) $\sin x+\cos x=1,$

$\sin(x+\frac{\pi}{4})=\frac{1}{\sqrt{2}}, x+\frac{\pi}{4}=(-1)^k\frac{\pi}{4}+\pi n,$

$x=(-1)^k\frac{\pi}{4}-\frac{\pi}{4}+\pi n;$

в) $\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x=1,$

$\cos\frac{\pi}{6}\cos x-\sin\frac{\pi}{6}\sin x=1,\ \cos\left(x+\frac{\pi}{6}\right)=1,$

$x+\frac{\pi}{6}=2\pi n,\quad x=-\frac{\pi}{6}+2\pi n;$

г) $\sqrt{3}\cos x-\sin x=1,$

$\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\sin x=\frac{1}{2},\ \cos\left(x+\frac{\pi}{6}\right)=\frac{1}{2},$

$x+\frac{\pi}{6}=\pm\frac{\pi}{3}+2\pi n,\ x=\pm\frac{\pi}{3}-\frac{\pi}{6}+2\pi n.$

19.26. а) $\sin x\cos 3x+\cos x\sin 3x>\frac{1}{2},\quad \sin 4x>\frac{1}{2},$

$4x\in(\frac{\pi}{6}+2\pi n;\ \frac{5\pi}{6}+2\pi n), x\in(\frac{\pi}{24}+\frac{\pi n}{2};\ \frac{5\pi}{24}+\frac{\pi n}{2});$

б) $\cos 2x\cos 5x-\sin 2x\sin 5x<-\frac{1}{3},\quad \cos 7x<-\frac{1}{3},$

$7x\in(\pi-\arccos\frac{1}{3}+2\pi n;\ \pi+\arccos\frac{1}{3}+2\pi n),$

$x\in(\frac{\pi}{7}-\frac{1}{7}\arccos\frac{1}{3}+\frac{2\pi n}{7};\ \frac{\pi}{7}+\frac{1}{7}\arccos\frac{1}{3}+\frac{2\pi n}{7});$

в) $\sin\frac{x}{4}\cos\frac{x}{2}-\cos\frac{x}{4}\sin\frac{x}{2}<\frac{1}{3},$

$\sin(\frac{x}{4}-\frac{x}{2})<\frac{1}{3},\quad \sin\frac{x}{4}>-\frac{1}{3},$

$\frac{x}{4}\in(-\arcsin\frac{1}{3}+2\pi n;\ \pi+\arcsin\frac{1}{3}+2\pi n),$

$x\in(-4\arcsin\frac{1}{3}+8\pi n;\ 4\pi+4\arcsin\frac{1}{3}+8\pi n);$

г) $\sin 2x \sin 5x + \cos 2x \cos 5x > -\frac{\sqrt{3}}{2}$,

$\cos 3x > -\frac{\sqrt{3}}{2}$,

$3x \in (-\frac{5\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n)$,

$x \in (-\frac{5\pi}{18} + \frac{2\pi n}{3}; \frac{5\pi}{18} + \frac{2\pi n}{3})$.

§ 20. Тангенс суммы и разности аргументов

20.1. а) $\operatorname{tg}\frac{\pi}{12} = \operatorname{tg}(45° - 30°) = \frac{\operatorname{tg} 45° - \operatorname{tg} 30°}{1 + \operatorname{tg}45° \ \operatorname{tg}30°} = \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$.

б) $\operatorname{tg} 105° = \operatorname{tg}(60° + 45°) = \frac{\operatorname{tg} 60° + \operatorname{tg} 45°}{1 - \operatorname{tg}60° \ \operatorname{tg}45°} = \frac{\sqrt{3} + 1}{1 - \sqrt{3}}$.

в) $\operatorname{tg}\frac{5\pi}{12} = \operatorname{tg}(45° + 30°) = \frac{\operatorname{tg} 45° + \operatorname{tg} 30°}{1 - \operatorname{tg}45° \ \operatorname{tg}30°} = \frac{1 + \frac{\sqrt{3}}{3}}{1 - \frac{\sqrt{3}}{3}}$.

г) $\operatorname{tg} 165° = -\operatorname{tg} 15° = -\frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$.

20.2. а) $\frac{\operatorname{tg} 25° + \operatorname{tg} 20°}{1 - \operatorname{tg}25° \ \operatorname{tg}20°} = \operatorname{tg} 45° = 1$; б) $\frac{1 - \operatorname{tg} 70° \operatorname{tg} 65°}{\operatorname{tg}70° + \operatorname{tg}65°} = \operatorname{ctg} 135° = -1$;

в) $\frac{\operatorname{tg} 9° + \operatorname{tg} 51°}{1 - \operatorname{tg}9° \ \operatorname{tg}51°} = \operatorname{tg} 60° = \sqrt{3}$; г) $\frac{1 + \operatorname{tg} 54° \operatorname{tg} 9°}{\operatorname{tg}54° - \operatorname{tg}9°} = \operatorname{ctg} 45° = 1$.

20.3. а) $\operatorname{tg}(\frac{\pi}{4} - \alpha)$, $\operatorname{tg}\alpha = \frac{2}{3}$, $\operatorname{tg}(\frac{\pi}{4} - \alpha) = \frac{1 - \operatorname{tg}\alpha}{1 + \operatorname{tg}\alpha} = \frac{1}{3} \cdot \frac{3}{5} = \frac{1}{5}$;

б) $\operatorname{tg}\alpha = \frac{4}{5}$,

$\operatorname{tg}(\alpha + \frac{\pi}{3}) = \frac{\operatorname{tg}\ \alpha + \operatorname{tg}\frac{\pi}{3}}{1 - \operatorname{tg}\ \alpha \operatorname{tg}\frac{\pi}{3}} = \frac{\frac{4}{5} + \sqrt{3}}{1 - \frac{4}{5}\cdot\sqrt{3}} = \frac{4 + 5\sqrt{3}}{5 - 4\sqrt{3}} = -\frac{41\sqrt{3} + 80}{23}$;

в) $\mathrm{ctg}\,\alpha=\frac{4}{3}$, $\mathrm{tg}\,(\frac{\pi}{2}+\alpha)=-\mathrm{ctg}\,\alpha=-\frac{4}{3}$;

г) $\mathrm{ctg}\,\alpha=\frac{8}{5}$, $\mathrm{tg}\,(\alpha-\frac{\pi}{4})=\frac{\mathrm{tg}\alpha-1}{1+\mathrm{tg}\alpha}=\frac{\frac{5}{8}-1}{1+\frac{5}{8}}=-\frac{3}{13}$.

20.4. $\mathrm{tg}\,\alpha=\frac{1}{2}$, $\mathrm{tg}\,\beta=\frac{1}{3}$;

а) $\mathrm{tg}\,(\alpha+\beta)=\frac{\mathrm{tg}\ \alpha+\mathrm{tg}\ \beta}{1-\mathrm{tg}\ \alpha\ \mathrm{tg}\ \beta}=\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{6}}=\frac{5}{6}\cdot\frac{6}{5}=1.$

б) $\mathrm{tg}\,(\alpha-\beta)=\frac{\frac{1}{2}-\frac{1}{3}}{1+\frac{1}{6}}=\frac{1}{6}\cdot\frac{6}{7}=\frac{1}{7}$.

20.5. $\mathrm{tg}\,\alpha=\frac{2}{5}$, $\mathrm{tg}\,(\frac{\pi}{2}+\beta)=-3$, $-\mathrm{ctg}\,\beta=-3$, $\mathrm{tg}\,\beta=\frac{1}{3}$;

а) $\mathrm{tg}\,(\alpha+\beta)=\frac{\mathrm{tg}\ \alpha+\mathrm{tg}\ \beta}{1-\mathrm{tg}\ \alpha\,\mathrm{tg}\ \beta}=\frac{\frac{2}{5}+\frac{1}{3}}{1-\frac{2}{15}}=\frac{11}{15}\cdot\frac{15}{13}=\frac{11}{13}$;

б) $\mathrm{tg}\,(\alpha-\beta)=\frac{\mathrm{tg}\ \alpha-\mathrm{tg}\ \beta}{1+\mathrm{tg}\ \alpha\,\mathrm{tg}\ \beta}=\frac{\frac{2}{5}-\frac{1}{3}}{1+\frac{2}{15}}=\frac{11}{15}\cdot\frac{15}{17}=\frac{1}{17}$.

20.6. а) $\frac{\mathrm{tg}\,2{,}22+\mathrm{tg}\,0{,}92}{1-\mathrm{tg}\,2{,}22\cdot\mathrm{tg}\,0{,}92}=\mathrm{tg}\,(2{,}22+0{,}92)=\mathrm{tg}\,3{,}14.$

б) $\frac{\mathrm{tg}\,1{,}47-\mathrm{tg}\,0{,}69}{1+\mathrm{tg}\,1{,}47\,\mathrm{tg}\,0{,}69}=\mathrm{tg}\,(1{,}47-0{,}69)=\mathrm{tg}\,0{,}78.$

20.7. а) $\frac{tg(\frac{\pi}{8}+\alpha)+tg(\frac{\pi}{8}-\alpha)}{1-tg(\frac{\pi}{8}+\alpha)\ tg(\frac{\pi}{8}-\alpha)}=\mathrm{tg}\,(\frac{\pi}{8}+\alpha+\frac{\pi}{8}-\alpha)=$

$\mathrm{tg}\,\frac{\pi}{4}=1.$

б) $\frac{\mathrm{tg}(45^\circ+\alpha)-\mathrm{tg}\,\alpha}{1+\mathrm{tg}(45^\circ+\alpha)\ \mathrm{tg}\,\alpha}=\mathrm{tg}\,(45^\circ+\alpha-\alpha)=\mathrm{tg}\,45^\circ=1.$

20.8. а) $\frac{1-tg\alpha}{1+tg\alpha}=\text{tg}\,(45°-\alpha)$, $\text{tg}\,(45°-\alpha)=\frac{1-tg\alpha}{1+tg\alpha}$.

б) $\frac{tg\alpha+tg\beta}{tg(\alpha+\beta)}+\frac{tg\alpha-tg\beta}{tg(\alpha-\beta)}=2$.

$\frac{tg\alpha+tg\beta}{tg(\alpha+\beta)}+\frac{tg\alpha-tg\beta}{tg(\alpha-\beta)}=1-\text{tg}\,\alpha\,\text{tg}\,\beta+1+\text{tg}\alpha\,\text{tg}\,\beta=2.$

20.9. а) $\frac{\text{tg}\ x+\text{tg}\ 3x}{1-\text{tg}\ x\ \ \text{tg}\ 3x}=1$,

$\text{tg}\,4x=1$, $4x=\frac{\pi}{4}+\pi n$, $x=\frac{\pi}{16}+\frac{\pi}{4}n$;

б) $\frac{\text{tg}\ 5x-\text{tg}\ 3x}{1+\text{tg}\ 3x\ \ \text{tg}\ 5x}=\sqrt{3}$,

$\text{tg}\ 2x=\sqrt{3}$, $2x=\frac{\pi}{3}+\pi n$, $x=\frac{\pi}{6}+\frac{\pi n}{2}$.

20.10. а) $\frac{\sqrt{3}-\text{tg}\,x}{1+\sqrt{3}\,\text{tg}\,x}=1$,

$\text{tg}\,(\frac{\pi}{3}-x)=1$, $x-\frac{\pi}{3}=-\frac{\pi}{4}+\pi n$, $x=\frac{\pi}{12}+\pi n$,

$x\in[-\pi,2\pi]$ $x=-\frac{11}{12}\pi,\frac{\pi}{12},\frac{13\pi}{12}$;

б) $\frac{\text{tg}\frac{\pi}{5}-\text{tg}\,2x}{\text{tg}\frac{\pi}{5}\text{tg}\ \ 2x+1}=\sqrt{3}$, $\text{tg}\,(2x-\frac{\pi}{5})=-\sqrt{3}$,

$2x-\frac{\pi}{5}=-\frac{\pi}{3}+\pi n$, $x=-\frac{\pi}{15}+\frac{\pi n}{2}$, $x\in[-\pi,2\pi]$

$x=-\frac{17\pi}{30},-\frac{\pi}{15},\frac{13\pi}{30},\frac{14\pi}{15},\frac{43\pi}{30},\frac{29\pi}{15}$.

20.11. а) $\text{tg}\,(\alpha-\frac{\pi}{4})=3$, $\frac{\text{tg}\,\alpha-1}{1+\text{tg}\,\alpha}=3$, $\text{tg}\,\alpha-1=3+3\,\text{tg}\,\alpha$,

$2\text{tg}\,\alpha=-4$, $\text{tg}\,\alpha=-2$;

б) $\text{tg}\,(\alpha+\frac{\pi}{4})=\frac{1}{5}$, $\frac{\text{tg}\,\alpha+1}{1-\text{tg}\,\alpha}=\frac{1}{5}$,

$5\,\text{tg}\,\alpha+5=1-\text{tg}\,\alpha$, $\text{tg}\,\alpha=-\frac{2}{3}$,

$\text{ctg}\,\alpha=-\frac{3}{2}$.

20.12. а) $\text{tg}\,\alpha = 3$, $\text{tg}\,(\alpha + \beta) = 1$.

$$\frac{\text{tg}\,\alpha + \text{tg}\beta}{1 - \text{tg}\,\alpha\ \text{tg}\beta} = 1,\quad 3 + \text{tg}\,\beta = 1 - 3\,\text{tg}\,\beta,\quad \text{tg}\,\beta = -\frac{1}{2};$$

б) $\text{tg}\,\alpha = \frac{1}{4}$, $\text{tg}\,(\alpha - \beta) = 2$.

$$\frac{\text{tg}\,\alpha - \text{tg}\beta}{1 + \text{tg}\,\alpha\ \text{tg}\beta} = 2,\quad \frac{1}{4} - \text{tg}\,\beta = 2 + \frac{1}{2}\,\text{tg}\,\beta,$$

$$\frac{3}{2}\,\text{tg}\,\beta = -\frac{7}{4},\quad \text{tg}\,\beta = -\frac{7}{6}.$$

20.13. $\sin\alpha = -\frac{12}{13}$, $\pi < \alpha < \frac{3\pi}{2}$; $\cos\alpha = -\frac{5}{13}$, $\text{tg}\,\alpha\,\beta = \frac{12}{5}$;

а) $$\text{tg}\,(\alpha + \frac{\pi}{4}) = \frac{\text{tg}\,\alpha + 1}{1 - \text{tg}\,\alpha} = \frac{\frac{12}{5} + 1}{1 - \frac{12}{5}} = -\frac{17}{5}\cdot\frac{5}{7} = -\frac{17}{7}.$$

б) $$\text{tg}\,(\alpha - \frac{\pi}{4}) = \frac{\text{tg}\,\alpha - 1}{1 + \text{tg}\,\alpha} = \frac{\frac{12}{5} - 1}{1 + \frac{12}{5}} = \frac{17}{5}\cdot\frac{5}{17} = \frac{7}{17}.$$

20.14. $\cos\alpha = \frac{3}{5}$, $0 < \alpha < \frac{\pi}{2}$, $\sin\alpha = \frac{4}{5}$, $\text{tg}\,\alpha = \frac{4}{3}$;

а) $$\text{tg}\,(\alpha + \frac{\pi}{3}) = \frac{\text{tg}\,\pi + \sqrt{3}}{1 - \sqrt{3}\,\text{tg}\,\pi} = \frac{\frac{4}{3} + \sqrt{3}}{1 - \frac{4}{3}\sqrt{3}} = \frac{4 + 3\sqrt{3}}{3 - 4\sqrt{3}} = -\frac{25\sqrt{3} - 48}{39};$$

б) $$\text{tg}\,(\alpha - \frac{\pi}{3}) = \frac{\text{tg}\,\alpha - \sqrt{3}}{1 + \sqrt{3}\,\text{tg}\,\alpha} = \frac{4 - 3\sqrt{3}}{3}\cdot\frac{3}{3 + 4\sqrt{3}} = \frac{4 - 3\sqrt{3}}{3 + 4\sqrt{3}}.$$

20.15. $y_1 = 3x + 1$ $y_2 = 6 - 2x$.

$\text{tg}\,y_1 = 3$ – тангенс угла наклона 1-й прямой.

$\text{tg}\,y_2$ – тангенс угла наклона 2-ой прямой.

$y_1 = \text{arctg}\,3$, $y_2 = -\,\text{arctg}\,2$

$y_1 - y_2 = \text{arctg}\,3 + \text{arctg}\,2$

$$\text{tg}\,(y_1 - y_2) = \frac{3 + 2}{1 - 6} = -1.$$

$$y_1 - y_2 = \frac{3\pi}{4}.$$

20.16. $\text{tg}\,\angle KBC = \frac{1}{2}$, $\angle KBC = \text{arctg}\,\frac{1}{2}$. (см. рис. 144)

$\angle BKC = 90° - \angle \text{KBC}$

$\angle KOC = 180° - 45° - 90° + \angle KBC = 45° + \angle KBC$.

$$\operatorname{tg} \angle KOC = \operatorname{tg}\left(45° + \operatorname{arctg}\frac{1}{2}\right) = \frac{1+\frac{1}{2}}{1-\frac{1}{2}} = \frac{3}{2}\cdot 2 = 3$$

$\angle KOC = \operatorname{arctg} 3$.

§ 21. Формулы двойного угла

21.1. а) $\frac{\sin 2t}{\cos t} - \sin t = 2\sin t - \sin t = \sin t$.

б) $\frac{\sin 6t}{\cos^2 3t} = 2\operatorname{tg} 3t$.

в) $\cos^2 t - \cos 2t = \cos^2 t - \cos^2 t + \sin^2 t = \sin^2 t$.

г) $\frac{\cos 2t}{\cos t - \sin t} - \sin t = \frac{(\cos - \sin t)(\cos t + \sin t)}{\cos t - \sin t} - \sin t = \cos t$.

21.2. а) $\frac{\sin 40^\circ}{\sin 20^\circ} = 2\cos 20°$; б) $\frac{\cos 80^\circ}{\cos 40^\circ + \sin 40^\circ} = \cos 40° - \sin 40°$;

в) $\frac{\sin 100^\circ}{2\cos 50^\circ} = \sin 50°$; г) $\frac{\cos 36^\circ + \sin^2 18^\circ}{\cos 18^\circ} = \frac{\cos^2 18^\circ}{\cos 18^\circ} = \cos 18°$.

21.3. а) $2\sin 15° \cos 15° = \sin 30° = \frac{1}{2}$.

б) $(\cos 75° - \sin 75°)^2 = 1 - 2\sin 75° \cos 75° = 1 - \sin 150° = \frac{1}{2}$.

в) $\cos^2 15° - \sin^2 15° = \cos 30° = \frac{\sqrt{3}}{2}$.

г) $(\cos 15° + \sin 15°)^2 = 1 + 2\sin 15° \cos 15° = \frac{3}{2}$.

21.4.

а) $2\sin\frac{\pi}{8}\cos\frac{\pi}{8} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

б) $\sin\frac{\pi}{8}\cos\frac{\pi}{8} + \frac{1}{4} = \frac{\sqrt{2}}{4} + \frac{1}{4} = \frac{\sqrt{2}+1}{4}$.

в) $\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8} = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

г) $\frac{\sqrt{2}}{2} - \left(\cos\frac{\pi}{8} + \sin\frac{\pi}{8}\right)^2 = \frac{\sqrt{2}}{2} - 1 - \sin\frac{\pi}{4} = -1$.

21.5. а) $\dfrac{\operatorname{tg}\frac{\pi}{8}}{1-\operatorname{tg}^2\frac{\pi}{8}} = \frac{1}{2}\operatorname{tg}\frac{\pi}{4} = \frac{1}{2}$;

б) $\dfrac{\operatorname{tg}75^\circ}{1-\operatorname{tg}^2 75^\circ} = \frac{1}{2}\operatorname{tg}150^\circ = -\frac{1}{2}\operatorname{tg}30^\circ = -\frac{\sqrt{3}}{6}$;

21.6. а) $\sin\frac{x}{2}\cos\frac{x}{2} = \frac{1}{2}\sin x$, $\frac{1}{2}\sin x = \sin\frac{x}{2}\cos\frac{x}{2}$.

б) $\cos^2\frac{x}{4} - \sin^2\frac{x}{4} = \cos\frac{x}{2}$, $\cos\frac{x}{2} = \cos^2\frac{x}{4} - \sin^2\frac{x}{4}$.

в) $\sin 2x\cos 2x = \frac{1}{2}\sin 4x$, $\frac{1}{2}\sin 4x = \sin 2x\cos 2x$.

г) $\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x$, $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$.

21.7. а) $\cos(2\alpha + 2\beta) = \cos^2(\alpha+\beta) - \sin^2(\alpha+\beta)$.
$\cos(2\alpha + 2\beta) = \cos 2(\alpha+\beta) = \cos^2(\alpha+\beta) - \sin^2(\alpha+\beta)$.
б) $\sin(2\alpha + 2\beta) = 2\sin(\alpha+\beta)\cos(\alpha+\beta)$.
$\sin(2\alpha + 2\beta) = \sin 2(\alpha+\beta) = 2\sin(\alpha+\beta)\cos(\alpha+\beta)$.

21.8. а) $\operatorname{tg}(2\alpha + 2\beta) = \dfrac{2tg(\alpha+\beta)}{1-tg^2(\alpha+\beta)}$

$\operatorname{tg}(2\alpha + 2\beta) = \operatorname{tg}2(\alpha+\beta) = \dfrac{2tg(\alpha+\beta)}{1-tg^2(\alpha+\beta)}$.

б) $\operatorname{tg}(\alpha+\beta) = \dfrac{2tg(\frac{\alpha}{2}+\frac{\beta}{2})}{1-tg^2(\frac{\alpha}{2}+\frac{\beta}{2})}$ $\quad \operatorname{tg}(\alpha+\beta) = \operatorname{tg}2(\frac{\alpha}{2}+\frac{\beta}{2}) =$

$= \dfrac{2\operatorname{tg}(\frac{\alpha}{2}+\frac{\beta}{2})}{1-\operatorname{tg}^2(\frac{\alpha}{2}+\frac{\beta}{2})}$.

21.9. $\sin t = \frac{5}{13}$, $\frac{\pi}{2} < t < \pi$;

$\cos t = -\frac{12}{13}$; $\operatorname{tg} t = \frac{5}{13}\cdot\left(-\frac{13}{12}\right) = -\frac{5}{12}$;

а) $\sin 2t = 2\sin t\cos t = 2\cdot\frac{5}{13}\cdot(-\frac{12}{13}) = -\frac{120}{169}$.

б) $\cos 2t = \cos^2 t - \sin^2 t = \frac{144}{169} - \frac{25}{169} = \frac{119}{169}$.

в) $\operatorname{tg} 2t = \dfrac{2\operatorname{tg} t}{1-\operatorname{tg}^2 t} = \dfrac{2\cdot\left(-\dfrac{5}{12}\right)}{1-\dfrac{25}{144}} = -\dfrac{5}{6}\cdot\dfrac{144}{119} = -\dfrac{120}{119}$.

г) $\operatorname{ctg} 2t = \dfrac{1}{\operatorname{tg} 2t} = -\dfrac{119}{120}$.

21.10. $\cos x = \dfrac{4}{5}$, $0 < x < \dfrac{\pi}{2}$; $\sin x = \dfrac{3}{5}$, $\operatorname{tg} x = \dfrac{3}{4}$, $\operatorname{ctg} x = \dfrac{4}{3}$;

а) $\sin 2x = 2\cdot\dfrac{4}{5}\cdot\dfrac{3}{5} = \dfrac{24}{25}$; б) $\cos 2x = \dfrac{16}{25} - \dfrac{9}{25} = \dfrac{7}{25}$.

в) $\operatorname{tg} 2x = \dfrac{2\operatorname{tg} x}{1-\operatorname{tg}^2 x} = \dfrac{\dfrac{3}{2}}{1-\dfrac{9}{16}} = \dfrac{3}{2}\cdot\dfrac{16}{7} = \dfrac{24}{7}$; г) $\operatorname{ctg} 2x = \dfrac{7}{24}$.

21.11. а) $\cos t = \dfrac{3}{4}$, $0 < t < \dfrac{\pi}{2}$;

$\cos\dfrac{t}{2} = \sqrt{\dfrac{\cos t + 1}{2}} = \dfrac{\sqrt{14}}{4}$;

$\sin\dfrac{t}{2} = \sqrt{\dfrac{1-\cos t}{2}} = \dfrac{\sqrt{2}}{4}$;

$\operatorname{tg}\dfrac{t}{2} = \dfrac{\sin\dfrac{t}{2}}{\cos\dfrac{t}{2}} = \dfrac{\sqrt{7}}{7}$; $\operatorname{ctg}\dfrac{t}{2} = \dfrac{\cos\dfrac{t}{2}}{\sin\dfrac{t}{2}} = \sqrt{7}$

б) $\operatorname{ctg} t = \dfrac{3}{4}$, $\pi < t < \dfrac{3\pi}{2}$;

$\cos t = -\dfrac{\operatorname{ctg} t}{\sqrt{1+\operatorname{ctg}^2 t}} = -\dfrac{3}{5}$; $\cos\dfrac{t}{2} = -\sqrt{\dfrac{\cos t + 1}{2}} = -\dfrac{1}{\sqrt{5}}$

$\sin\dfrac{t}{2} = \sqrt{\dfrac{1-\cos t}{2}} = \dfrac{2}{\sqrt{5}}$; $\operatorname{tg}\dfrac{t}{2} = \dfrac{\sin\dfrac{t}{2}}{\cos\dfrac{t}{2}} = -2$; $\operatorname{ctg}\dfrac{t}{2} = \dfrac{\cos\dfrac{t}{2}}{\sin\dfrac{t}{2}} = -\dfrac{1}{2}$

21.12. а) $\sin 2x = -\dfrac{3}{5}$, $\dfrac{\pi}{2} < x < \dfrac{3\pi}{4}$

$\cos 2x = -\sqrt{1-\sin^2 2x} = -\dfrac{4}{5}$; $\cos x = -\sqrt{\dfrac{\cos 2x + 1}{2}} = -\dfrac{1}{\sqrt{10}}$;

$\sin x = \sqrt{\dfrac{1-\cos 2x}{2}} = \dfrac{3}{\sqrt{10}}$; $\operatorname{tg} x = \dfrac{\sin x}{\cos x} = -3$ $\operatorname{ctg} x = \dfrac{\cos x}{\sin x} = -\dfrac{1}{3}$

б) $\operatorname{tg}2x=\frac{3}{4},\ \pi<x<\frac{5\pi}{4}$

$\cos 2x=\frac{1}{\sqrt{1+\operatorname{tg}^2 2x}}=\frac{4}{5};\ \cos x=-\sqrt{\frac{\cos 2x+1}{2}}=-\frac{3}{\sqrt{10}};$

$\sin x=-\sqrt{\frac{1-\cos 2x}{2}}=-\frac{1}{\sqrt{10}};\ \operatorname{tg}x=\frac{\sin x}{\cos x}=\frac{1}{3},\ \operatorname{ctg}x=\frac{\cos x}{\sin x}=3$

21.13. а) $\frac{\sin t}{2\cos^2\frac{t}{2}}=\frac{2\sin\frac{t}{2}(\cos\frac{t}{2})}{2\cos^2\frac{t}{2}}=\operatorname{tg}\frac{t}{2}.$

б) $\frac{\cos t}{\cos\frac{t}{2}+\sin\frac{t}{2}}=\frac{\cos^2\frac{t}{2}-\sin^2\frac{t}{2}}{\cos\frac{t}{2}+\sin\frac{t}{2}}=\cos\frac{t}{2}-\sin\frac{t}{2}.$

в) $\frac{\sin 4t}{\cos 2t}=\frac{2\sin 2t\cos 2t}{\cos 2t}=2\sin 2t.$

г) $\frac{\cos 2t-\sin 2t}{\cos 4t}=\frac{\cos 2t-\sin 2t}{(\sin 2t-\sin 2t)\ (\cos 2t+\cos 2t)}=$

$=\frac{1}{\cos 2t+\sin 2t}.$

21.14. а) $\frac{\sin 2t-2\sin t}{\cos t-1}=\frac{2\sin t(\cos t-1)}{\cos t-1}.=2\sin t.$

б) $\frac{\cos 2t-\cos^2 t}{1-\cos^2 t}=\frac{\cos^2 t-\sin^2 t-\cos^2 t}{\sin^2 t}=-1$

в) $\sin 2t\operatorname{ctg}t-1=2\sin t\cos t\cdot\frac{\cos t}{\sin t}-1=2\cos^2 t-1=\cos 2t.$

г) $(\operatorname{tg}t+\operatorname{ctg}t)\sin 2t=\frac{1}{\sin t\ \cos t}\cdot 2\ \sin t\ \cos t=2.$

21.15. а) $\frac{2}{\operatorname{tg}t+\operatorname{ctg}t}=\frac{2}{\frac{1}{\sin t\cos t}}=\sin 2t.$

б) $\frac{2}{\operatorname{tg}t-\operatorname{ctg}t}=\frac{2}{\frac{-\cos 2t}{\sin t\cos t}}=\frac{\sin 2t}{-\cos 2t}=-\operatorname{tg}2t.$

21.16. а) $(1-\operatorname{tg}^2 t)\cos^2 t=\cos^2 t-\sin^2 t=\cos 2t.$

б) $2\cos^2\frac{\pi+t}{4}-2\sin^2\frac{\pi+t}{4}=2\cos(\frac{\pi}{2}+\frac{t}{2})=-2\sin\frac{t}{2}.$

21.17. а) $(\sin t - \cos t)^2 = 1 - \sin 2t$, $\sin^2 t - 2\sin t\cos t + \cos^2 t = 1 - \sin 2t$.

б) $2\cos^2 t = 1 + \cos 2t$, $1 + \cos 2t = \sin^2 t + \cos^2 t - \sin^2 t + \cos^2 t = 2\cos^2 t$.

в) $(\sin t + \cos t)^2 = 1 + \sin 2t$, $\sin^2 t + \cos^2 t + 2\sin t\cos t = 1 + \sin 2t$.

г) $2\sin^2 t = 1 - \cos 2t$, $1 - \cos 2t = \sin^2 t + \cos^2 t - \cos^2 t + \sin^2 t = 2\sin^2 t$.

21.18. а) $\cos^4 t - \sin^4 t = \cos 2t$.

$\cos 2t = (\cos^2 t - \sin^2 t)\cdot 1 = (\cos^2 t - \sin^2 t)(\cos^2 t + \sin^2 t) = \cos^4 t - \sin^4 t$.

б) $\cos^4 t + \sin^4 t = 1 - \frac{1}{2}\cdot\sin^2 2t$.

$\cos^4 t + \sin^4 t + 2\sin^2 t\cos^2 t - 2\sin^2 t\cos^2 t = (\sin^2 t + \cos^2 t)^2 - \frac{1}{2}\sin^2 2t =$

$= 1 - \frac{1}{2}\sin^2 2t$.

21.19. а) $\operatorname{ctg} t - \sin 2t = \operatorname{ctg} t\cos 2t$.

$$\frac{\cos t - 2\sin^2 t\cos t}{\sin t} = \frac{\cos t\,(\cos^2 t - \sin^2 t)}{\sin t} = \frac{\cos t\cdot\cos 2t}{\sin t} = \operatorname{ctg} t\cos 2t$$

б) $\sin 2t - \operatorname{tg} t = \cos 2t\operatorname{tg} t$;

$$\frac{2\sin t\cos^2 t - \sin t}{\cos t} = \frac{(\cos^2 t - \sin^2 t)\sin t}{\cos t} = \frac{\cos 2t\sin t}{\cos t} = \cos 2t\operatorname{tg} t$$

21.20. а) $\sin^2 2t = \frac{1-\cos 4t}{2}$; $\frac{1-\cos 4t}{2} = \frac{1}{2}(1 - \cos^2 2t + \sin^2 2t) =$

$= \frac{1}{2}(2\sin^2 2t) = \sin^2 2t$

б) $2\sin^2\frac{t}{2} + \cos t = 1$. $2(1-\cos t)\cdot\frac{1}{2} + \cos t = 1$.

в) $2\sin^2 2t = 1 + \sin\left(\frac{3\pi}{2} - 4t\right)$. $1 - \cos 4t = 1 + \sin\left(\frac{3\pi}{2} - 4t\right)$.

г) $2\cos^2 t - \cos 2t = 1$. $2\cos^2 t - \cos 2t = 2\cos^2 t - \cos^2 t + \sin^2 t = 1$.

21.21. а) $\cos^2 3t = \dfrac{1+\sin\left(\frac{\pi}{2} - 6t\right)}{2}$, $\cos^2 3t = \dfrac{1+\cos 6t}{2} = \dfrac{1+\sin\left(\frac{\pi}{2} - 6t\right)}{2}$.

б) $\dfrac{1-\cos t}{1+\cos t} = \operatorname{tg}^2\frac{t}{2}$. $\operatorname{tg}^2\frac{t}{2} = \dfrac{\sin^2\frac{t}{2}}{\cos^2\frac{t}{2}} = \dfrac{1-\cos t}{1+\cos t}$.

в) $\cos^2 3t = \dfrac{1-\cos(6t-3\pi)}{2} = \cos^2 3t = \dfrac{1+\cos 6t}{2} = \dfrac{1-\cos(6t-3\pi)}{2}$

г) $\frac{1-\cos t}{\sin t}=\operatorname{tg}\frac{t}{2}$, $\frac{1-\cos t}{2}=\frac{2\sin^2\frac{t}{2}}{2\sin\frac{t}{2}\cos\frac{t}{2}}=\operatorname{tg}\frac{t}{2}$.

21.22. а) $1+\sin\alpha=2\cos^2(45°-\frac{1}{2})$,

$2\cos^2(45°-\frac{\alpha}{2})=1+\cos(90°-\alpha)=1+\sin\alpha$.

б) $2\sin^2(45°-\alpha)+\sin 2\alpha=1$,
$1-\cos(90°-2\alpha)+\sin 2\alpha=1-\sin 2\alpha+\sin 2\alpha=1$;

в) $1-\sin\alpha=2\sin^2(45°-\frac{\alpha}{2})$,

$2\sin^2(45°-\frac{\alpha}{2})=1-\cos(90°-\alpha)=1-\sin\alpha$;

г) $2\cos^2(45°+\alpha)+\sin 2\alpha=1$,
$1+\cos(90°+2\alpha)+\sin 2\alpha=1-\sin 2\alpha+\sin 2\alpha=1$.

21.23. а) $\sin 22{,}5°=\sqrt{\frac{1-\cos 45°}{2}}=\sqrt{\frac{2-\sqrt{2}}{4}}$.

б) $\cos 22{,}5°=\sqrt{\frac{1+\cos 45°}{2}}=\sqrt{\frac{2+\sqrt{2}}{4}}$.

в) $\sin\frac{3\pi}{8}=\sqrt{\frac{1-\cos\frac{3\pi}{4}}{2}}=\sqrt{\frac{2+\sqrt{2}}{4}}$.

г) $\cos\frac{3\pi}{8}=\sqrt{\frac{1+\cos\frac{3\pi}{4}}{2}}=\sqrt{\frac{2-\sqrt{2}}{4}}$.

21.24. а) $\sin 2x-2\cos x=0$.

$2\cos x(\sin x-1)=0$, $\cos x=0$, $x=\frac{\pi}{2}+\pi n$, $\sin x=1$, $x=\frac{\pi}{2}+2\pi n$, =>

=> $x=\frac{\pi}{2}+\pi n$;

б) $\sin 2x-\sin x=0$,
$\sin x(2\cos x-1)=0$, $\sin x(2\cos x-1)=0$, $\sin x=0$,
$x=\pi n$, $\cos x=\frac{1}{2}$, $x=\pm\frac{\pi}{3}+2\pi n$;

в) $2\sin x=\sin 2x$,
$2\sin x(\cos x-1)=0$, $\sin x=0$, $\cos x=1$;
$x=\pi n$, $x=2\pi n$, => $x=\pi n$;

г) $\sin 2x+\cos x=0$;

$$\cos x(2\sin x+1)=0 \Rightarrow \left[\begin{array}{l}\cos x=0\\ \sin x=-\dfrac{1}{2}\end{array}\right. \Rightarrow$$

$$\Rightarrow x=\frac{\pi}{2}+\pi n,\ n\in Z \text{ и } x=(-1)^{k+1}\frac{\pi}{6}+\pi k,\ k\in Z$$

21.25. а) $\sin x\cos x=\dfrac{1}{4}$.

$2\sin x\cos x=\dfrac{1}{2}$, $\quad \sin 2x=\dfrac{1}{2}$; $\quad 2x=(-1)^n\dfrac{\pi}{6}+\pi n$,

$x=(-1)^n\dfrac{\pi}{12}+\dfrac{\pi n}{2}$;

б) $\sin 4x\cos 4x=\dfrac{1}{2}$.

$\sin 8x=1$, $\quad 8x=\dfrac{\pi}{2}+2\pi n$, $\quad x=\dfrac{\pi}{16}+\dfrac{\pi n}{4}$;

в) $\cos^2\dfrac{x}{3}-\sin^2\dfrac{x}{3}=\dfrac{1}{2}$,

$\cos\dfrac{2x}{3}=\dfrac{1}{2}$, $\quad \dfrac{2x}{3}=\pm\dfrac{\pi}{3}+2\pi n$, $x=\pm\dfrac{\pi}{2}+3\pi n$;

г) $\sin^2 x-\cos^2 x=\dfrac{1}{2}$,

$\cos 2x=-\dfrac{1}{2}$, $\quad 2x=\pm\frac{2\pi}{3}+2\pi n$, $\quad x=\pm\dfrac{\pi}{3}+\pi n$.

21.26. а) $1-\cos x=2\sin\dfrac{x}{2}$,

$1-(1-2\sin^2\dfrac{x}{2})=2\sin\dfrac{x}{2}$; $\sin^2\dfrac{x}{2}=\sin\dfrac{x}{2}$; $(\sin\cdot\dfrac{x}{2}-1)\cdot\sin\dfrac{x}{2}=0$;

$$\left[\begin{array}{l}\sin\dfrac{x}{2}=0\\ \sin\dfrac{x}{2}=1\end{array}\right.;\quad \left[\begin{array}{l}x=2\pi n\\ x=\pi+4\pi k\end{array}\right.$$

б) $1+\cos x=2\cos\dfrac{x}{2}$.

$1+(2\cos^2\dfrac{x}{2}-1)=2\cos\dfrac{x}{2}$;

$\cos^2\dfrac{x}{2}=\cos\dfrac{x}{2}$; $(\cos\dfrac{x}{2}-1)\cdot\cos\dfrac{x}{2}=0$;

$$\left[\begin{array}{l}\cos\frac{x}{2}=0\\ \cos\frac{x}{2}=1\end{array}\right.;\quad \left[\begin{array}{l}x=\pi+2\pi n\\ x=4\pi k\end{array}\right.$$

21.27. а) $1-\cos x=\sin x\sin\frac{x}{2}$.

$1-(1-2\sin^2\frac{x}{2})=\sin x\sin\frac{x}{2}$; $\quad 2\sin^2\frac{x}{2}=2\sin^2\frac{x}{2}\cos\frac{x}{2}$;

$\sin^2\frac{x}{2}(1-\cos\frac{x}{2})=0$; $\left[\begin{array}{l}\sin\frac{x}{2}=0\\ \cos\frac{x}{2}=1\end{array}\right.$; $\left[\begin{array}{l}x=2\pi n\\ x=4\pi k\end{array}\right.$; $\quad x=2\pi n$.

б) $\sin x=\operatorname{tg}^2\frac{x}{2}(1+\cos x)$,

$\sin x=\frac{1-\cos x}{1+\cos x}(1+\cos x)$; $\begin{cases}\sin x=1-\cos x\\ 1+\cos x\neq 0\end{cases}$;

$\begin{cases}\sqrt{2}\sin x\left(x+\frac{\pi}{4}\right)=1\\ x\neq\pi+2\pi n\end{cases}$; $\begin{cases}\left[\begin{array}{l}x+\frac{\pi}{4}=\frac{\pi}{4}+2\pi k\\ x+\frac{\pi}{4}=\frac{3\pi}{4}+2\pi k\end{array}\right.\\ x\neq\pi+2\pi n\end{cases}$; $\begin{cases}\left[\begin{array}{l}x=2\pi k\\ x=\frac{\pi}{2}+2\pi k\end{array}\right.;\\ x\neq\pi+2\pi n\end{cases}$ $\left[\begin{array}{l}x=2\pi k\\ x=\frac{\pi}{2}+2\pi k\end{array}\right.$.

21.28. а) $\sin^2 2x=1$;

$\left[\begin{array}{l}\sin 2x=1\\ \sin 2x=-1\end{array}\right.$ $\left[\begin{array}{l}2x=\frac{\pi}{2}+2\pi n\\ 2x=-\frac{\pi}{2}+2\pi k\end{array}\right.$; $\quad 2x=\frac{\pi}{2}+\pi\ell$; $x=\frac{\pi}{4}+\frac{\pi\ell}{2}$

б) $\cos^2 4x=\frac{1}{2}$,

$2\cos^2 4x-1=0$; $\cos 8x=0$; $8x=\frac{\pi}{2}+\pi n$; $x=\frac{\pi}{16}+\frac{\pi n}{8}$.

в) $\sin^2\frac{x}{2}=\frac{3}{4}$.

$\left[\begin{array}{l}\sin\frac{x}{2}=\frac{\sqrt{3}}{2}\\ \sin\frac{x}{2}=-\frac{\sqrt{3}}{2}\end{array}\right.$; $\left[\begin{array}{l}\frac{x}{2}=(-1)^n\frac{\pi}{3}+\pi n\\ \frac{x}{2}=(-1)^k\left(-\frac{\pi}{3}\right)+\pi k\end{array}\right.$; $\left[\begin{array}{l}x=(-1)^n\frac{2\pi}{3}+2\pi n\\ x=(-1)^k\left(-\frac{2\pi}{3}\right)+2\pi k\end{array}\right.$;

$x=\pm\frac{2\pi}{3}+2\pi n$

г) $\cos^2\frac{\pi}{11}=\frac{1}{4}$.

$$\left[\begin{array}{l}\cos\frac{x}{4}=\frac{1}{2}\\ \cos\frac{x}{4}=-\frac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}\frac{x}{4}=\pm\frac{\pi}{3}+2\pi n\\ \frac{x}{4}=\pm\frac{2\pi}{3}+2\pi k\end{array}\right.;\quad \frac{x}{4}=\pm\frac{\pi}{3}+\pi n;\ x=\pm\frac{4\pi}{3}+4\pi n$$

21.29. а) $\cos 2x+3\sin x=1$.

$1-2\sin^2 x+3\sin x=1$, $\sin x(2\sin x-3)=0$, $\sin x=0$,

$\sin x=\frac{3}{2}$ – не существует,

$x=\pi n$, $x\in[0,2\pi]$ $x=0,\pi,2\pi$;

б) $\sin^2 x=-\cos 2x$,

$\sin^2 x+1-2\sin^2 x=0$, $\sin^2 x=1$, $\sin x=\pm 1$,

$x=\frac{\pi}{2}+\pi n$, $x\in[0,2\pi]$ $x=\frac{\pi}{2},\frac{3}{2}\pi$;

в) $\cos 2x=\cos^2 2x$, $2\cos^2 2x-1-\cos^2 x=0$, $\cos^2 2x=1$, $\cos x=\pm 1$,

$x=\pi n$, $x\in[0,2\pi]$ $x=0,\pi,2\pi$;

г) $\cos 2x=2\sin^2 x$,

$1-2\sin^2 x=2\sin^2 x$, $\sin x=\pm\frac{1}{2}$,

$x=\pm\frac{\pi}{6}+\pi k$, $x\in[0,2\pi]$ $x=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$.

21.30. а) $\sin 11°15'\cdot\cos 11°15'\cos 22°30'\cos 45°=$

$=\frac{1}{2}\sin 22°30'\cos 22°30'\cos 45°=\frac{1}{8}\sin 90°=\frac{1}{8}$.

б) $\sin\frac{\pi}{48}\cos\frac{\pi}{48}\cos\frac{\pi}{24}\cos\frac{\pi}{12}=\frac{1}{8}\sin\frac{\pi}{6}=\frac{1}{16}$.

21.31. а) $\frac{1+\cos 40°+\cos 80°}{\sin 80°+\sin 40°}\mathrm{tg}40°=$

$=\frac{2\cos^2 40°+\cos 40°}{\sin 40°(2\cos 40°+1)}\cdot\mathrm{tg}40°=\mathrm{tg}\,40°\ \mathrm{ctg}\,40°=1$

б) $\frac{1-\cos 25°+\cos 50°}{\sin 50°-\sin 25°}-tg65°=$

$\frac{2\cos^2 25°-\cos 25°}{2\sin 25°\cos 25°-\sin 25°}-tg65°=\frac{\cos 25°}{\sin 25°}\cdot\frac{2\cos 25°-1}{2\cos 25°-1}-\mathrm{tg}65°=$

$=\mathrm{ctg}25°-\mathrm{tg}65°=\mathrm{tg}65°-\mathrm{tg}65°=0$.

21.32. а) $\sin 3x=3\sin x-4\sin^3 x$.

$\sin 3x=\sin(x+2x)=\sin x\cos 2x+\sin 2x\cos x=$

$=\sin x-2\sin^3 x+2\sin x-2\sin^3 x=3\sin x-4\sin^3 x$.

б) $\cos 3x = 4\cos^3 x - 3\cos x$.
$\cos(x + 2x) = \cos x \cos 2x - \sin x \sin 2x = 2\cos^3 x - \cos x - 2\cos x \sin^2 x =$
$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x = 4\cos^3 x - 3\cos x$.

21.33. а) $f(x) = 2\cos 2x + \sin^2 x = 2\cos 2x + \frac{1}{2}(1 - 2\cos 2x) =$

$= \frac{3}{2}\cos 2x + \frac{1}{2}$.

Поскольку наибольшее значение функции $y = \cos 2x$ равно 1, а наименьшее -1, то наибольшее значение функции $f(x)$ равно 2, а наименьшее -1.
б) $f(x) = 2\sin^2 3x - \cos 6x = (1 - \cos 6x) - \cos 6x = 1 - 2\cos 6x$.
Поскольку наибольшее значение функции $y = \cos 6x$ равно 1, а наименьшее -1, то наибольшее значение функции $f(x)$ равно 3, а наименьшее -1.

21.34. а) $\cos x = \dfrac{\sin 22{,}5^\circ \cos 22{,}5^\circ}{\cos^2 67{,}5^\circ - \sin^2 67{,}5^\circ} = \dfrac{\frac{1}{2}\sin 45^\circ}{\cos 135^\circ} = -\dfrac{1}{2} \Rightarrow$

$x = \pm\dfrac{2\pi}{3} + 2\pi k \Rightarrow x = -\dfrac{2\pi}{3} = -120^\circ$;

б) $\sin x = \dfrac{\sin^2 75^\circ - \cos^2 75^\circ}{4\sin 15^\circ \cos 15^\circ} = \dfrac{-\cos 150^\circ}{2\sin 30^\circ} = +\dfrac{\sqrt{3}}{2}$;

$x = (-1)^k \dfrac{2\pi}{3} + 2\pi k = -\pi - \dfrac{\pi}{3} = -240^\circ$.

21.35. а) $3\sin 2x + \cos 2x = 1$.
$6\sin x \cos x - 2\sin^2 x = 0$;
$2\sin x(3\cos x - \sin x) = 0$;
$\sin x = 3\cos x$;
$\operatorname{tg} x = 3$; $\quad x = \operatorname{arctg} 3 + \pi n$, $\quad \sin x = 0$; $x = \pi n$;
б) $\cos 4x + 2\sin 4x = 1$.
$-2\sin^2 2x + 4\sin 2x \cos 2x = 0$;
$\sin 2x(2\cos 2x - \sin 2x) = 0$.

$\cos 2x = \frac{1}{2}\sin 2x$, $\operatorname{tg} 2x = 2$,

$2x = \operatorname{arctg} 2 + \pi n$, $x = \dfrac{1}{2}\operatorname{arctg} 2 + \dfrac{tg\frac{\pi}{10}}{\cos\frac{\pi}{5}}$,

$\sin 2x = 0$, $2x = \pi n$, $x = \dfrac{\pi n}{2}$.

21.36. а) $4\sin x + \sin 2x = 0$, $\quad x \in [0; 2\pi]$
$2\sin x(2 + \cos x) = 0$, $\sin x = 0$,
$\cos x = -2$ – нет решений
$x = 0, x = \pi, x = 2\pi$.

б) $\cos^2(3x+\frac{\pi}{4})-\sin^2(3x+\frac{\pi}{4})+\frac{\sqrt{3}}{2}=0,\ x\in[\frac{3\pi}{4},\pi]$

$\cos(6x+\frac{2\pi}{4})=-\frac{\sqrt{3}}{2},\quad \sin 6x=\frac{\sqrt{3}}{2},$

$6x=(-1)^k\frac{\pi}{3}+\pi k,\quad x=(-1)^k\frac{\pi}{18}+\frac{\pi k}{6}\ \Rightarrow$

$\Rightarrow x=\frac{5\pi}{6}-\frac{\pi}{18}=\frac{14\pi}{18}=\frac{7\pi}{9}.$

21.37. $2\cos^2\frac{x}{2}-\cos\frac{\pi}{9}=1$

$(2\cos^2\frac{x}{2}-1)=\cos\frac{\pi}{9};\ \cos x=\cos\frac{\pi}{9};$

$x=\pm\arccos(\cos\frac{\pi}{9})+2\pi n$

Отсюда имеем, что уравнение на отрезке $[-2\pi, 2\pi]$ имеет 4 корня:

$\frac{\pi}{9}-2\pi,\ -\frac{\pi}{9},\ \frac{\pi}{9},\ 2\pi-\frac{\pi}{9}.$

21.38. а) $(\cos x-\sin x)^2=1-2\sin 2x=2x,\ x\in\left[\frac{20\pi}{9},\frac{28\pi}{9}\right]$

$1-\sin 2x=1-2\sin 2x,\quad \sin 2x=0,\ x=\frac{\pi n}{2},$ 2 корня.

б) $2\cos^2(2x-\frac{\pi}{4})-2\sin^2(2x-\frac{\pi}{4})+1=0,\ x\in\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$

$\cos(4x-\frac{\pi}{2})=-\frac{1}{2},\quad \sin 4x=-\frac{1}{2},\quad 4x=(-1)^{k+1}\frac{\pi}{6}+\pi k,$

$x=(-1)^{k+1}\frac{\pi}{24}+\frac{\pi k}{4},$ 3 корня.

§ 22. Преобразование сумм тригонометрических функций в произведения

22.1. а) $\sin 40^\circ+\sin 16^\circ=2\sin\frac{40^\circ+16^\circ}{2}\cos\frac{40^\circ-16^\circ}{2}=2\sin 28^\circ\cos 12^\circ$

б) $\sin 20^\circ-\sin 40^\circ=2\sin\frac{20^\circ-40^\circ}{2}\cos\frac{20^\circ+40^\circ}{2}=-2\sin 10^\circ\cos 30^\circ=$

$=-\sqrt{3}\sin 10^\circ.$

в) $\sin 10°+\sin 50° = 2\sin\dfrac{10°+50°}{2}\cos\dfrac{50°-10°}{2} = 2\sin 30°\cos 20° =$

$= \cos 20°.$

г) $\sin 52° - \sin 36° = 2\sin\dfrac{52°-36°}{2}\cos\dfrac{52°+36°}{2} = 2\sin 8°\cos 44°.$

22.2.

а) $\cos 15° + \cos 45° = 2\cos\dfrac{15°+45°}{2}\cos\dfrac{45°-15°}{2} = 2\cos 30°\cos 15° =$

$= \sqrt{3}\cos 15°.$

б) $\cos 46° - \cos 74° = 2\sin\dfrac{46°+74°}{2}\sin\dfrac{74°-46°}{2} = 2\sin 60°\sin 14° =$

$= \sqrt{3}\sin 14°.$

в) $\cos 20° + \cos 40° = 2\cos\dfrac{20°+40°}{2}\cos\dfrac{40°-20°}{2} = 2\cos 30°\cos 10° =$

$= \sqrt{3}\cos 10°.$

г) $\cos 75° - \cos 15° = 2\sin\dfrac{75°+15°}{2}\sin\dfrac{15°-75°}{2} = -2\sin 45°\sin 30° = -\dfrac{\sqrt{2}}{2}.$

22.3. а) $\sin\dfrac{\pi}{5} - \sin\dfrac{\pi}{10} = 2\sin\dfrac{\frac{\pi}{5}-\frac{\pi}{10}}{2}\cos\dfrac{\frac{\pi}{5}+\frac{\pi}{10}}{2} = 2\sin\dfrac{\pi}{20}\cos\dfrac{3\pi}{20}.$

б) $\sin\dfrac{\pi}{3} + \sin\dfrac{\pi}{4} = 2\sin\dfrac{\frac{\pi}{3}+\frac{\pi}{4}}{2}\cos\dfrac{\frac{\pi}{3}-\frac{\pi}{4}}{2} = 2\sin\dfrac{7\pi}{24}\cos\dfrac{\pi}{24}.$

в) $\sin\dfrac{\pi}{6} + \sin\dfrac{\pi}{7} = 2\sin\dfrac{\frac{\pi}{6}+\frac{\pi}{7}}{2}\cos\dfrac{\frac{\pi}{6}-\frac{\pi}{7}}{2} = 2\sin\dfrac{13\pi}{84}\cos\dfrac{\pi}{84}$

г) $\sin\dfrac{\pi}{3} - \sin\dfrac{\pi}{11} = 2\sin\dfrac{\frac{\pi}{3}-\frac{\pi}{11}}{2}\cos\dfrac{\frac{\pi}{3}+\frac{\pi}{11}}{2} = 2\sin\dfrac{4\pi}{33}\cos\dfrac{7\pi}{33}$

22.4. а) $\cos\dfrac{\pi}{10} - \cos\dfrac{\pi}{20} = 2\sin\dfrac{\frac{\pi}{10}+\frac{\pi}{20}}{2}\sin\dfrac{\frac{\pi}{20}-\frac{\pi}{10}}{2} = -2\sin\dfrac{3\pi}{40}\sin\dfrac{\pi}{40}$

б) $\cos\dfrac{11\pi}{12} + \cos\dfrac{3\pi}{4} = 2\cos\dfrac{\frac{11\pi}{12}+\frac{3\pi}{4}}{2}\cos\dfrac{\frac{11\pi}{12}-\frac{3\pi}{4}}{2} = 2\cos\dfrac{5\pi}{6}\cos\dfrac{\pi}{12} =$

$= -\sqrt{3}\cos\dfrac{\pi}{12}.$

в) $\cos\frac{\pi}{5}-\cos\frac{\pi}{11}=2\sin\frac{\frac{\pi}{5}+\frac{\pi}{11}}{2}\sin\frac{\frac{\pi}{11}-\frac{\pi}{5}}{2}=-2\sin\frac{8\pi}{55}\sin\frac{3\pi}{55}$.

г) $\cos\frac{3\pi}{8}+\cos\frac{5\pi}{4}=2\cos\frac{\frac{3\pi}{8}+\frac{5\pi}{4}}{2}\cos\frac{\frac{3\pi}{8}-\frac{5\pi}{4}}{2}=2\cos\frac{13\pi}{16}\cos\frac{7\pi}{16}$

22.5. а) $\sin 3t-\sin t=2\sin\frac{3t-t}{2}\cos\frac{3t+t}{2}=2\sin t\cos 2t$

б) $\cos(\alpha-2\beta)-\cos(\alpha+2\beta)=2\sin\frac{\alpha-2\beta+\alpha+2\beta}{2}\sin\frac{\alpha+2\beta-\alpha+2\beta}{2}=$

$=2\sin\alpha\sin 2\beta$.

в) $\cos 6t+\cos 4t=2\cos\frac{6t+4t}{2}\cos\frac{6t-4t}{2}=2\cos 5t\cos t$.

г) $\sin(\alpha-2\beta)-\sin(\alpha+2\beta)=2\sin\frac{\alpha-2\beta-\alpha-2\beta}{2}\cos\frac{\alpha-2\beta+\alpha+2\beta}{2}=$

$=-2\sin 2\beta\cos\alpha$.

22.6. а) $\operatorname{tg}25°+\operatorname{tg}35°=\frac{\sin(25°+35°)}{\cos 25°\cos 35°}=\frac{\sin 60°}{\cos 25°\cos 35°}=$

$=\frac{\sqrt{3}}{2\cos 25°\cos 35°}$.

б) $\operatorname{tg}\frac{\pi}{5}-\operatorname{tg}\frac{\pi}{10}=\frac{\sin\left(\frac{\pi}{5}-\frac{\pi}{10}\right)}{\cos\frac{\pi}{5}\cos\frac{\pi}{10}}=\frac{\sin\frac{\pi}{10}}{\cos\frac{\pi}{5}\cos\frac{\pi}{10}}=\frac{tg\frac{\pi}{10}}{\cos\frac{\pi}{5}}$.

в) $\operatorname{tg}20°+\operatorname{tg}40°=\frac{\sin(20°+40°)}{\cos 20°\cos 40°}=\frac{\sin 60°}{\cos 20°\cos 40°}=\frac{\sqrt{3}}{2\cos 20°\cos 40°}$

г) $\operatorname{tg}\frac{\pi}{3}-\operatorname{tg}\frac{\pi}{4}=\frac{\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)}{\cos\frac{\pi}{3}\cos\frac{\pi}{4}}=\frac{\sin\frac{\pi}{12}}{\frac{1}{2}\cdot\frac{\sqrt{2}}{2}}=2\sqrt{2}\sin\frac{\pi}{12}$.

22.7. а) $\frac{\cos 68°-\cos 22°}{\sin 68-\sin 22°}=\frac{2\sin\frac{68°+22°}{2}\sin\frac{22°-68°}{2}}{2\sin\frac{68°-22°}{2}\sin\frac{68°+22°}{2}}=$

$=\frac{-\sin 45°\sin 23°}{\sin 23°\cos 45°}=-\operatorname{tg}45°=-1$.

б) $\dfrac{\sin 130^\circ + \sin 110^\circ}{\cos 130^\circ + \cos 110^\circ} = \dfrac{2\sin\frac{130^\circ + 110^\circ}{2}\cos\frac{130^\circ - 110^\circ}{2}}{2\cos\frac{130^\circ + 110^\circ}{2}\cos\frac{130^\circ - 110^\circ}{2}} =$

$= \dfrac{\sin 120^\circ \cos 10^\circ}{\cos 120^\circ \cos 10^\circ} = \dfrac{\sin 120^\circ}{\cos 120^\circ} = \text{tg } 120^\circ = -\sqrt{3}$.

22.8. а) $\sin 35^\circ + \sin 25^\circ = \cos 5^\circ$.

$\sin 35^\circ + \sin 25^\circ = 2\sin\dfrac{35^\circ + 25^\circ}{2}\cos\dfrac{35^\circ - 25^\circ}{2} = 2\sin 30^\circ \cos 5^\circ = \cos 5^\circ$

б) $\sin 40^\circ + \cos 70^\circ = \cos 10^\circ$.

$\sin 40^\circ + \cos 70^\circ = \sin 40^\circ + \sin 20^\circ = 2\sin\dfrac{40^\circ + 20^\circ}{2}\cos\dfrac{40^\circ - 20^\circ}{2} =$

$= 2\sin 30^\circ \cos 10^\circ = \cos 10^\circ$

в) $\cos 12^\circ - \cos 48^\circ = \sin 18^\circ$.

$\cos 12^\circ - \cos 48^\circ = 2\sin\dfrac{12^\circ + 48^\circ}{2}\sin\dfrac{48^\circ - 12^\circ}{2} = 2\sin 30^\circ \sin 18^\circ = \sin 18^\circ$

г) $\cos 20^\circ - \sin 50^\circ = \sin 10^\circ$.

$\cos 20^\circ - \sin 50^\circ = \cos 20^\circ - \cos 40^\circ = 2\sin\dfrac{20^\circ + 40^\circ}{2}\sin\dfrac{40^\circ - 20^\circ}{2} =$

$= 2\sin 30^\circ \sin 10^\circ = \sin 10^\circ$.

22.9. а) $\dfrac{\sin 2\alpha + \sin 6\alpha}{\cos 2\alpha + \cos 6\alpha} = \text{tg } 4\alpha$.

$\dfrac{\sin 2\alpha + \sin 6\alpha}{\cos 2\alpha + \cos 6\alpha} = \dfrac{2\sin\frac{2\alpha + 6\alpha}{2}\cos\frac{6\alpha - 2\alpha}{2}}{2\cos\frac{2\alpha + 6\alpha}{2}\cos\frac{6\alpha - 2\alpha}{2}} = \dfrac{\sin 4\alpha \cos 2\alpha}{\cos 4\alpha \cos 2\alpha} =$

$= \dfrac{\sin 4\alpha}{\cos 4\alpha} = \text{tg } 4\alpha$.

б) $\dfrac{\cos 2\alpha - \cos 4\alpha}{\cos 2\alpha + \cos 4\alpha} = \text{tg } 3\alpha \, \text{tg } \alpha$.

$\dfrac{\cos 2\alpha - \cos 4\alpha}{\cos 2\alpha + \cos 4\alpha} = \dfrac{2\sin\frac{2\alpha + 4\alpha}{2}\sin\frac{4\alpha - 2\alpha}{2}}{2\cos\frac{2\alpha + 4\alpha}{2}\cos\frac{4\alpha - 2\alpha}{2}} = \dfrac{\sin 3\alpha \sin \alpha}{\cos 3\alpha \cos \alpha} = \text{tg } 3\alpha \, \text{tg } \alpha$.

22.10.

а) $\cos x + \cos 3x = 0$.

$2\cos\dfrac{x + 3x}{2}\cos\dfrac{3x - x}{2} = 0$; $\cos 2x \cos x = 0$;

$$\left[\begin{array}{l}\cos 2x = 0 \\ \cos x = 0\end{array}\right.; \quad \left[\begin{array}{l}2x = \frac{\pi}{2} + \pi n \\ x = \frac{\pi}{2} + \pi k\end{array}\right.; \quad \left[\begin{array}{l}x = \frac{\pi}{4} + \frac{\pi n}{2} \\ x = \frac{\pi}{2} + \pi k\end{array}\right.$$

б) $\sin 12x + \sin 4x = 0.$

$2 \sin \frac{12x + 4x}{2} \cos \frac{12x - 4x}{2} = 0; \qquad \sin 8x \cos 4x = 0;$

$$\left[\begin{array}{l}\sin 8x = 0 \\ \cos 4x = 0\end{array}\right.; \quad \left[\begin{array}{l}8x = \pi n \\ 4x = \frac{\pi}{2} + \pi k\end{array}\right.; \quad \left[\begin{array}{l}x = \frac{\pi n}{8} \\ x = \frac{\pi}{8} + \frac{\pi k}{4}\end{array}\right.; \ x = \frac{\pi n}{8}$$

в) $\cos x = \cos 5x.$
$\cos x - \cos 5x = 0;$

$2 \sin \frac{x + 5x}{2} \sin \frac{5x - x}{2} = 0; \sin 3x \sin 2x = 0;$

$$\left[\begin{array}{l}\sin 3x = 0 \\ \sin 2x = 0\end{array}\right.; \quad \left[\begin{array}{l}3x = \pi n \\ 2x = \pi k\end{array}\right.; \quad \left[\begin{array}{l}x = \frac{\pi n}{3} \\ x = \frac{\pi k}{2}\end{array}\right.;$$

г) $\sin 3x = \sin 17x.$
$\sin 17x - \sin 3x = 0;$

$2 \sin \frac{17x - 3x}{2} \cos \frac{17x + 3x}{2} = 0;$

$$\sin 7x \cos 10x = 0; \left[\begin{array}{l}\sin 7x = 0 \\ \cos 10x = 0\end{array}\right.; \quad \left[\begin{array}{l}7x = \pi n \\ 10x = \frac{\pi}{2} + \pi k\end{array}\right.; \quad \left[\begin{array}{l}x = \frac{\pi n}{7} \\ x = \frac{\pi}{20} + \frac{\pi k}{10}\end{array}\right.$$

22.11. а) $\sin x + \sin 2x + \sin 3x = 0.$

$\sin 2x + 2 \sin \frac{x + 3x}{2} \cos \frac{3x - x}{2} = 0;$

$\sin 2x + 2\sin 2x \cos x = 0;$
$\sin 2x\,(1 + 2 \cos x) = 0;$

$$\left[\begin{array}{l}\sin 2x = 0 \\ \cos x = -\frac{1}{2}\end{array}\right.; \quad \left[\begin{array}{l}2x = \pi n \\ x = \pm\frac{2\pi}{3} + 2\pi k\end{array}\right.; \quad \left[\begin{array}{l}x = \frac{\pi n}{2} \\ x = \pm\frac{2\pi}{3} + 2\pi k\end{array}\right.$$

б) $\cos 3x - \cos 5x = \sin 4x.$

$2 \sin \frac{3x + 5x}{2} \sin \frac{5x - 3x}{2} = \sin 4x;$

$2\sin 4x \sin x - \sin 4x = 0;$
$\sin 4x\,(2\sin x - 1) = 0;$

$$\left[\begin{array}{l}\sin 4x = 0\\ \sin x = \dfrac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}4x = \pi n\\ x = (-1)^k\dfrac{\pi}{6} + \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \dfrac{\pi n}{4}\\ x = (-1)^k\dfrac{\pi}{6} + \pi k\end{array}\right.$$

22.12. а) $\dfrac{1}{2} - \cos t = \cos\dfrac{\pi}{3} - \cos t = 2\sin\dfrac{\frac{\pi}{3}+t}{2}\sin\dfrac{t-\frac{\pi}{3}}{2} =$

$= 2\sin(\dfrac{\pi}{6} + \dfrac{t}{2})\sin(\dfrac{t}{2} - \dfrac{\pi}{6}).$

б) $\dfrac{\sqrt{3}}{2} + \sin t = \sin\dfrac{\pi}{3} + \sin t = 2\sin\dfrac{\frac{\pi}{3}+t}{2}\cos\dfrac{\frac{\pi}{3}-t}{2} =$

$= 2\sin(\dfrac{\pi}{6} + \dfrac{t}{2})\cos(\dfrac{\pi}{6} - \dfrac{t}{2}).$

в) $1 + 2\cos t = 2\left(\dfrac{1}{2} + \cos t\right) = 2\left(\cos\dfrac{\pi}{3} + \cos t\right) = 4\cos\dfrac{\frac{\pi}{3}+t}{2}\cos\dfrac{\frac{\pi}{3}-t}{2} =$

$= 4\cos(\dfrac{\pi}{6} + \dfrac{t}{2})\cos(\dfrac{\pi}{6} - \dfrac{t}{2}).$

г) $\cos t + \sin t = \cos t + \cos(\dfrac{\pi}{2} - t) = 2\cos\dfrac{t + \frac{\pi}{2} - t}{2}\cos\dfrac{t - \frac{\pi}{2} + t}{2} =$

$= 2\cos\dfrac{\pi}{4}\cos(t - \dfrac{\pi}{4}) = \sqrt{2}\cos\left(t - \dfrac{\pi}{4}\right).$

22.13. а) $\sin 5x + 2\sin 6x + \sin 7x = (\sin 5x + \sin 7x) + 2\sin 6x =$

$= 2\sin\dfrac{5x+7x}{2}\cos\dfrac{7x-5x}{2} + 2\sin 6x =$

$= 2\sin 6x\cos x + 2\sin 6x = 2\sin 6x\,(1 + \cos x) = 4\sin 6x\cos^2\dfrac{x}{2}.$

б) $2\cos x + \cos 2x + \cos 4x = 2\cos x + 2\cos\dfrac{2x+4x}{2}\cos\dfrac{4x-2x}{2} =$

$= 2\cos x + 2\cos x\cos 3x = 2\cos x\,(1+\cos 3x) = 4\cos x\cos^2\dfrac{3x}{2}.$

22.14. а) $\sin t + \sin 2t + \sin 3t + \sin 4t = (\sin t + \sin 4t) + (\sin 2t + \sin 3t) =$

$= 2\sin\dfrac{t+4t}{2}\cos\dfrac{4t-t}{2} + 2\sin\dfrac{2t+3t}{2}\cos\dfrac{3t-2t}{2} =$

$$= 2\sin\frac{5t}{2}\cos\frac{3t}{2} + 2\sin\frac{5t}{2}\cos\frac{t}{2} =$$

$$= 2\sin\frac{5t}{2}\left(\cos\frac{3t}{2} + \cos\frac{t}{2}\right) = 4\sin\frac{5t}{2}\cos\frac{\frac{3t}{2}+\frac{t}{2}}{2}\cos\frac{\frac{3t}{2}-\frac{t}{2}}{2} =$$

$$= 4\sin\frac{5t}{2}\cos t\cos\frac{t}{2}.$$

б) $\cos 2t - \cos 4t - \cos 6t + \cos 8t = (\cos 2t + \cos 8t) - (\cos 4t + \cos 6t) =$

$$= 2\cos\frac{2t+8t}{2}\cos\frac{8t-2t}{2} - 2\cos\frac{4t+6t}{2}\cos\frac{6t-4t}{2} =$$

$$= 2\cos 5t\cos 3t - 2\cos 5t\cos t =$$

$$= 2\cos 5t(\cos 3t - \cos t) = 4\cos 5t\sin\frac{3t+t}{2}\sin\frac{t-3t}{2} =$$

$$= -4\cos 5t\sin 2t\sin t.$$

22.15.

а) $\sin 20° + \sin 40° - \cos 10° = 0.$

$$\sin 20° + \sin 40° - \cos 10° = 2\sin\frac{20° + 40°}{2}\cos\frac{40° - 20°}{2} - \cos 10° =$$

$$= 2\sin 30°\cos 10° - \cos 10° = 2\cdot\frac{1}{2}\cdot\cos 10° - \cos 10° = 0.$$

б) $\cos 85° + \cos 35° - \cos 25° = 0.$

$$\cos 85° + \cos 35° - \cos 25° = 2\cos\frac{85° + 35°}{2}\cos\frac{85° - 35°}{2} - \cos 25° =$$

$$= 2\cos 60°\cos 25° - \cos 25° = 2\cdot\frac{1}{2}\cdot\cos 25° - \cos 25° = 0.$$

22.16. а) $\sin 3x = \cos 2x,$

$$\sin 3x - \sin\left(\frac{\pi}{2} - 2x\right) = 0;$$

$$2\sin\frac{3x - \frac{\pi}{2} + 2x}{2}\cos\frac{3x + \frac{\pi}{2} - 2x}{2} = 0;$$

$$\sin\left(\frac{5x}{2} - \frac{\pi}{4}\right)\cos\left(\frac{x}{2} + \frac{\pi}{4}\right) = 0;$$

$$\left[\begin{array}{l}\sin\left(\frac{5x}{2} - \frac{\pi}{4}\right) = 0\\ \cos\left(\frac{x}{2} + \frac{\pi}{4}\right) = 0\end{array}\right.;\quad \left[\begin{array}{l}\frac{5x}{2} - \frac{\pi}{4} = \pi n\\ \frac{x}{2} + \frac{\pi}{4} = \frac{\pi}{2} + \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \frac{\pi}{10} + \frac{2\pi n}{5}\\ x = \frac{\pi}{2} + 2\pi k\end{array}\right.;\quad x = \frac{\pi}{10} + \frac{2\pi n}{5}$$

б) $\sin(5\pi - x) = \cos(2x + 7\pi)$.

$\sin x = -\cos 2x$;

$$\cos 2x + \cos\left(\frac{\pi}{2} - x\right) = 0;$$

$$2\cos\frac{2x + \frac{\pi}{2} - x}{2}\cos\frac{2x + \frac{\pi}{2} - x}{2} = 0;$$

$$\cos\left(\frac{x}{2} + \frac{\pi}{4}\right)\cos\left(\frac{3x}{2} - \frac{\pi}{4}\right) = 0;$$

$$\left[\begin{array}{l}\cos\left(\frac{x}{2} + \frac{\pi}{4}\right) = 0\\ \cos\left(\frac{3x}{2} - \frac{\pi}{4}\right) = 0\end{array}\right.;\quad \left[\begin{array}{l}\frac{x}{2} + \frac{\pi}{4} = \frac{\pi}{2} + \pi n\\ \frac{3x}{2} - \frac{\pi}{4} = \frac{\pi}{2} + \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \frac{\pi}{2} + 2\pi n\\ x = \frac{\pi}{2} + \frac{2\pi k}{3}\end{array}\right.;\quad x = \frac{\pi}{2} + \frac{2\pi k}{3}.$$

в) $\cos 5x = \sin 15x$

$$\cos 5x - \cos\left(\frac{\pi}{2} - 2x\right) = 0;$$

$$2\sin\frac{5x + \frac{\pi}{2} - 15x}{2}\sin\frac{\frac{\pi}{2} - 15x - 5x}{2} = 0;$$

$$\sin\left(\frac{5x}{2} - \frac{\pi}{4}\right)\sin\left(\frac{\pi}{4} - 10x\right) = 0;\quad \sin\left(5x - \frac{\pi}{4}\right)\sin\left(10x - \frac{\pi}{4}\right) = 0;$$

$$\left[\begin{array}{l}\sin\left(\frac{5x}{2} - \frac{\pi}{4}\right) = 0\\ \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) = 0\end{array}\right.;\quad \left[\begin{array}{l}\frac{5x}{2} - \frac{\pi}{4} = \pi n\\ \frac{x}{2} + \frac{\pi}{4} = \frac{\pi}{2} + \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \frac{\pi}{20} + \frac{\pi n}{5}\\ x = \frac{\pi}{40} + \frac{\pi k}{10}\end{array}\right..$$

г) $\sin(7\pi + x) = \cos(9\pi + 2x)$.

$$-\sin x = -\cos 2x;\quad \cos 2x - \sin x = 0;\quad \cos 2x - \cos\left(\frac{\pi}{2} - x\right) = 0;$$

$$2\sin\frac{2x + \frac{\pi}{2} - x}{2}\sin\frac{2x + \frac{\pi}{2} - x}{2} = 0;$$

$$\sin\left(\frac{x}{2} + \frac{\pi}{4}\right)\sin\left(\frac{\pi}{4} - \frac{3x}{2}\right) = 0;\quad \sin\left(\frac{x}{2} + \frac{\pi}{4}\right)\sin\left(\frac{3x}{2} - \frac{\pi}{4}\right) = 0;$$

$$\left[\begin{array}{l}\sin\left(\frac{x}{2} + \frac{\pi}{4}\right) = 0\\ \sin\left(\frac{3x}{2} - \frac{\pi}{4}\right) = 0\end{array}\right.;\quad \left[\begin{array}{l}\frac{x}{2} + \frac{\pi}{4} = \pi n\\ \frac{3x}{2} - \frac{\pi}{4} = \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = -\frac{\pi}{2} + 2\pi n\\ x = \frac{\pi}{6} + \frac{2\pi k}{3}\end{array}\right.;\quad x = \frac{\pi}{6} + \frac{2\pi k}{3}.$$

22.17. а) $1 + \cos 6x = 2\sin^2 5x$

$1 + \cos 6x = 1 - \cos 10x$; $\cos 6x + \cos 10x = 0$;

$$2\cos\frac{6x+10x}{2}\cos\frac{10x-6x}{2} = 0;$$

$\cos 8x \cos 2x = 0$;

$$\left[\begin{array}{l}\cos 8x = 0\\ \cos 2x = 0\end{array}\right.;\quad \left[\begin{array}{l}8x = \dfrac{\pi}{2} + \pi k\\ 2x = \dfrac{\pi}{2} + \pi n\end{array}\right.;\quad \left[\begin{array}{l}x = \dfrac{\pi}{16} + \dfrac{\pi k}{8}\\ x = \dfrac{\pi}{4} + \dfrac{\pi n}{2}\end{array}\right..$$

б) $\cos^2 2x = \cos^2 4x$.

$$\frac{1}{2}(1 + \cos 4x) = \frac{1}{2}(1 + \cos 8x);$$

$\cos 4x - \cos 8x = 0$;

$$2\sin\frac{4x+8x}{2}\sin\frac{8x-4x}{2} = 0; \qquad \sin 6x \sin 2x = 0;$$

$$\left[\begin{array}{l}\sin 6x = 0\\ \sin 2x = 0\end{array}\right.;\quad \left[\begin{array}{l}6x = \pi n\\ 2x = \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \dfrac{\pi n}{6}\\ x = \dfrac{\pi k}{2}\end{array}\right.;\quad x = \frac{\pi n}{6}.$$

в) $\sin^2\dfrac{x}{2} = \cos^2\dfrac{7x}{2}$

$$\frac{1}{2}(1 - \cos x) = \frac{1}{2}(1 + \cos 7x); \qquad \cos 7x + \cos x = 0;$$

$$2\cos\frac{7x+x}{2}\cos\frac{7x-x}{2} = 0; \qquad \cos 4x \cos 3x = 0;$$

$$\left[\begin{array}{l}\cos 4x = 0\\ \cos 3x = 0\end{array}\right.;\quad \left[\begin{array}{l}4x = \dfrac{\pi}{2} + \pi n\\ 3x = \dfrac{\pi}{2} + \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \dfrac{\pi}{8} + \dfrac{\pi n}{4}\\ x = \dfrac{\pi}{6} + \dfrac{\pi k}{3}\end{array}\right..$$

г) $\sin^2 x + \sin^2 3x = 1$.

$$\frac{1}{2}(1 - \cos 2x) + \frac{1}{2}(1 - \cos 6x); \qquad \cos 2x + \cos 6x = 0;$$

$$2\cos\frac{2x+6x}{2}\cos\frac{2x+6x}{2} = 0; \qquad \cos 4x \cos 2x = 0;$$

$$\left[\begin{array}{l}\cos 4x = 0\\ \cos 2x = 0\end{array}\right.;\quad \left[\begin{array}{l}4x = \dfrac{\pi}{2} + \pi n\\ 2x = \dfrac{\pi}{2} + \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \dfrac{\pi}{8} + \dfrac{\pi n}{4}\\ x = \dfrac{\pi}{4} + \dfrac{\pi k}{2}\end{array}\right..$$

22.18. а) $2\sin^2 x + \cos 5x = 1$.

$1 - \cos 2x + \cos 5x = 1$;

$\cos 5x - \cos 2x = 0$;

$$\left[\begin{array}{l} \sin\frac{7x}{2} = 0 \\ \sin\frac{3x}{2} = 0 \end{array}\right.;\quad \left[\begin{array}{l} \frac{7x}{2} = \pi n \\ \frac{3x}{2} = \pi k \end{array}\right.;\quad \left[\begin{array}{l} x = \frac{2\pi n}{7} \\ x = \frac{2\pi k}{3} \end{array}\right..$$

б) $2\sin^2 3x - 1 = \cos^2 4x - \sin^2 4x$

$-\cos 6x = \cos 8x$;

$\cos 6x + \cos 8x = 0$;

$$2\cos\frac{6x + 10x}{2}\cos\frac{8x - 6x}{2} = 0;$$

$\cos 7x \cos x = 0$;

$$\left[\begin{array}{l} \cos 8x = 0 \\ \cos 2x = 0 \end{array}\right.;\quad \left[\begin{array}{l} 8x = \frac{\pi}{2} + \pi k \\ 2x = \frac{\pi}{2} + \pi n \end{array}\right.;\quad \left[\begin{array}{l} x = \frac{\pi}{16} + \frac{\pi k}{8} \\ x = \frac{\pi}{4} + \frac{\pi n}{2} \end{array}\right..$$

22.19. а) $\operatorname{tg} x + \operatorname{tg} 5x = 0$.

$$\frac{\sin(x + 5x)}{\cos x \cos 5x} = 0;\quad \frac{\sin 6x}{\cos x \cos 5x} = 0;$$

$$\begin{cases} \sin 6x = 0 \\ \cos x \neq 0 \\ \cos 5x \neq 0 \end{cases};\quad \begin{cases} 6x = \pi n \\ x \neq \frac{\pi}{2} + \pi k \\ 5x \neq \frac{\pi}{2} + \pi \ell \end{cases};\quad \begin{cases} x = \frac{\pi n}{6} \\ x \neq \frac{\pi}{2} + \pi k \\ x \neq \frac{\pi}{10} + \frac{\pi \ell}{5} \end{cases};\quad \begin{cases} x = \frac{\pi n}{6} \\ x \neq \frac{\pi}{2} + \pi k \end{cases}.$$

б) $\operatorname{tg} 3x = \operatorname{ctg} x$.

$$\frac{\sin 3x}{\cos 3x} = \frac{\cos x}{\sin x};\quad \begin{cases} \sin 3x \sin x = \cos 3x \cos x \\ \cos 3x \neq 0 \\ \sin x \neq 0 \end{cases};$$

$$\begin{cases} \cos 3x \cos x - \sin 3x \sin x = 0 \\ \cos 3x \neq 0 \\ \sin x \neq 0 \end{cases};\quad \begin{cases} \cos(3x + x) = 0 \\ \cos 3x \neq 0 \\ \sin x \neq 0 \end{cases};$$

$$\begin{cases} \cos 4x = 0 \\ \cos 3x \neq 0; \\ \sin x \neq 0 \end{cases}\quad \begin{cases} 4x = \frac{\pi}{2} + \pi n \\ 3x \neq \frac{\pi}{2} + \pi k; \\ x \neq \pi \ell \end{cases}\quad \begin{cases} x = \frac{\pi}{8} + \frac{\pi n}{4} \\ x \neq \frac{\pi}{6} + \frac{\pi k}{3}; \\ x \neq \pi \ell \end{cases}\quad x = \frac{\pi}{8} + \frac{\pi n}{4}.$$

в) tg $2x$ = tg $4x$.

$$\frac{\sin 2x}{\cos 2x}=\frac{\sin 4x}{\cos 4x};\quad \begin{cases}\sin 2x\cos 4x=\sin 4x\cos 2x\\ \cos 2x\neq 0\\ \cos 4x\neq 0\end{cases};$$

$$\begin{cases}\sin(4x-2x)=0\\ \cos 2x\neq 0\\ \cos 4x\neq 0\end{cases};\ \begin{cases}\sin 2x=0\\ \cos 2x\neq 0;\\ \cos 4x\neq 0\end{cases}\ \begin{cases}2x=\pi n\\ 2x\neq\frac{\pi}{2}+\pi k;\\ 4x\neq\frac{\pi}{2}+\pi\ell\end{cases}\ \begin{cases}x=\frac{\pi n}{2}\\ x\neq\frac{\pi}{4}+\frac{\pi k}{2};\\ x\neq\frac{\pi}{8}+\frac{\pi\ell}{4}\end{cases}\ x=\frac{\pi n}{2}.$$

г) ctg $\frac{x}{2}$ + ctg $\frac{3x}{2}$ = 0.

$$\frac{\cos\frac{x}{2}}{\sin\frac{x}{2}}+\frac{\cos\frac{3x}{2}}{\sin\frac{3x}{2}}=0;$$

$$\begin{cases}\cos\frac{x}{2}\sin\frac{3x}{2}+\cos\frac{3x}{2}\sin\frac{x}{2}=0\\ \sin\frac{x}{2}\neq 0\\ \sin\frac{3x}{2}\neq 0\end{cases};\quad \begin{cases}\sin\left(\frac{3x}{2}+\frac{x}{2}\right)=0\\ \sin\frac{x}{2}\neq 0\\ \sin\frac{3x}{2}\neq 0\end{cases};$$

$$\begin{cases}\sin 2x=0\\ \sin\frac{x}{2}\neq 0\\ \sin\frac{3x}{2}\neq 0\end{cases};\ \begin{cases}2x=\pi n\\ \frac{x}{2}\neq\pi k\\ \frac{3x}{2}\neq\pi\ell\end{cases};\ \begin{cases}x=\frac{\pi n}{2}\\ x\neq 2\pi k\\ x\neq\frac{2\pi\ell}{3}\end{cases};\ \begin{cases}x=\frac{\pi n}{2}\\ n\neq 4k\ \varepsilon\\ 3n\neq 4\ell\end{cases};\ \left[\begin{array}{l}x=\frac{\pi}{2}+\pi k\\ x=\pi+2\pi n\end{array}\right.$$

22.20. а) $\sin x+\sin 3x+\cos x+\cos 3x=0$.

$$2\sin\frac{x+3x}{2}\cos\frac{3x-x}{2}+2\cos\frac{x+3x}{2}\cos\frac{3x-x}{2}=0;$$

$\sin 2x\cos x+\cos 2x\cos x=0$; $\cos x(\sin 2x+\cos 2x)=0$;

$$\sqrt{2}\cos x\cdot\sin\left(\frac{\pi}{4}+2x\right)=0;$$

$$\left[\begin{array}{l}\cos x=0\\ \sin\left(\frac{\pi}{4}+2x\right)=0\end{array}\right.;\ \left[\begin{array}{l}x=\frac{\pi}{2}+\pi n\\ \frac{\pi}{4}+2x=\pi k\end{array}\right.;\ \left[\begin{array}{l}x=\frac{\pi}{2}+\pi n\\ x=-\frac{\pi}{8}+\frac{\pi k}{2}\end{array}\right.\varepsilon.$$

б) $\sin 5x + \sin x + 2\sin^2 x = 1$.

$\sin 5x + \sin x - \cos 2x = 0$;

$$2\sin\frac{5x+x}{2}\cos\frac{5x-x}{2} - \cos 2x = 0;$$

$2\sin 3x\cos 2x - \cos 2x = 0$; $\qquad \cos 2x\,(2\sin 3x - 1) = 0$;

$$\left[\begin{array}{l}\cos 2x = 0\\ \sin 3x = \dfrac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}2x = \dfrac{\pi}{2} + \pi n\\ 3x = (-1)^k\dfrac{\pi}{6} + \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \dfrac{\pi}{4} + \dfrac{\pi n}{2}\\ x = (-1)^k\dfrac{\pi}{18} + \dfrac{\pi k}{3}\end{array}\right..$$

22.21. а) $\sin 2x + \sin 6x = \cos 2x$

$$2\sin\frac{x+3x}{2}\cos\frac{6x-2x}{2} - \cos 2x = 0;$$

$2\sin 4x\cos 2x - \cos 2x = 0$; $\ \cos 2x\,(2\sin 4x - 1) = 0$;

$$\left[\begin{array}{l}\cos 2x = 0\\ \sin 4x = \dfrac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}2x = \dfrac{\pi}{2} + \pi n\\ 4x = (-1)^k\dfrac{\pi}{6} + \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \dfrac{\pi}{4} + \dfrac{\pi n}{2}\\ x = \dfrac{\pi}{24} + 2\pi k\\ x = \dfrac{5\pi}{24} + 2\pi k\end{array}\right..$$

Отсюда видно, что данное уравнение имеет на отрезке $[0, \frac{\pi}{2}]$ 3 корня:

$x = \dfrac{\pi}{4}$, $x = \dfrac{\pi}{24}$ и $x = \dfrac{5\pi}{24}$.

б) $2\cos^2 x - 1 = \sin 3x$

$\cos 2x = \sin 3x$; $\qquad \cos 2x - \cos\left(\dfrac{\pi}{2} - 3x\right) = 0$;

$$2\sin\frac{2x + \frac{\pi}{2} - 3x}{2}\sin\frac{\frac{\pi}{2} - 3x - 2x}{2} = 0;$$

$$\sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\sin\left(\frac{\pi}{4} - \frac{5x}{2}\right) = 0;\quad \sin\left(\frac{\pi}{4} - \frac{x}{2}\right)\sin\left(\frac{5x}{2} - \frac{\pi}{4}\right) = 0;$$

$$\left[\begin{array}{l}\sin\left(\dfrac{x}{2} - \dfrac{\pi}{4}\right) = 0\\ \sin\left(\dfrac{5x}{2} - \dfrac{\pi}{4}\right) = 0\end{array}\right.;\quad \left[\begin{array}{l}\dfrac{x}{2} - \dfrac{\pi}{4} = \pi n\\ \dfrac{5x}{2} - \dfrac{\pi}{4} = \pi k\end{array}\right.;\quad \left[\begin{array}{l}x = \dfrac{\pi}{2} + 2\pi n\\ x = \dfrac{\pi}{10} + \dfrac{2\pi k}{5}\end{array}\right..$$

Отсюда видно, что данное уравнение имеет на отрезке $[0, \frac{\pi}{2}]$ 2 корня:

$x = \dfrac{\pi}{4}$ и $x = \dfrac{\pi}{2}$.

22.22. а) $\cos 6x + \cos 8x = \cos 10x + \cos 12x$

$2\cos\dfrac{6x+8x}{2}\cos\dfrac{8x-6x}{2} = 2\cos\dfrac{10x+12x}{2}\cos\dfrac{12x-10x}{2}$;

$\cos 7x \cos x = \cos 11x \cos x$;

$\cos x(\cos 7x - \cos 11x) = 0$;

$2\cos x \sin\dfrac{7x+11x}{2}\sin\dfrac{11x-7x}{2} = 0$;

$\cos x \sin 9x \sin 2x = 0$;

$\left[\begin{array}{l}\cos x = 0\\ \sin 9x = 0,\\ \sin 2x = 0\end{array}\right.\left[\begin{array}{l}x = \dfrac{\pi}{2}+\pi n\\ x = \dfrac{\pi k}{9}\\ x = \dfrac{\pi l}{2}\end{array}\right.,\left[\begin{array}{l}x = \dfrac{\pi k}{2}\\ x = \dfrac{\pi n}{9}\end{array}\right.$ т.к. $x \in (0; 2{,}5)$, то

$x = \dfrac{\pi}{2}, \dfrac{\pi}{9}, \dfrac{2\pi}{9}, \dfrac{\pi}{3}, \dfrac{4\pi}{9}, \dfrac{5\pi}{9}, \dfrac{2\pi}{3}, \dfrac{7\pi}{9}$;

б) $\sin 2x + 5\sin 4x + \sin 6x = 0$. $(\sin 2x + \sin 6x) + 5\sin 4x = 0$;
$2\sin 4x\cos 2x + 5\sin 4x = 0$; $\sin 4x(2\cos 2x + 5) = 0$;

$\sin 4x = 0$ (т.к. $2\cos 2x + 5 > 0$ при всех x); $4x = \pi n$; $x = \dfrac{\pi n}{4}$.

Т.к. $x \in (0; 2{,}5)$, то $x = \dfrac{\pi}{4}, \dfrac{\pi}{2}, \dfrac{3\pi}{4}$.

§ 23. Преобразование произведения тригонометрических функций в суммы

23.1. а) $\sin 23^\circ \sin 32^\circ = \dfrac{1}{2}(\cos(32^\circ - 23^\circ) - \cos(32^\circ + 23^\circ)) =$

$= \dfrac{1}{2}(\cos 9^\circ - \cos 55^\circ)$.

б) $\cos\dfrac{\pi}{12}\cos\dfrac{\pi}{8} = \dfrac{1}{2}\left(\cos\left(\dfrac{\pi}{8}-\dfrac{\pi}{12}\right)+\cos\left(\dfrac{\pi}{8}+\dfrac{\pi}{12}\right)\right) =$

$= \dfrac{1}{2}\left(\cos\dfrac{\pi}{24}+\cos\dfrac{5\pi}{24}\right)$.

в) $\sin 14^\circ \sin 16^\circ = \dfrac{1}{2}(\cos(16^\circ - 14^\circ) - \cos(16^\circ + 14^\circ)) =$

$= \dfrac{1}{2}(\cos 2^\circ - \cos 30^\circ) = \dfrac{1}{2}\left(\cos 2^\circ - \dfrac{\sqrt{3}}{2}\right)$.

г) $2\sin\frac{\pi}{8}\cos\frac{\pi}{5}=\sin\left(\frac{\pi}{8}+\frac{\pi}{5}\right)+\sin\left(\frac{\pi}{8}-\frac{\pi}{5}\right)=\sin\frac{13\pi}{40}-\sin\frac{3\pi}{40}$.

23.2. а) $\sin(\alpha+\beta)\sin(\alpha-\beta)=\sin2\alpha-\sin2\beta=\cos2\beta-\cos2\alpha=$

$=\frac{1}{2}(\cos2\beta-\cos2\alpha)$.

б) $\cos(\alpha+\beta)\cos(\alpha-\beta)=\cos^2\beta-\sin^2\alpha=\cos^2\alpha-\sin^2\beta=$

$=\frac{1}{2}(\cos2\beta+\cos2\alpha)$;

в) $\cos\left(\frac{\alpha}{2}+\frac{\beta}{2}\right)\cos\left(\frac{\alpha}{2}-\frac{\beta}{2}\right)=\cos^2\frac{\beta}{2}-\sin^2\frac{\alpha}{2}=\cos^2\frac{\alpha}{2}-\sin^2\frac{\beta}{2}=$

$=\frac{1}{2}(\cos\beta+\cos\alpha)$;

г) $2\sin(\alpha+\beta)\cos(\alpha-\beta)=\sin2\alpha+\sin2\beta$.

23.3. а) $\cos\alpha\sin(\alpha+\beta)=\frac{1}{2}(\sin(2\alpha+\beta)+\sin(\alpha+\beta-\alpha))=$

$=\frac{1}{2}(\sin(2\alpha+\beta)+\sin\beta))$.

б) $\sin(60^\circ+\alpha)\sin(60^\circ-\alpha)=\frac{1}{2}(\cos2\alpha-\cos120^\circ)=\frac{1}{2}\left(\cos2\alpha+\frac{1}{2}\right)$.

в) $\sin\beta\cos(\alpha+\beta)=\frac{1}{2}(\sin(\alpha+2\beta)+\sin(\beta-\alpha-\beta))=$

$=\frac{1}{2}(\sin(\alpha+2\beta)-\sin\alpha))$.

г) $\cos\left(\alpha+\frac{\pi}{4}\right)\cos\left(\alpha-\frac{\pi}{4}\right)=\frac{1}{2}\left(\cos2\alpha+\cos\frac{\pi}{2}\right)=\frac{1}{2}\cos2\alpha$.

23.4. а) $\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)-0{,}25=0$;

$\frac{1}{2}\left(\cos\left(\frac{\pi}{8}-\frac{\pi}{12}\right)+\cos\left(\frac{\pi}{8}+\frac{\pi}{12}\right)\right)=\frac{1}{4}$;

$\frac{1}{2}\left(\cos\frac{2\pi}{3}+\cos2x\right)=\frac{1}{4}$; $\frac{1}{2}\left(-\frac{1}{2}+\cos2x\right)=\frac{1}{4}$;

$\cos2x=1$; $2x=2\pi n$; $x=\pi n$;

б) $\sin\left(x+\frac{\pi}{3}\right)\cos\left(x+\frac{\pi}{3}\right)=1$.

$\frac{1}{2}\left(\sin\left(x+\frac{\pi}{3}+x-\frac{\pi}{6}\right)+\sin\left(x+\frac{\pi}{3}-x+\frac{\pi}{6}\right)\right)=1$;

$$\frac{1}{2}\left(\sin\left(2x+\frac{\pi}{6}\right)+\sin\frac{\pi}{2}\right)=1;$$

$$\frac{1}{2}\sin\left(2x+\frac{\pi}{6}\right)+\frac{1}{2}=1; \qquad \sin\left(2x+\frac{\pi}{6}\right)+\frac{1}{2}=1;$$

$$2x+\frac{\pi}{6}=\frac{\pi}{2}+2\pi n;\ x=\frac{\pi}{6}+\pi n$$

23.5. а) $2\sin x\cos 3x+\sin 4x=0$.
$\sin(x+3x)+\sin(x-3x)+\sin 4x=0$;
$2\sin 4x-\sin 2x=0$;
$4\sin 2x\cos 2x-\sin 2x=0$;
$\sin 2x(4\cos 2x-1)=0$;

$$\left[\begin{array}{l}\sin 2x=0\\ \cos 2x=\frac{1}{4}\end{array}\right.;\quad \left[\begin{array}{l}2x=\pi n\\ 2x=\pm\arccos\frac{1}{4}+2\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi n}{2}\\ x=\pm\frac{1}{2}\arccos\frac{1}{4}+\pi k\end{array}\right.$$

б) $\sin\frac{x}{2}\sin\frac{3x}{2}=\frac{1}{2}$, $\frac{1}{2}\left(\cos\left(\frac{x}{2}-\frac{3x}{2}\right)-\cos\left(\frac{x}{2}+\frac{3x}{2}\right)\right)=\frac{1}{2}$;

$\cos x-\cos 2x=1$; $2\cos^2 x-\cos x=0$; $\cos x(2\cos x-1)=0$;

$$\left[\begin{array}{l}\cos x=0\\ \cos x=\frac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi}{2}+\pi n\\ x=\pm\frac{\pi}{3}+2\pi k\end{array}\right.$$

23.6. а) $2\sin t\sin 2t+\cos 3t=\cos t$.
$2\sin t\sin 2t+\cos 3t=\cos(2t-t)-\cos(2t+t)+\cos 3t=\cos t$.

б) $\sin\alpha-2\sin\left(\frac{\alpha}{2}-15^\circ\right)\cos\left(\frac{\alpha}{2}+15^\circ\right)=\frac{1}{2}$.

$$\sin\alpha-2\sin\left(\frac{\alpha}{2}-15^\circ\right)\cos\left(\frac{\alpha}{2}+15^\circ\right)=\sin\alpha-$$

$$-\sin\left(\frac{\alpha}{2}-15^\circ+\frac{\alpha}{2}+15^\circ\right)-\sin\left(\frac{\alpha}{2}-15^\circ-\frac{\alpha}{2}-15^\circ\right)=$$

$$=\sin\alpha-\sin\alpha-\sin(-30^\circ)=\sin 30^\circ=\frac{1}{2}.$$

23.7. а) $\sin 10^\circ\cos 8^\circ\cos 6^\circ=\frac{1}{2}\sin 10^\circ(\cos(8^\circ-6^\circ)+\cos(8^\circ+6^\circ))=$

$$=\frac{1}{2}\sin 10^\circ\cos 2^\circ+\frac{1}{2}\sin 10^\circ\cos 14^\circ=$$

$$=\frac{1}{4}(\sin 12^\circ+\sin 8^\circ+\sin 24^\circ-\sin 4^\circ).$$

б) $4\sin 25^\circ \cos 15^\circ \sin 5^\circ = 2\sin 25^\circ (\sin 20^\circ + \sin(-10^\circ)) =$
$= 2(\sin 25^\circ \sin 20^\circ - \sin 25^\circ \sin 10^\circ) =$
$= \cos 5^\circ - \cos 45^\circ - \cos 15^\circ + \cos 35^\circ =$

$= \cos 5^\circ - \cos 15^\circ + \cos 35^\circ - \dfrac{1}{\sqrt{2}}$

23.8. а) $\cos^2 3^\circ + \cos^2 1^\circ - \cos 4^\circ \cos 2^\circ = \cos^2 3^\circ + \cos^2 1^\circ - \cos(3^\circ + 1^\circ)\cos(3^\circ - 1^\circ) =$
$= \cos^2 3^\circ + \cos^2 1^\circ - \cos^2 1^\circ + \sin^2 3^\circ = \cos^2 3^\circ + \sin^2 3^\circ = 1$
б) $\sin^2 10^\circ + \cos 50^\circ \cos 70^\circ = \sin^2 10^\circ + \cos(60^\circ - 10^\circ)\cos(60^\circ + 10^\circ) =$

$= \sin^2 10^\circ + \cos^2 60^\circ - \sin^2 10^\circ = \cos^2 60^\circ = \dfrac{1}{4}.$

23.9. а) $\dfrac{1}{2\sin 10^\circ} - 2\sin 70^\circ = \dfrac{1 - 4\sin 10^\circ \sin 70^\circ}{2\sin 10^\circ} =$

$$= \frac{1 - 2\cos(70^\circ - 10^\circ) + 2\cos(70^\circ + 10^\circ)}{2\sin 10^\circ} = \frac{1 - 2\cos 60^\circ + 2\cos 80^\circ}{2\sin 10^\circ} =$$

$$= \frac{1 - 2\cdot\frac{1}{2} + 2\sin 10^\circ}{2\sin 10^\circ} = 1.$$

б) $\dfrac{\operatorname{tg} 60^\circ}{\sin 40^\circ} + 4\cos 100^\circ = \dfrac{\operatorname{tg} 60^\circ + 4\sin 40^\circ \cos 100^\circ}{\sin 40^\circ} =$

$$= \frac{\operatorname{tg} 60^\circ + 2\sin 140^\circ + 2\sin(-60^\circ)}{\sin 40^\circ} = \frac{\sqrt{3} + 2\sin 40^\circ - 2\cdot\frac{\sqrt{3}}{2}}{\sin 40^\circ} =$$

$$= \frac{2\sin 40^\circ}{\sin 40^\circ} = 2.$$

23.10. а) $\sin 3x \cos x = \sin\dfrac{5x}{2}\cos\dfrac{3x}{2}.$

$$\frac{1}{2}(\sin(3x + x) + \sin(3x - x)) = \frac{1}{2}\left(\sin\left(\frac{5x}{2} + \frac{3x}{2}\right) + \sin\left(\frac{5x}{2} - \frac{3x}{2}\right)\right);$$

$\sin 4x + \sin 2x = \sin 4x + \sin x;$
$\sin 2x - \sin x = 0;$

$2\sin\dfrac{2x - x}{2}\cos\dfrac{2x + x}{2} = 0;$

$\sin\dfrac{x}{2}\cos\dfrac{3x}{2} = 0;$

$$\left[\begin{array}{l} \sin\frac{x}{2} = 0 \\ \cos\frac{3x}{2} = 0 \end{array}\right.; \quad \left[\begin{array}{l} \frac{x}{2} = \pi n \\ \frac{3x}{2} = \frac{\pi}{2} + \pi k \end{array}\right.; \quad \left[\begin{array}{l} x = 2\pi n \\ x = \frac{\pi}{3} + \frac{2\pi k}{3} \end{array}\right.$$

б) $2\sin\left(\frac{\pi}{4}+x\right)\sin\left(\frac{\pi}{4}-x\right)+\sin^2 x=0.$

$2\left(\sin^2\frac{\pi}{4}-\sin^2 x\right)+\sin^2 x=0;$

$1-\sin^2 x=0;\qquad -\frac{1}{2}(1-\cos 2x)+1=0;\ \cos 2x=-1;$

$2x=\pi+2\pi n;\qquad x=\frac{\pi}{2}+\pi n.$

в) $\sin 2x\cos x=\sin x\cos 2x.$

$\frac{1}{2}(\sin 3x+\sin x)=\frac{1}{2}(\sin 3x-\sin x);\quad \sin x=0;\quad x=\pi n.$

г) $\cos 2x\cos x=\cos 2{,}5x\cos 0{,}5x.$

$\frac{1}{2}(\cos x+\cos 3x)=\frac{1}{2}(\cos 2x+\cos 3x);$

$\cos x=\cos 2x;\qquad \cos x-\cos 2x=0;$

$2\sin\frac{x+2x}{2}\sin\frac{2x-x}{2}=0;\qquad \sin\frac{3x}{2}\sin\frac{x}{2}=0;$

$$\left[\begin{array}{l}\sin\frac{3x}{2}=0\\ \sin\frac{x}{2}=0\end{array}\right.;\quad \left[\begin{array}{l}\frac{3x}{2}=\pi n\\ \frac{x}{2}=\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{2\pi n}{3}\\ x=2\pi k\end{array}\right..$$

23.11. а) $\sin x\sin 3x=0{,}5.$

$\frac{1}{2}(\cos 2x-\cos 4x)=\frac{1}{2};\qquad \cos 2x=1+\cos 4x;$

$\cos 2x=2\cos^2 2x;\quad \cos 2x(2\cos 2x-1)=0;$

$$\left[\begin{array}{l}\cos 2x=0\\ \cos 2x=\frac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}2x=\frac{\pi}{2}+\pi n\\ 2x=\pm\frac{\pi}{3}+2\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi}{4}+\frac{\pi n}{2}\\ x=\pm\frac{\pi}{6}+\pi k\end{array}\right.$$

, отсюда наименьший положительный корень $\frac{\pi}{6}$, наибольший отрицательный корень $-\frac{\pi}{6}$;

б) $\cos x\cos 3x+0{,}5=0$

$\frac{1}{2}(\cos 2x+\cos 4x)+\frac{1}{2}=0;\ \cos 2x+(1+\cos 4x)=0;$

$\cos 2x+2\cos^2 2x=0;\ \cos 2x(2\cos 2x+1)=0;$

$$\left[\begin{array}{l}\cos 2x=0\\ \cos 2x=-\frac{1}{2}\end{array}\right.;\quad \left[\begin{array}{l}2x=\frac{\pi}{2}+\pi n\\ 2x=\pm\frac{\pi}{3}+2\pi k\end{array}\right.;\quad \left[\begin{array}{l}x=\frac{\pi}{4}+\frac{\pi n}{2}\\ x=\pm\frac{\pi}{6}+\pi k\end{array}\right.$$

, отсюда наименьший положительный корень $+\frac{\pi}{4}$, наибольший отрицательный корень $-\frac{\pi}{4}$.

23.12. а) $f(x) = \sin\left(x+\frac{\pi}{8}\right)\cos\left(x-\frac{\pi}{24}\right) =$

$= \frac{1}{2}\left(\sin\left(x+\frac{\pi}{8}+x-\frac{\pi}{24}\right)+\sin\left(x+\frac{\pi}{8}-x+\frac{\pi}{24}\right)\right) =$

$= \frac{1}{2}\sin\left(2x+\frac{\pi}{12}\right)+\frac{1}{2}\sin\frac{\pi}{6} = \frac{1}{2}\sin\left(2x+\frac{\pi}{12}\right)+\frac{1}{4}$.

Поскольку наибольшее и наименьшее значения функции

$y=\sin\left(2x+\frac{\pi}{12}\right)$ равны 1 и -1 соответственно, то наибольшее и наименьшее значения функции $f(x)$ равны $\frac{3}{4}$ и $-\frac{1}{4}$ соответственно.

б) $f(x) =$

$\sin\left(x+\frac{\pi}{8}\right)\sin\left(x+\frac{\pi}{3}\right)=\frac{1}{2}\left(\cos\left(x+\frac{\pi}{3}-x+\frac{\pi}{3}\right)-\cos\left(x+\frac{\pi}{3}+x-\frac{\pi}{3}\right)\right)=$

$=\frac{1}{2}\left(\cos\frac{2\pi}{3}-\cos 2x\right)=-\frac{1}{4}-\frac{1}{2}\cos 2x$.

Поскольку наибольшее и наименьшее значения функции $y=\cos 2x$ равны 1 и -1 соответственно, то наибольшее и наименьшее значения функции $f(x)$ равны $\frac{1}{4}$ и $-\frac{3}{4}$ соответственно.

23.13. $\cos^2(45^\circ-\alpha)-\cos^2(60^\circ+\alpha)-\cos 75^\circ\sin(75^\circ-2\alpha)=\sin 2\alpha$.

$\cos^2(45^\circ-\alpha)-\cos^2(60^\circ+\alpha)-\cos 75^\circ\sin(75^\circ-2\alpha)=$

$=\cos^2(45^\circ-\alpha)-\cos^2(60^\circ+\alpha)-\frac{1}{2}(\sin(75^\circ-2\alpha+75^\circ)+$

$+\sin(75^\circ-2\alpha-75^\circ))=\frac{1}{2}(1+\cos(90^\circ-2\alpha))-\frac{1}{2}(1+\cos(120^\circ+$

$+2\alpha))-\frac{1}{2}\sin(150^\circ-2\alpha)+\frac{1}{2}\sin 2\alpha=\sin 2\alpha-\frac{1}{2}(\cos 120^\circ\cos 2\alpha-$

$-\sin 120^\circ\sin 2\alpha)-\frac{1}{2}(\sin 150^\circ\cos 2\alpha-\sin 2\alpha\cos 150^\circ)=$

$=\sin 2\alpha-\frac{1}{2}(-\frac{1}{2}\cos 2\alpha-\frac{\sqrt{3}}{2}\sin 2\alpha+\frac{1}{2}\cos 2\alpha+$

$+\frac{\sqrt{3}}{2}\sin 2\alpha)=\sin 2\alpha$.

Глава 5. Производная

§ 24. Предел последовательности

24.1. а) $y_n = 3 - 2n$.

$y_1 = 1,\quad y_2 = -1,\quad y_3 = -3,\quad y_4 = -5,\quad y_5 = -7.$

б) $y_n = 2n^2 - n$.

$y_1 = 1,\quad y_2 = 6,\quad y_3 = 15,\quad y_4 = 28,\quad y_5 = 45.$

в) $y_n = n^3 - 1$.

$y_1 = 0,\quad y_2 = 7,\quad y_3 = 26,\quad y_4 = 63,\quad y_5 = 124.$

г) $\frac{3n-1}{2n}$..

$y_1 = 1,\quad y_2 = \frac{5}{4} = 1\frac{1}{4},\quad y_3 = \frac{8}{6} = 1\frac{1}{3},$

$y_4 = \frac{11}{8} = 1\frac{3}{8},\quad y_5 = \frac{7}{5} = 1\frac{2}{5}.$

24.2. а) $y_n = (-1)^n$.

$y_1 = -1,\quad y_2 = 1,\quad y_3 = -1,\quad y_4 = 1,\quad y_5 = -1.$

б) $y_n = \frac{(-2)^n}{n^2+1}$.

$y_1 = -1,\quad y_2 = \frac{4}{5},\quad y_3 = -\frac{8}{10} = -\frac{4}{5},$

$y_4 = \frac{16}{17},\quad y_5 = -\frac{32}{26} = -\frac{16}{13} = -1\frac{3}{13}.$

в) $y_n = (-1)^n \frac{1}{10^n}$.

$y_1 = -\frac{1}{10},\quad y_2 = \frac{1}{100},\quad y_3 = -\frac{1}{1000},$

$y_4 = \frac{1}{10000},\quad y_5 = \frac{1}{10000}.$

г) $y_n = \frac{(-1)^n + 2}{3n-2}$.

$y_1 = 1,\quad y_2 = \frac{3}{4},\quad y_3 = \frac{1}{7},\quad y_4 = \frac{3}{10},\quad y_5 = \frac{1}{13}.$

24.3. а) $y_n = 3\cos\frac{2\pi}{n}$.

$y_1 = 3,\quad y_2 = -3,\quad y_3 = -\frac{3}{2},\quad y_4 = 0,\quad y_5 = 3\cos\frac{2\pi}{5}.$

б) $y_n = \text{tg}\left((-1)^n \frac{\pi}{4}\right)$.

$y_1 = -1,$ $y_2 = 1,$ $y_3 = -1,$ $y_4 = 1,$ $y_5 = -1.$

в) $y_n = 1 - \cos^2 \frac{\pi}{n}$.

$y_1 = 0,$ $y_2 = 1,$ $y_3 = \frac{3}{4},$ $y_4 = \frac{1}{2},$ $y_5 = \sin^2 \frac{\pi}{5}$.

г) $y_n = \sin \pi n - \cos \pi n = -\cos \pi n$.

$y_1 = 1,$ $y_2 = -1,$ $y_3 = 1,$ $y_4 = -1,$ $y_5 = 1.$

24.4. $2^2 + 3^2 + 5^2 + 7^2 + 11^2 + 13^2 + 17^2 + 19^2 =$
$= 4 + 9 + 25 + 49 + 121 + 169 + 289 + 361 = 1027$

24.5. а) $y_n = n - 1, n \in \mathrm{N}$. б) $y_n = -n, n \in \mathrm{N}$.
в) $y_n = n + 4, n \in \mathrm{N}$. г) $y_n = 11 - n, n \in \mathrm{N}$.

24.6. а) $y_n = 5n, n \in \mathrm{N}$. б) $y_n = 6n, n \in \mathrm{N}$.
в) $y_n = 4n, n \in \mathrm{N}$. г) $y_n = 3n, n \in \mathrm{N}$.

24.7. а) $y_n = 3^n,$ $n \in \mathrm{N}$.
б) $y_n = (n + 2)^2,$ $n \in \mathrm{N}$.
в) $y_n = n^3,$ $n \in \mathrm{N}$.
г) $y_n = n^3 + 1,$ $n \in \mathrm{N}$.

24.8. а) $y_n = \frac{2}{2^n} = \frac{1}{2^{n-1}}$; б) $y_n = \frac{2n+1}{2n+2}$;

в) $y_n = \frac{1}{n^3}$; г) $y_n = \frac{1}{(2n+1)(2n+3)}$.

24.9. а) 1; 1,4; 1,41; 1,414; 1,4142.
б) 2; 1,5; 1,42; 1,415; 1,4143.

24.10. $y_n = \frac{2-n}{5n+1}$.

а) $y_n = 0,$ $n = 2.$

б) $y_n = -\frac{3}{26},$ $n = 5.$

в) $y_n = -\frac{1}{6},$ $\frac{2-n}{5n+1} = -\frac{1}{6}$.

$6n - 12 = 5n + 1,$ $n = 13.$

г) $-\frac{43}{226} = y_n.$ $\frac{2-n}{5n+1} = -\frac{43}{226}$

$226n - 452 = 215n + 43,$ $11n = 495,$ $n = 45.$

24.11. $a_n = (2n - 1)(3n + 2)$ а) $a_n = 0$.

нет, т.к. n не может быть равным $\frac{1}{2}, -\frac{2}{3}$.

б) $a_n = 24$. $6n^2 + n - 2 = 24$.
$6n^2 + n - 26 = 0$.

$n = \frac{-1+25}{12} = 2$, или $n = -\frac{26}{12}$. (не подходит, т.к. $n \in N$).

Ответ: является.

в) $a_n = 153$. $6n^2 + n - 155 = 0$.

$n = \frac{-1+61}{12} = 5$ $n = -\frac{62}{12}$ (не подходит).

Ответ: является.

г) $a_n = -2$. $6n^2 + n = 0$.
$n(6n + 1) = 0$.
Решения в натуральных числах нет $\Rightarrow$ не является.

24.12. а) 1; $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$… ограничена снизу.

б) –1; 2; –3; 4; –5… не ограничена снизу.

в) $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$… ограничена снизу.

г) 5; 4; 3; 2; 1; 0; –1… не ограничена снизу.

24.13. а) –3; –2; –1; 0; 1… не ограничена сверху.

б) 1; –1; 1; –2; 1; –3… ограничена сверху.

в) $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$; $\frac{1}{5}$… ограничена сверху.

г) $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$… ограничена сверху.

24.14. а) $\frac{1}{2}$; $\frac{1}{3}$; $\frac{1}{4}$;… $\frac{1}{n}$… ограничена, т.к. $0 < y_n < 1$.

б) $\frac{1}{2}$; $\frac{3}{4}$; $\frac{5}{6}$;… $\frac{2n-1}{2n}$…ограничена, т.к. $0 < y_n < 1$.

в) 5; –5; 5; –5; … $(-1)^{n-1}5$ ограничена, т.к. $-10 < y_n < 10$.

г) –2; 3; –4; 5; … $(-1)^n(n+1)$, … не ограничена.

24.15. а) $y_n = 2n - 1$; $y_{n+1} = 2n + 1$; $y_{n+1} > y_n \Rightarrow$ возрастающая.

б) $y_n = 5^{-n}$; $y_{n+1} = 5^{-n-1} = \frac{5^{-n}}{5}$; $y_{n+1} < y_n \Rightarrow$ убывающая.

в) $y_n = n^2 + 8$, $y_{n+1} = n^2 + 2n + 9$; $y_{n+1} > y_n \Rightarrow$ возрастающая.

г) $y_n = \frac{2}{3n+1}$, $y_{n+1} = \frac{2}{3n+4}$, $y_{n+1} < y_n \Rightarrow$ убывающая.

24.16.

а) $x_n = (-2)^n$ $x_1 = -2$, $x_2 = 4$, $x_3 = -8$

$\Rightarrow$ последовательность не является монотонной.

б) $y_n = \cos\frac{\pi}{n+5}$.

$y_1 = \cos\frac{\pi}{6}$, $y_2 = \cos\frac{\pi}{7}$, $y_3 = \cos\frac{\pi}{8}$.

$y_1 < y_2 < y_3 \ldots$ $\Rightarrow$ возрастающая.

в) $y_n = n^3 - 5$

$y_1 = -4$, $y_2 = 3$, $y_3 = 22$. $\Rightarrow$ возрастающая.

г) $y_n = \sqrt{n+8}$.

$y_1 = 3$, $y_2 = \sqrt{10}$, $y_3 = \sqrt{11}$ $\Rightarrow$ возрастающая.

24.17. а) $y_n = 2^n$; б) $y_n = 2^n$;

в) $y_n = \left(\frac{1}{2}\right)^n$; г) $y_n = -n$.

24.18. а) $\lim\limits_{n\to\infty}\frac{5}{n^2} = 0$. б) $\lim\limits_{n\to\infty}\left(-\frac{17}{n^3}\right) = 0$.

в) $\lim\limits_{n\to\infty}\left(-\frac{15}{n^2}\right) = 0$. г) $\lim\limits_{n\to\infty}\frac{3}{\sqrt{n}} = 0$.

24.19. а) $\lim\limits_{n\to\infty}\left(\frac{7}{n} + \frac{8}{\sqrt{n}} + \frac{9}{n^3}\right) = 0 + 0 + 0 = 0$.

б) $\lim\limits_{n\to\infty}\left(6 - \frac{7}{n^2} - \frac{3}{n} - \frac{3}{\sqrt{n}}\right) = 6 - 0 - 0 - 0 = 6$.

в) $\lim\limits_{n\to\infty}\left(\frac{3}{n} + \frac{7}{n^2} - \frac{5}{n^3} + \frac{13}{n^4}\right) = 0 + 0 - 0 + 0 = 0$.

г) $\lim\limits_{n\to\infty}\left(\frac{1}{n} + \frac{3}{\sqrt{n}} - 4 + \frac{7}{n^2}\right) = 0 + 0 - 4 + 0 = -4$.

24.20. а) $\lim\limits_{n\to\infty}\frac{5n+3}{n+1} = \lim\limits_{n\to\infty}\frac{5+\frac{3}{n}}{1+\frac{1}{n}} = 5$. б) $\lim\limits_{n\to\infty}\frac{7n-5}{n+2} = \lim\limits_{n\to\infty}\frac{7-\frac{5}{n}}{1+\frac{2}{n}} = 7$.

в) $\lim\limits_{n\to\infty}\frac{3n+1}{n+2} = \lim\limits_{n\to\infty}\frac{3+\frac{1}{n}}{1+\frac{2}{n}} = 3$. г) $\lim\limits_{n\to\infty}\frac{2n+1}{3n-1} = \lim\limits_{n\to\infty}\frac{2+\frac{1}{n}}{3-\frac{1}{n}} = \frac{2}{3}$.

24.21. а) $\lim\limits_{n\to\infty}\frac{5}{2^n} = 0$. б) $\lim\limits_{n\to\infty}\frac{1}{2}\cdot 5^{-n} = 0$.

в) $\lim\limits_{n\to\infty}7\cdot 3^{-n} = 0$. г) $\lim\limits_{n\to\infty}\frac{4}{3^{n+1}} = 0$.

24.22. а) $\lim\limits_{n\to\infty}\frac{2n^2-1}{n^2}=\lim\limits_{n\to\infty}\left(2-\frac{1}{n^2}\right)=2.$

б) $\lim\limits_{n\to\infty}\frac{\left(1+2n+n^2\right)}{n^2}=\lim\limits_{n\to\infty}\left(\frac{1}{n^2}+\frac{2}{n}+1\right)=1.$

в) $\lim\limits_{n\to\infty}\frac{3-n^2}{n^2}=\lim\limits_{n\to\infty}\left(\frac{3}{n^2}-1\right)=-1.$

г) $\lim\limits_{n\to\infty}\frac{3n-4-2n^2}{n^2}=\lim\limits_{n\to\infty}\left(\frac{3}{n}-\frac{4}{n^2}-2\right)=-2.$

§ 25. Сумма бесконечной геометрической прогрессии

25.1. а) $b_1=3;\ q=\frac{1}{3}.\ \ S_n=\frac{3}{1-\frac{1}{3}}=4\frac{1}{2}.$

б) $b_1=-5;\ q=-0{,}1.\ S_n=\frac{-5}{1+0{,}1}=-\frac{50}{11}=-4\frac{6}{11}.$

в) $b_1=-1;\ q=0{,}2.\ \ S_n=\frac{-1}{0{,}8}=-\frac{5}{4}=-1\frac{1}{4}.$

г) $b_1=2;\ q=-\frac{1}{3}.\ \ S_n=\frac{2}{1+\frac{1}{3}}=\frac{6}{4}=\frac{3}{2}.$

25.2. а) 32, 16, 8, 4…

$b_1=32;\ q=\frac{1}{2}.\ \ S_n=\frac{32}{1-\frac{1}{2}}=64.$

б) $24,\ -8,\ \frac{8}{3},\ \ -\frac{8}{9}\ldots$

$b_1=24;\ q=-\frac{1}{3}.\ \ S_n=\frac{24}{1+\frac{1}{3}}=\frac{24\cdot 3}{4}=18.$

в) $27,\ 9,\ 3,\ 1,\ \frac{1}{3}\ldots$

$b_1=27;\ q=\frac{1}{3}.\ \ S_n=\frac{27}{\frac{2}{3}}=\frac{81}{2}=40{,}5.$

г) 18, –6, 2, $-\frac{1}{3}$...

$b_1 = 18;\ q = -\frac{1}{3}.\quad S_n = \frac{18}{1+\frac{1}{3}} = \frac{27}{2} = 13{,}5.$

25.3. а) $2 + 1 + \frac{1}{2} + \frac{1}{4}$...

$b_1 = 2 \qquad q = \frac{1}{2}. \qquad S_n = \frac{2}{1-\frac{1}{2}} = 4.$

б) $49 + 7 + 1 + \frac{1}{7}$...

$b_1 = 49 \qquad q = \frac{1}{7}. \qquad S_n = \frac{49}{1-\frac{1}{7}} = 57\frac{1}{6}.$

в) $\frac{3}{2} - 1 + \frac{2}{3} - \frac{4}{9}$...

$b_1 = \frac{3}{2} \qquad q = -\frac{2}{3}. \qquad S_n = \frac{\frac{3}{2}}{1+\frac{2}{3}} = \frac{3}{2}\cdot\frac{3}{5} = \frac{9}{10}.$

г) $125 + 25 + 5 + 1$...

$b_1 = 125 \qquad q = \frac{1}{5}. \qquad S_n = \frac{125}{\frac{4}{5}} = \frac{625}{4} = 156{,}25.$

25.4. а) $-6 + \frac{2}{3} - \frac{2}{27} + \frac{2}{243}$...

$b_1 = -6 \qquad q = -\frac{1}{9}. \qquad S_n = \frac{-6}{1+\frac{1}{9}} = -\frac{54}{10} = -\frac{27}{5}.$

б) $3 + \sqrt{3} + 1 + \frac{1}{\sqrt{3}}$...

$b_1 = 3 \qquad q = \frac{1}{\sqrt{3}}. \qquad S_n = \frac{3}{1-\frac{1}{\sqrt{3}}} = \frac{3\sqrt{3}}{\sqrt{3}-1} = \frac{3\sqrt{3}(\sqrt{3}+1)}{2}.$

в) $49 - 14 + 4 - \frac{8}{7}$...

$b_1 = 49 \qquad q = -\frac{2}{7}. \qquad S_n = \frac{49}{1+\frac{2}{7}} = \frac{343}{9} = 38\frac{1}{9}.$

г) $4 + 2\sqrt{2} + 2 + \sqrt{2}...$

$b_1 = 4 \qquad q = \frac{1}{\sqrt{2}}. \qquad S_n = \frac{4}{1-\frac{1}{\sqrt{2}}} = \frac{4\sqrt{2}}{\sqrt{2}-1} = 4\sqrt{2}(\sqrt{2}+1).$

25.5. а) $b_1 = -2;\ b_2 = 1.$

$q = -\frac{1}{2}. \quad S_n = \frac{-2}{1+\frac{1}{2}} = -\frac{4}{3} = -1\frac{1}{3}.$

б) $b_1 = 3;\ b_2 = \frac{1}{3}.$

$q = \frac{1}{9}. \quad S_n = \frac{3}{1-\frac{1}{9}} = \frac{27}{8} = 3\frac{3}{8}.$

в) $b_1 = 7;\ b_2 = -1.$

$q = -\frac{1}{7}. \quad S_n = \frac{7}{1+\frac{1}{7}} = \frac{49}{8} = 6\frac{1}{8}.$

г) $b_1 = -20;\ b_2 = 4.$

$q = -\frac{1}{5}. \quad S_n = \frac{-20}{1+\frac{1}{5}} = -\frac{100}{6} = -\frac{50}{3}.$

25.6. а) $S_n = 2;\ b_1 = 3.$

$S_n = \frac{b_1}{1-q};\ 1 - q = \frac{b_1}{S_n}.\ q = 1 - \frac{b_1}{S_n} = 1 - \frac{3}{2} = -\frac{1}{2}.$

б) $S_n = -10;\ b_1 = -5.$

$q = 1 - \frac{5}{10} = \frac{1}{2}.$

в) $S_n = -\frac{9}{4};\ b_1 = -3.$

$q = 1 - \frac{3}{9/4} = 1 - \frac{4}{3} = -\frac{1}{3}.$

г) $S_n = 1{,}5;\ b_1 = 2.$

$q = 1 - \frac{2}{3/2} = 1 - \frac{4}{3} = -\frac{1}{3}.$

25.7. а) $S = 10,\ q = \frac{1}{10}.$

$S = \frac{b_1}{1-q},\ b_1 = S(1-q) = 10 \cdot \frac{9}{10} = 9.$

б) $S=-3,\ q=-\frac{1}{3},\quad b_1=-3\cdot(1+\frac{1}{3})=-4.$

в) $S=6,\ q=-\frac{1}{2},\quad b_1=6\cdot\frac{3}{2}=9.$

г) $S=-21,\quad q=\frac{1}{7},\quad b_1=-21\cdot\left(1-\frac{1}{7}\right)=-18.$

25.8. а) $S=15,\quad q=-\frac{1}{3},\quad n=3.$

$b_1=15\cdot\frac{4}{3}=20.\quad b_3=20\cdot\left(-\frac{1}{3}\right)^2=\frac{20}{9}=2\frac{2}{9}.$

б) $S=-20,\ b_1=-16,\quad n=4.\quad q=1-\frac{16}{20}=\frac{1}{5}.$

$b_4=-16\cdot\left(\frac{1}{5}\right)^3=-\frac{16}{125}.$

в) $S=20,\quad b_1=22,\ n=4,\quad q=1-\frac{22}{20}=-\frac{1}{10}.$

$b_4=22\cdot\left(-\frac{1}{10}\right)^3=-\frac{11}{500}.$

г) $S=21,\ q=\frac{2}{3},\quad n=3,\quad b_1=21\cdot\left(1-\frac{2}{3}\right)=7.$

$b_3=7\cdot\left(\frac{2}{3}\right)^2=\frac{28}{9}=3\frac{1}{9}.$

25.9.

а) $b_n=\frac{25}{3^n}\quad b_1=\frac{25}{3}\quad b_2=\frac{25}{9}\quad q=\frac{1}{3}.$

$S_n=\frac{25/3}{2/3}=12{,}5.$

б) $b_n=(-1)^n\frac{13}{2^{n-1}}\quad b_1=-13\quad b_2=\frac{13}{2}\quad q=-\frac{1}{2}.$

$S_n=\frac{-13}{3/2}=-\frac{26}{3}=-8\frac{2}{3}.$

в) $b_n=\frac{45}{3^n}\quad b_1=\frac{45}{3}\quad b_2=\frac{45}{9}\quad q=\frac{1}{3}.$

$S_n=\frac{45/3}{2/3}=\frac{45}{2}=22{,}5.$

5-10561

г) $b_n = (-1)^n \frac{7}{6^{n-2}}$ $b_1 = -42$ $b_2 = 7$ $q = -\frac{1}{6}$.

$S_n = \frac{-42}{1+1/6} = -36.$

25.10. $\begin{cases} b_1 + b_3 = 29 \\ b_2 + b_4 = 11{,}6 \end{cases}$ $\begin{cases} b_1 + b_1q^2 = 29 \\ b_1q + b_1q^3 = 11{,}6 \end{cases}$

$\begin{cases} b_1(1+q^2) = 29 \\ b_1q(1+q^2) = 11{,}6 \end{cases}$ $q = \frac{2}{5}$.

$b_1 = \frac{29}{1+\frac{4}{25}} = 25.$ $S_n = \frac{25}{1-\frac{2}{5}} = \frac{25\cdot 5}{3} = 41\frac{2}{3}.$

Ответ: $S_n = 41\frac{2}{3}$.

25.11. $S_n = 24$ $S_3 = 21.$

$\begin{cases} \frac{b_1}{1-q} = 24 \\ \frac{b_1(q^3-1)}{q-1} = 21 \end{cases}$ $q^3 - 1 = -\frac{7}{8}$ $q^3 = \frac{1}{8}$ $q = \frac{1}{2}$ $\begin{cases} \frac{b_1}{1/2} = 24 \\ q = \frac{1}{2} \end{cases}$ $b_1 = 12$

Ответ: $b_1 = 12$, $q = \frac{1}{2}$.

25.12. $\begin{cases} S_n = 18 \\ b_1^2 + b_1^2q^2 + b_1^2q^4 \ldots = 162 \end{cases}$

$\begin{cases} \frac{b_1}{q-1} = -18 \\ \frac{b_1^2}{1-q^2} = 162 \end{cases}$ $\begin{cases} \frac{b_1}{1-q} = 18 \\ \frac{b_1^2}{1-q^2} = 162 \end{cases}$ $\begin{cases} b_1 = 18(1-q) \\ 324(1-2q+q^2) = 162 - 162q^2 \end{cases}$

$2q^2 - 4q + 2 = 1 - q^2$ $3q^2 - 4q + 1 = 0$

$q = \frac{2+1}{3} = 1$ — но такого быть не может.

$q = \frac{1}{3}$, $b_1 = 12.$

25.13. а) $\sin x + \sin^2 x + \sin^3 x + \ldots = \frac{\sin x}{1-\sin x}.$

б) $\cos x - \cos^2 x + \cos^3 x + \ldots = \dfrac{\cos x}{1+\cos x}$.

в) $\cos^2 x + \cos^4 x + \cos^6 x + \ldots = \dfrac{\cos^2 x}{1-\cos^2 x} = \mathrm{ctg}^2\, x$.

г) $1 - \sin^3 x + \sin^6 x - \sin^9 x + \ldots = \dfrac{1}{1+\sin^3 x}$.

25.14. а) $x + x^2 + x^3 + \ldots = 4$.

$\dfrac{x}{1-x} = 4$; $\quad x = 4 - 4x$; $\quad x = \dfrac{4}{5}$.

б) $2x - 4x^2 + 8x^3 - 16x^4 + \ldots = \dfrac{3}{8}$.

$\dfrac{2x}{1+2x} = \dfrac{3}{8}$; $\quad 2x = \dfrac{3}{8} + \dfrac{3}{4}x$; $\quad 10x = 3$; $\quad x = \dfrac{3}{10}$.

25.15. а) $0,(15) = 0,15 + \dfrac{0,(15)}{100} \Rightarrow 0,(15) = \dfrac{0,15}{0,99} = \dfrac{5}{33}$

б) $0,1(2) = 0,1 + \dfrac{0,1(2)+0,1}{10} \Rightarrow 0,1(2) = \dfrac{0,11}{0,9} = \dfrac{11}{90}$

в) $0,(18) = 0,18 + \dfrac{0,(18)}{100} \Rightarrow 0,(18) = \dfrac{0,18}{0,99} = \dfrac{2}{11}$

г) $0,2(34) = 0,23 + \dfrac{0,2(34)+0,2}{100} \Rightarrow 0,2(34) = \dfrac{0,232}{0,99} = \dfrac{116}{495}$

§ 26. Предел функции

26.1. а) при $x \to +\infty$ рис. 23, 25 учебника.
б) при $x \to -\infty$ рис. 24, 25 учебника.
в) при $x \to \infty$ рис. 25 учебника.

26.2. а) $y = 3$ – горизонт. асимптота на луче $(-\infty; 4]$

$\lim\limits_{x\to-\infty} f(x) = 3$; $\lim\limits_{x\to+\infty} f(x)$, $\lim\limits_{x\to\infty} f(x)$ не существуют

б) $y = -2$ – горизонт. асимптота на луче $[-6; +\infty)$

$\lim\limits_{x\to\infty} f(x)$, $\lim\limits_{x\to-\infty} f(x)$ не существуют; $\lim\limits_{x\to+\infty} f(x) = -2$.

в) $y = -5$ – горизонт. асимптота на луче $(-\infty; 3]$

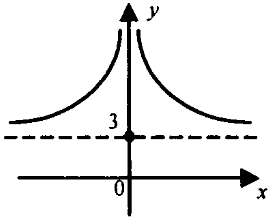
$\lim\limits_{x\to-\infty} f(x) = -5$; $\lim\limits_{x\to+\infty} f(x)$, $\lim\limits_{x\to\infty} f(x)$ не существуют

г) $y = 5$ – горизонт. асимптота на луче $[4; +\infty)$

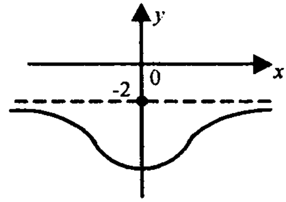
$\lim\limits_{x\to\infty} f(x)$, $\lim\limits_{x\to-\infty} f(x)$ не существуют; $\lim\limits_{x\to+\infty} f(x) = 5$.

26.3.

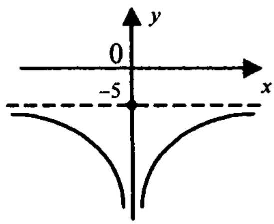
а) $\lim\limits_{x\to\infty} f(x)=5.$



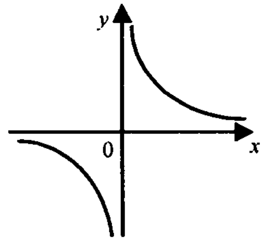
б) $\lim\limits_{x\to\infty} f(x)=-2.$



в) $\lim\limits_{x\to\infty} f(x)=-5.$

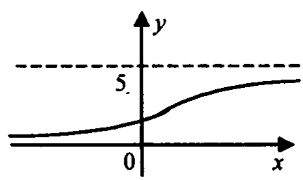


г) $\lim\limits_{x\to\infty} f(x)=0.$

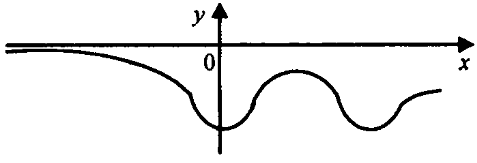


26.4.

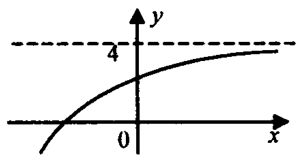
а) $\lim\limits_{x\to+\infty} f(x)=5.$ $f(x)>0.\ x\in\mathfrak{R}.$



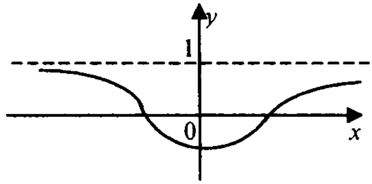
б) $\lim\limits_{x\to-\infty} f(x)=0,$ $f(x)<0.\ x\in\mathfrak{R}.$



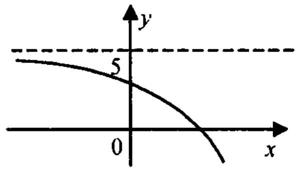
26.5. а) $\lim\limits_{x\to+\infty} h(x)=4$ и функция возрастает.



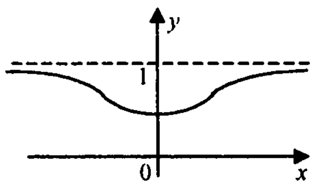
б) $\lim\limits_{x\to-\infty} h(x) = 1$ и функция ограничена сверху.



в) $\lim\limits_{x\to-\infty} h(x) = 5$ и функция убывает.



г) $\lim\limits_{x\to\infty} h(x) = 1$ и функция ограничена



26.6. $\lim\limits_{x\to+\infty} f(x) = -3.$

а) $\lim\limits_{x\to\infty} f(x) = -18.$ б) $\lim\limits_{x\to-\infty} \dfrac{f(x)}{3} = -1.$

в) $\lim\limits_{x\to-\infty} 8f(x) = -24.$ г) $\lim\limits_{x\to\infty} 0{,}4f(x) = -\dfrac{6}{5}.$

26.7. $\lim\limits_{x\to\infty} f(x) = 2.$ $\lim\limits_{x\to\infty} g(x) = -3.$ $\lim\limits_{x\to\infty} h(x) = 9.$

а) $\lim\limits_{x\to\infty}(f(x) + g(x)) = \lim\limits_{x\to\infty} f(x) + \lim\limits_{x\to\infty} g(x) = 2 - 3 = -1.$

б) $\lim\limits_{x\to\infty}(g(x)\cdot f(x)) = \lim\limits_{x\to\infty} g(x) \lim\limits_{x\to\infty} f(x) = -6$

в) $\lim\limits_{x\to\infty} \dfrac{f(x)g(x)}{h(x)} = \dfrac{\lim\limits_{x\to\infty}(f(x)g(x))}{\lim\limits_{x\to\infty} h(x)} = -\dfrac{2}{3}$

г) $\lim\limits_{x\to\infty} \dfrac{2h(x)}{3g(x)} = \dfrac{2}{3}\dfrac{\lim\limits_{x\to\infty} h(x)}{\lim\limits_{x\to\infty} g(x)} = -2$

26.8. а) $\lim\limits_{x\to\infty}\left(\frac{1}{x^2}+\frac{3}{x^3}\right)=0.$

б) $\lim\limits_{x\to\infty}\left(\frac{5}{x^3}+1\right)\left(-\frac{8}{x^2}-2\right)=1\cdot(-2)=-2.$

в) $\lim\limits_{x\to\infty}\left(\frac{2}{x^2}+\frac{8}{x^3}\right)=0$

г) $\lim\limits_{x\to\infty}\left(\frac{7}{x^6}-2\right)\left(-\frac{6}{x^{10}}-3\right)=6.$

26.9. а) $\lim\limits_{x\to\infty}\frac{x+1}{x-2}=1.$

б) $\lim\limits_{x\to\infty}\frac{3x-4}{2x+7}=\lim\limits_{x\to\infty}\frac{3-4/x}{2+7/x}=\frac{3}{2}.$

в) $\lim\limits_{x\to\infty}\frac{x-4}{x+3}=1.$

г) $\lim\limits_{x\to\infty}\frac{7x+9}{6x-1}=\frac{7}{6}=1\frac{1}{6}.$

26.10. а) $\lim\limits_{x\to\infty}\frac{4x^2+9}{x^2+2}=\lim\limits_{x\to\infty}\frac{4+9/x^2}{1+2/x^2}=4.$

б) $\lim\limits_{x\to\infty}\frac{3x-1}{x^2+7x+5}=\lim\limits_{x\to\infty}\frac{3/x-1/x^2}{1+7/x+5/x^2}=0.$

в) $\lim\limits_{x\to\infty}\frac{-2x-1}{3x^2-4x+1}=\lim\limits_{x\to\infty}\frac{-2/x-1/x^2}{3-4/x+1/x^2}=0.$

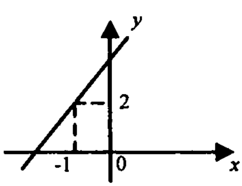
г) $\lim\limits_{x\to\infty}\frac{10x^2+4x-3}{5x^2+2x+1}=\lim\limits_{x\to\infty}\frac{10+4/x-3/x^2}{5+2/x+1/x^2}=2.$

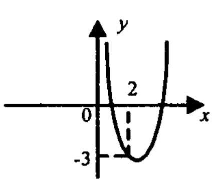
26.11. при $x\to 3$

$f(x)$ на рис. 25 имеет предел и он равен 4.
$f(x)$ на рис. 23 имеет предел и он равен 3.
$f(x)$ на рис. 24 имеет предел и он равен 4.
$f(x)$ на рис. 30 имеет предел и он равен 0.

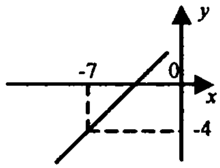
26.12. а) $\lim\limits_{x\to -1} g(x)=2.$

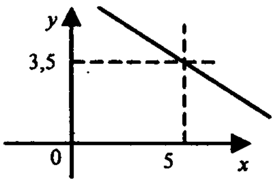
б) $\lim\limits_{x\to 2} g(x)=-3.$





в) $\lim\limits_{x\to -7} g(x) = -4.$ г) $\lim\limits_{x\to 5} g(x) = 3{,}5.$





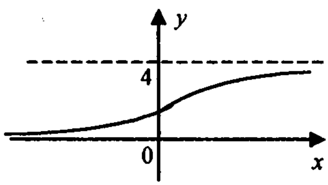
26.13.

а) $\lim\limits_{x\to -\infty} f(x) = 0.$ б) $\lim\limits_{x\to 0} f(x) = 4.$

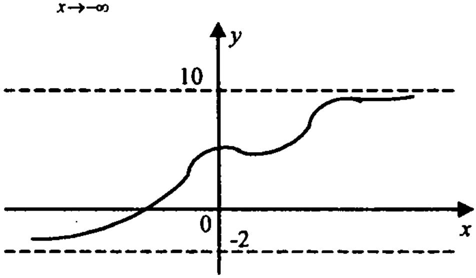
в) $\lim\limits_{x\to 3} f(x) = 9.$ г) $\lim\limits_{x\to \infty} f(x)$ не существует.

26.14.

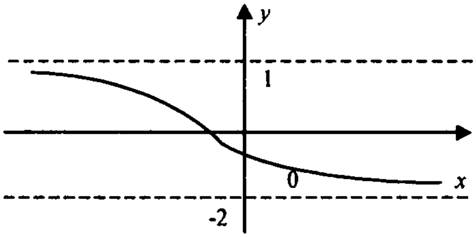
а) $\lim\limits_{x\to +\infty} f(x) = 4$ и $\lim\limits_{x\to -\infty} (x) = 0$



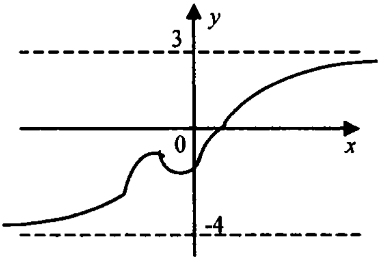
б) $\lim\limits_{x\to +\infty} f(x) = 10$ и $\lim\limits_{x\to -\infty} f(x) = -2$



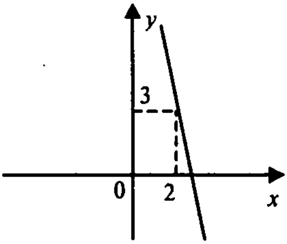
в) $\lim\limits_{x\to +\infty} f(x) = -2$ и $\lim\limits_{x\to -\infty} f(x) = 1$



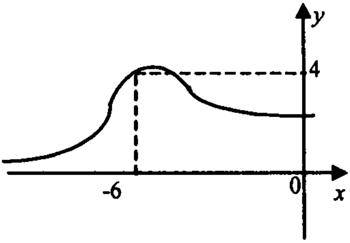
г) $\lim\limits_{x\to+\infty} f(x) = 3$ и $\lim\limits_{x\to-\infty} f(x) = -4$



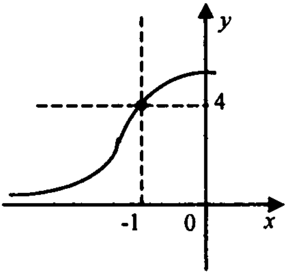
26.15. а) $\lim\limits_{x\to 2} f(x) = 3$ и $f(2) = 3$



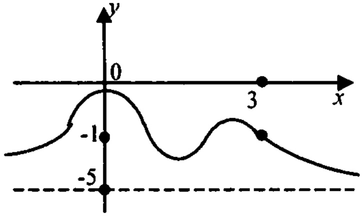
б) $\lim\limits_{x\to-6} f(x) = 4$ и $\lim\limits_{x\to-\infty} f(x) = 0$



в) $\lim\limits_{x\to-1} f(x) = 4$ и $f(-1)$ не существует



г) $\lim\limits_{x\to 3} f(x)=-1$ и $\lim\limits_{x\to +\infty} f(x)=-5$



26.16. а) $\lim\limits_{x\to 1}(x^2-3x+5)=1-3+5=3$.

б) $\lim\limits_{x\to \frac{1}{2}}\left(\frac{2x+3}{4x+2}\right)=\frac{1+3}{2+2}=1$.

в) $\lim\limits_{x\to -1}(x^2+6x-8)=1-6-8=-13$.

г) $\lim\limits_{x\to -\frac{1}{3}}\frac{7x-14}{21x+2}=\frac{-7/3-14}{-7+2}=\frac{49}{15}$.

26.17. а) $\lim\limits_{x\to 5}\sqrt{x+4}=\sqrt{9}=3$.

б) $\lim\limits_{x\to 1}\frac{3+4x}{2x^2+6x-3}=\frac{3+4}{2+6-3}=\frac{7}{5}=1\frac{2}{5}$.

в) $\lim\limits_{x\to 0}\frac{4x+7}{x^2-5x+3}=\frac{7}{3}=2\frac{1}{3}$.

г) $\lim\limits_{x\to -1}\frac{5-2x}{3x^2-2x+4}=\frac{5+2}{3+2+4}=\frac{7}{9}$.

26.18. а) $\lim\limits_{x\to 0}\frac{x^2}{x^2-x}=\lim\limits_{x\to 0}\frac{x}{x-1}=0$.

б) $\lim\limits_{x\to -2}\frac{x^2-4}{2+x}=\lim\limits_{x\to -2}(x-2)=-4$.

в) $\lim\limits_{x\to 5}\frac{x^2-25}{x-5}=\lim\limits_{x\to 5}(x+5)=10$.

г) $\lim\limits_{x\to -3}\frac{3+x}{x^2-9}=\lim\limits_{x\to -3}\frac{1}{x-3}=-\frac{1}{6}$.

26.19. а) $\lim\limits_{x\to \frac{\pi}{2}}\frac{\sin 3x+\sin x}{\cos 3x+\cos x}=\lim\limits_{x\to \frac{\pi}{2}}\frac{\sin 2x\cos x}{\cos 2x\cos x}=\lim\limits_{x\to \frac{\pi}{2}}\operatorname{tg} 2x=0$.

б) $\lim\limits_{x\to 0}\frac{\cos 5x-\cos 3x}{\sin 5x+\sin 3x}=\lim\limits_{x\to 0}\left(-\frac{\sin 4x\sin x}{\sin 4x\cos x}\right)=0$.

26.20. $y = 2x - 3$ $\quad x_0 = 3.$ $\quad y(x_0) = 3$

а) $x_1 = 3{,}2.$ $y(x_1) = 3{,}4.$ $\Delta y = 0{,}4.$ б) $x_1 = 2{,}9.$ $y(x_1) = 2{,}8.$ $\Delta y = -0{,}2.$

в) $x_1 = 3{,}5.$ $y(x_1) = 4.$ $\Delta y = 1.$ г) $x_1 = 2{,}5.$ $y(x_1) = 2.$ $\Delta y = -1.$

26.21. $y = x^2 + 2x$ $\quad x_0 = -2.$ $\quad y(x_0) = 0.$

а) $x_1 = -1{,}9.$ $y(x_1) = -0{,}19.$ $\Delta y = -0{,}19.$

б) $x_1 = -2{,}1.$ $y(x_1) = 0{,}21.$ $\Delta y = 0{,}21.$

в) $x_1 = -1{,}5.$ $y(x_1) = -0{,}75.$ $\Delta y = -0{,}75.$

г) $x_1 = -2{,}5.$ $y(x_1) = 1{,}25.$ $\Delta y = 1{,}25.$

26.22. $y = \sqrt{x}$ $\quad x_0 = 1.$

а) $\Delta x = 0{,}44.$ $\quad y(x + \Delta x) - y(x) = 1{,}2 - 1 = 0{,}2.$

б) $\Delta x = -0{,}19.$ $\quad y(x + \Delta x) - y(x) = 0{,}9 - 1 = -0{,}1.$

в) $\Delta x = 0{,}21.$ $\quad y(x + \Delta x) - y(x) = 1{,}1 - 1 = 0{,}1.$

г) $\Delta x = 0{,}1025$ $\quad y(x + \Delta x) - y(x) = 1{,}05 - 1 = 0{,}05.$

26.23. а) $f(x_1) - f(x_0) = 1{,}4 - 2 = -0{,}6$ $\Delta x = x_1 - x_0 = -2$.

б) $f(x_1) - f(x_0) = 1 - 6 = -5$ $\Delta x = x_1 - x_0 = 2$.

26.24. а) $f(x) = 3x + 5.$ $f(x + \Delta x) = 3x + 3\Delta x + 5.$ $f(x + \Delta x) - f(x) = 3\Delta x.$

б) $f(x) = -x^2.$ $f(x + \Delta x) = -x^2 - 2x\Delta x - (\Delta x)^2.$

$f(x + \Delta x) - f(x) = -2x\Delta x - (\Delta x)^2.$

в) $f(x) = 4 - 2x.$ $f(x + \Delta x) = 4 - 2x - 2\Delta x.$ $f(x + \Delta x) - f(x) = -2\Delta x.$

г) $f(x) = 2x^2.$ $f(x + \Delta x) = 2x^2 + 4x\Delta x + 2(\Delta x)^2.$ $f(x + \Delta x) - f(x) =$
$= 4x\Delta x + 2(\Delta x)^2.$

26.25. а) $f(x) = ax^2,$ $f(x + \Delta x) = ax^2 + 2a\Delta x\cdot x + a\cdot\Delta x^2,$

$f(x + \Delta x) - f(x) = 2a\Delta x\cdot x + a\cdot(\Delta x)^2;$

б) $f(x) = \dfrac{1}{x},$ $f(x + \Delta x) = \dfrac{1}{x + \Delta x},$

$$f(x + \Delta x) - f(x) = \frac{x - x - \Delta x}{(x + \Delta x)x} = -\frac{\Delta x}{x^2 + x\Delta x}.$$

§ 27. Определение производной

27.1. $s(t) = 2t + 1.$ $\quad t_1 = 2.$

а) $t_2 = 3.$

$S(3) - S(2) = 7 - 5 = 2 = \Delta S.$ $\Delta t = 1.$

$$V_{ср} = \frac{\Delta S}{\Delta t} = 2 \text{ (м/с)}.$$

б) $t_2 = 2{,}5.$

$S(2{,}5) - S(2) = 6 - 5 = 1.$ $\quad \Delta t = 0{,}5.$

$$\frac{\Delta S}{\Delta t} = V_{ср} = \frac{1}{0{,}5} = 2 \text{ (м/с)}.$$

в) $t_2 = 2{,}1.$

$S(2{,}1) - S(2) = 0{,}2.$ $\quad \Delta t = 0{,}1.$ $\quad \dfrac{\Delta S}{\Delta t} = \dfrac{0.2}{0{,}1} = 2 \text{ (м/с)}.$

г) $t_2 = 2{,}05$.

$S(2{,}05) - S(2) = 0{,}1$. $\Delta t = 0{,}05$. $\frac{\Delta S}{\Delta t} = \frac{0{,}1}{0{,}05} = 2$ (м/с).

27.2. $s(t) = t^2$. $t_1 = 0$.

а) $t_2 = 0{,}1$. $S(0{,}1) - S(0) = 0{,}01$. $\frac{\Delta S}{\Delta t} = 0{,}1$ (м/с).

б) $t_2 = 0{,}01$. $S(0{,}01) - S(0) = 0{,}0001$. $\frac{\Delta S}{\Delta t} = 0{,}01$ (м/с).

в) $t_2 = 0{,}2$. $S(0{,}2) - S(0) = 0{,}04$. $\frac{\Delta S}{\Delta t} = 0{,}2$ (м/с).

г) $t_2 = 0{,}02$. $S(0{,}02) - S(0) = 0{,}0004$. $\frac{\Delta S}{\Delta t} = 0{,}02$ (м/с).

27.3. а) $S(t) = 4t + 1$. $V_{мгнов} = 4$ (м/с).
б) $S(t) = 6t - 2$. $V_{мгнов} = 6$ (м/с).
в) $S(t) = 3t + 2$. $V_{мгнов} = 3$ (м/с).
г) $S(t) = 5t - 1$. $V_{мгнов} = 5$ (м/с).

27.4. а) $f'(x_1) = \text{tg } 60° = \sqrt{3}$ $f'(x_2) = \text{tg } 45° = 1$.

б) $\rho'(x_1) = 30° = \frac{\sqrt{3}}{3}$; $f'(x_2) = \text{tg } 0° = 0$.

в) $f'(x_1) = 0$. $f'(x_2) = -\frac{\sqrt{3}}{3}$.

г) $f'(x_1) = 0$. $f'(x_2) = 0$.

27.5. а) $y = 9{,}5x - 3$. $f'(x) = 9{,}5$. б) $y = -16x + 3$. $f'(x) = -16$.
в) $y = 6{,}7x - 13$. $f'(x) = 6{,}7$. г) $y = -9x + 4$. $f'(x) = -9$.

27.6. а) $f(x) = x^2$. $x_0 = 2$. $f'(x_0) = 2x_0 = 4$. б) $f(x) = x^2$. $x_0 = -1$. $f'(x_0) = -2$.
в) $f(x) = x^2$. $x_0 = -2$. $f'(x_0) = -4$. г) $f(x) = x^2$. $x_0 = 9$. $f'(x_0) = 18$.

27.7. а) $f(x) = \frac{1}{x}$. $x_0 = 2$. $f'(x_0) = -\frac{1}{4}$. $f'(x_0) = -\frac{1}{x_0^2}$.

б) $f(x) = \frac{1}{x}$. $x_0 = -1$. $f'(x_0) = -1$.

в) $f(x) = \frac{1}{x}$. $x_0 = 5$. $f'(x_0) = -\frac{1}{25}$.

г) $f(x) = \frac{1}{x}$. $x_0 = -0{,}5$. $f'(x_0) = -4$.

27.8. $S(t) = t^2$. $S'(t) = 2t$. $S''(t) = 2$.
а) $t = 1$. $V = 2$ (м/с). $a = 2$ (м/с^2).
б) $t = 2{,}1$. $V = 4{,}2$ (м/с). $a = 2$ (м/с^2).
в) $t = 2$. $V = 4$ (м/с). $a = 2$ (м/с^2).
г) $t = 3{,}5$. $V = 7$ (м/с). $a = 2$ (м/с^2).

27.9. $S_1' = 4$ км/ч. $S_2' = 3$ км/ч

$S_{max} = 14$ км

$S_3' = 11$ ч, $t_{отл} = 3$ ч.

27.10. $S(t) = 2t^2 + t$. $t_1 = 0$.

а) $t_2 = 0{,}6$. $S(t_2) - S(t_1) = 1{,}32$. $\frac{\Delta S}{\Delta t} = \frac{1{,}32}{0.6} = 2{,}2$. (м/с)

б) $t_2 = 0{,}2$. $S(t_2) - S(t_1) = 0{,}28$. $\frac{\Delta S}{\Delta t}\sigma = 1{,}4$. (м/с)

в) $t_2 = 0.5$. $S(t_2) - S(t_1) = 1$. $\frac{\Delta S}{\Delta t}\sigma\sigma = 2$. (м/с)

г) $t_2 = 0.1$. $S(t_2) - S(t_1) = 0{,}12$. $\frac{\Delta S}{\Delta t}\sigma = 1{,}2$. (м/с)

27.11. а) $S(t) = t^2 + 3$. $S'(t) = 2t$. $V_{мгнов} = 2t$. (м/с)

б) $S(t) = t^2 - t$. $S'(t) = 2t - 1$. $V_{мгнов} = 2t - 1$. (м/с)

в) $S(t) = t^2 + 4$. $S'(t) = 2t$. $V_{мгнов} = 2t$. (м/с)

г) $S(t) = t^2 - 2t$. $S'(t) = 2t - 2$. $V_{мгнов} = 2t - 2$. (м/с)

27.12. а) $f'(x_1) > 0, f'(x_2) > 0$. $x_1 = 0$ $x_2 = 1$.

б) $f'(x_1) < 0, f'(x_2) > 0$. $x_1 = -6$; $x_2 = 0$.

в) $f'(x_1) < 0, f'(x_2) < 0$. $x_1 = -5$; $x_2 = -4$.

г) $f'(x_1) > 0, f'(x_2) < 0$. $x_1 = 2$; $x_2 = 4$.

27.13. а) $f'(-7) < f'(-2)$. б) $f'(-4) < f'(2)$.

в) $f'(-9) < f'(0)$. г) $f'(-1) > f'(5)$.

27.14. а) $\varphi'(x) > 0$; $x = -7, -6, -5$. б) $\varphi'(x) < 0$ и $x > 0$; $x = 4, 5$.

в) $\varphi'(x) < 0$; $x = -3, -2$. г) $\varphi'(x) > 0$ и $x < 0$; $x = -5, -6$.

§ 28. *Вычисление производных*

28.1. а) $y = 7x + 4$. $y' = 7$. б) $y = x^2$. $y' = 2x$.

в) $y = -6x + 1$. $y' = -6$. г) $y = \frac{1}{x}$. $y' = -\frac{1}{x^2}$.

28.2. а) $y = \sin x$. $y' = \cos x$. б) $y = \sqrt{x}$. $y' = \frac{1}{2\sqrt{x}}$.

в) $y = \cos x$. $y' = -\sin x$ г) $y = 10^{10}$. $y' = 0$.

28.3. а) $g(x) = \sqrt{x}$. $x_0 = 4$. $g'(x) = \frac{1}{2\sqrt{x}}$. $g'(x_0) = \frac{1}{4}$.

б) $g(x) = x^2$. $x_0 = -7$. $g'(x) = 2x$. $g'(x_0) = -14$.

в) $g(x) = -3x - 1$. $x_0 = -3$. $g'(x) = -3$. $g'(x_0) = -3$.

г) $g(x) = \frac{1}{x}$. $x_0 = 0{,}5$. $g'(x) = -\frac{1}{x^2}$. $g'(x_0) = -4$.

28.4. а) $g(x) = \sin x$. $x_0 = -\frac{\pi}{2}$. $g'(x) = \cos x$. $g'(x_0) = 0$.

б) $g(x) = \cos x$. $x_0 = \frac{\pi}{6}$. $g'(x) = -\sin x$. $g'(x_0) = -\frac{1}{2}$.

в) $g(x) = \cos x$. $x_0 = -3\pi$. $g'(x) = -\sin x$. $g'(x_0) = 0$.

г) $g(x) = \sin x$. $x_0 = 0$. $g'(x) = \cos x$. $g'(x_0) = 1$.

28.5. а) $h(x) = 7x - 19$. $x_0 = -2$. $h'(x_0) = 7$.

б) $h(x) = \sqrt{x}$. $x_0 = 16$. $h'(x) = \frac{1}{2\sqrt{x}}$. $h'(x_0) = \frac{1}{8}$.

в) $h(x) = -6x + 4$. $x_0 = 0,5$. $h'(x_0) = -6$. г) $h(x) = \sqrt{x}$. $x_0 = 9$.

$h'(x) = \frac{1}{2\sqrt{x}}$. $h'(x_0) = \frac{1}{6}$.

28.6. а) $h(x) = \frac{1}{x}$. $x_0 = -2$. $h'(x) = -\frac{1}{x^2}$. $h'(x_0) = -\frac{1}{4}$.

б) $h(x) = \sin x$. $x_0 = \frac{\pi}{2}$. $h'(x) = \cos x$. $h'(x_0) = 0$.

в) $h(x) = x^2$. $x_0 = -0,1$. $h'(x) = 2x$. $h'(x_0) = -\frac{1}{5}$.

г) $h(x) = \cos x$. $x_0 = \pi$. $h'(x) = -\sin x$. $h'(x_0) = 0$.

28.7. а) $f(x) = x^2$. $x_0 = -4$. $f'(x) = 2x$. $f'(x_0) = -8$.

б) $f(x) = \frac{1}{x}$. $x_0 = -\frac{1}{3}$. $f'(x) = -\frac{1}{x^2}$. $f'(x_0) = -9$.

в) $f(x) = \frac{1}{x}$. $x_0 = \frac{1}{2}$. $f'(x) = -\frac{1}{x^2}$. $f'(x_0) = -4$.

г) $f(x) = x^2$. $x_0 = 2$. $f'(x) = 2x$. $f'(x_0) = 4$.

28.8. а) $f(x) = \sin x$. $x_0 = \frac{\pi}{3}$ $f'(x) = \cos x$. $f'(x_0) = \frac{1}{2}$.

б) $f(x) = \cos x$. $x_0 = -\frac{\pi}{4}$. $f'(x) = -\sin x$. $f'(x_0) = \frac{\sqrt{2}}{2}$.

в) $f(x) = \cos x$. $x_0 = \frac{\pi}{3}$. $f'(x) = -\sin x$. $f'(x_0) = -\frac{\sqrt{3}}{2}$.

г) $f(x) = \sin x$. $x_0 = -\frac{\pi}{6}$. $f'(x) = \cos x$. $f'(x_0) = \frac{\sqrt{3}}{2}$.

28.9. а) $f'(x) = 2x$. $f(x) = x^2$.

б) $f'(x) = \cos x$. $f(x) = \sin x$.

в) $f'(x) = 3$. $f(x) = 3x$.

г) $f'(x) = -\sin x$. $f(x) = \cos x$.

28.10.

а) $y = x^2 - 7x$. $y' = 2x - 7$.

б) $y = \sqrt{x} - 9x^2$. $y' = \frac{1}{2\sqrt{x}} - 18x$.

в) $y = 7x^2 + 3x$. $y' = 14x + 3$.

г) $y = \sqrt{x} - 5x^2$. $y' = \frac{1}{2\sqrt{x}} - 10x$.

28.11. а) $y = \frac{1}{x} + 4x$. $y' = \frac{-1}{x^2} + 4$.

б) $y = -2\sqrt{x} - \frac{1}{x}$. $y' = -\frac{1}{\sqrt{x}} + \frac{1}{x^2}$.

в) $y = \frac{1}{x} - 6x$. $y' = -\frac{1}{x^2} - 6$.

г) $\frac{4}{\sqrt{x}} - \frac{1}{x^2}$

28.12. а) $y = \sin x + 3$. $y' = \cos x$.

б) $y = 4\cos x$. $y' = -4\sin x$.

в) $y = \cos x - 6$. $y' = -\sin x$.

г) $y = -2\sin x$. $y' = -2\cos x$.

28.13. а) $y = \cos x + 2x$. $y' = -\sin x + 2$.

б) $y = 3\sin x + \cos x$. $y' = 3\cos x - \sin x$.

в) $y = \sin x - 3x$. $y' = \cos x - 3$.

г) $y = 2\cos x + \sin x$. $y' = -2\sin x + \cos x$.

28.14. а) $y = x^9 \Rightarrow y' = 9x^8$;

б) $y = x^{10}$. $y' = 10x^9$.

в) $y = x^{39} \Rightarrow y' = 39x^{38}$,

г) $y = x^{201}$. $y' = 201x^{200}$.

28.15. а) $y = x^3 + 2x^5$. $y' = 3x^2 + 10x^4$.

б) $y = x^4 - x^9$. $y' = 4x^3 - 9x^8$.

в) $y = x^3 + 4x^{100}$. $y' = 3x^2 + 400x^{99}$.

г) $y = x^4 - 7x^9$. $y' = 4x^3 - 63x^8$.

28.16. а) $y=(x^2 - 1)(x^4 + 2)$.
$y' = (x^4 + 2)2x + (x^2 - 1)(4x^3)$.

б) $y = \sqrt{x}\,(x^3 + 1)$. $y' = \frac{x^3+1}{2\sqrt{x}} + 3x^2\sqrt{x} = \frac{4x^3+1}{2\sqrt{x}}$.

в) $y = (x^2 + 3)(x^4 - 1)$. $y' = 2x(x^4 - 1) + (x^2 + 3)4x^3$.

г) $y = \sqrt{x}\,(x^4 + 2)$. $y' = \frac{x^4+2}{2\sqrt{x}} + 4x^3\sqrt{x} = \frac{9x^4+2}{2\sqrt{x}}$.

28.17. а) $y = \left(\frac{1}{x} + 1\right)(2x - 3)$. $y' = -\frac{1}{x^2}(2x - 3) + \frac{2}{x} + 2 = 2 + \frac{3}{x^2}$.

б) $y = \sqrt{x}\cos x$. $y' = \frac{\cos x}{2\sqrt{x}} - \sqrt{x}\sin x = \frac{\cos x - 2x\sin x}{2\sqrt{x}}$.

в) $y = \left(\frac{1}{x} + 8\right)(5x - 2)$. $y' = -\frac{1}{x^2}(5x - 2) + \frac{5}{x} + 40 = 40 + \frac{2}{x^2}$.

г) $y = \sqrt{x}\sin x$. $y' = \frac{\sin x}{2\sqrt{x}} + \sqrt{x}\cos x = \frac{\sin x + 2x\cos x}{2\sqrt{x}}$.

28.18.) $y = \dfrac{x^3}{2x+4}$.

$$y' = \frac{3x^2(2x+4)-2x^3}{4x^2+16x+16} = \frac{4x^3+12x^2}{4x^2+16x+16} = \frac{x^3+3x^2}{x^2+4x+4} = \frac{x^2(x+3)}{(x+2)^2}.$$

б) $y = \dfrac{x^2}{x^2-1}$. $\quad y' = \dfrac{2x(x^2-1)-2x^3}{(x^2-1)^2} = \dfrac{-2x}{(x^2-1)^2}$.

в) $y = \dfrac{x^2}{3-4x}$. $\quad y' = \dfrac{2x(3-4x)+4x^2}{(3-4x)^2} = \dfrac{6x-4x^2}{(3-4x)^2} = \dfrac{2x(3-2x)}{(3-4x)^2}$.

г) $y = \dfrac{x}{x^2+1}$. $\quad y' = \dfrac{x^2+1-2x^2}{(x^2+1)^2} = \dfrac{1-x^2}{(x^2+1)^2}$.

28.19. а) $y = 3\sin x + \operatorname{ctg} x$.

$y' = 3\cos x - \dfrac{1}{\sin^2 x}$.

б) $y = \operatorname{tg} x - \cos x$. $\quad y' = \dfrac{1}{\cos^2 x} + \sin x$.

в) $y = \cos x + \operatorname{tg} x$. $\quad y' = -\sin x + \dfrac{1}{\cos^2 x}$.

г) $y = 6\operatorname{tg} x - \sin x$. $\quad y' = \dfrac{6}{\cos^2 x} - \cos x$.

28.20. а) $y = x\operatorname{tg} x$. $\quad y' = \operatorname{tg} x + \dfrac{x}{\cos^2 x}$.

б) $y = \sin x \operatorname{tg} x$. $\quad y' = \sin x + \dfrac{\sin x}{\cos^2 x}$.

в) $y = x\operatorname{ctg} x$. $\quad y' = \operatorname{ctg} x - \dfrac{x}{\sin^2 x}$.

г) $y = \cos x \operatorname{ctg} x$. $\quad y' = -\cos x - \dfrac{\cos x}{\sin^2 x}$.

28.21.

а) $y = x^2 + 2x - 1$. $\quad x_0 = 0$. $\quad y' = 2x + 2$. $\quad y'(x_0) = 2$.

б) $y = x^3 - 3x + 2$. $\quad x_0 = -1$. $\quad y' = 3x^2 - 3$. $\quad y'(x_0) = 3 - 3 = 0$.

в) $y = x^2 + 3x - 4$. $\quad x_0 = 1$. $\quad y' = 2x + 3$. $\quad y'(x_0) = 5$.

г) $y = x^3 - 9x^2 + 7$. $\quad x_0 = 2$. $\quad y' = 3x^2 - 18x$. $\quad y'(x_0) = 12 - 36 = -24$.

28.22. а) $y = \dfrac{2}{x} - 1$. $x_0 = 4$. $y' = -\dfrac{2}{x^2}$. $y'(x_0) = -\dfrac{1}{8}$.

б) $y = \sqrt{x} + 4$. $x_0 = 9$. $y' = \dfrac{1}{2\sqrt{x}}$. $y'(x_0) = \dfrac{1}{6}$.

в) $y=\frac{8}{x}-6$. $x_0=1$. $y'=-\frac{8}{x^2}$. $y'(x_0)=-8$.

г) $y=\sqrt{x}+5$. $x_0=4$. $y'=\frac{1}{2\sqrt{x}}$. $y'(x_0)=\frac{1}{4}$.

28.23. а) $g=x^3+2x$. $x_0=2$.
$g'(x)=3x^2+2$. $g'(x_0)=14$.

б) $g=\left(\sqrt{x}+1\right)\sqrt{x}$. $x_0=1$.

$g'(x)=\frac{\sqrt{x}}{2\sqrt{x}}+\frac{\sqrt{x}+1}{2\sqrt{x}}=\frac{1}{2}+\frac{1}{2}+\frac{1}{2\sqrt{x}}$. $g'(x_0)=\frac{3}{2}$.

в) $g'(x_0)=2x_0+\frac{2}{\sqrt{x_0}}-4=5$

г) $g=\frac{1}{x}\left(\frac{4}{x}-2\right)$.

$x_0=-\frac{1}{2}$. $g'(x)=-\frac{1}{x^2}\left(\frac{4}{x}-2\right)+\frac{1}{x}\left(-\frac{4}{x^2}\right)=-\frac{4}{x^3}+\frac{2}{x^2}-\frac{4}{x^3}$.

$g'(x_0)=4\cdot8+2\cdot4+4\cdot8=72$.

28.24. а) $y'(x)=2\cos x-4\Rightarrow y'\left(\frac{\pi}{3}\right)=-3$.

б) $g=\frac{\operatorname{tg}x}{3}$. $x_0=-\frac{\pi}{3}$. $g'(x)=\frac{1}{3\cos^2 x}$. $g'(x_0)=\frac{4}{3}$.

в) $g=-3\cos x+x$. $x_0=-\frac{\pi}{6}$. $g'(x)=3\sin x+1$. $g'(x_0)=-\frac{1}{2}$.

г) $g=\frac{\operatorname{ctg}x}{5}$. $x_0=\frac{\pi}{3}$. $g'(x)=-\frac{1}{5\sin^2 x}$. $g'(x_0)=-\frac{4}{15}$.

28.25. а) $h(x)=x^6-4x$. $x_0=1$.
$h'(x)=6x^5-4$. $h'(x_0)=2$. $\operatorname{tg}\alpha=2$.

б) $h(x)=\sqrt{x}-3$. $x_0=\frac{1}{4}$.

$h'(x)=\frac{1}{2\sqrt{x}}$. $h'(x_0)=1$. $\operatorname{tg}\alpha=1$.

в) $h(x)=-x^5-2x^2+2$. $x_0=-1$.
$h'(x)=-5x^4-4x$. $h'(x_0)=-5+4=-1$. $\operatorname{tg}\alpha=-1$.

г) $h(x)=\frac{25}{x}+2$. $x_0=\frac{5}{4}$.

$h'(x)=-\frac{25}{x^2}$. $h'(x_0)=-16$. $\operatorname{tg}\alpha=-16$

28.26. а) $f(x) = x^2\sin x$.

$f'(x) = 2x\sin x + x^2\cos x$. $\quad f'\left(\frac{\pi}{2}\right) = \pi$.

б) $f(x) = \sqrt{3}\sin x + \frac{x^2}{\pi} + x\sin\frac{\pi}{6}$.

$f'(x) = \sqrt{3}\cos x + \frac{2x}{\pi} + \frac{1}{2}$. $\quad f'\left(\frac{\pi}{6}\right) = \frac{3}{2} + \frac{1}{3} + \frac{1}{2} = 2\frac{1}{3}$.

в) $f(x) = x(1 + \cos x)$.

$f'(x) = 1 + \cos x + x(-\sin x)$. $f'(\pi) = 1 - 1 + 0 = 0$.

г) $f(x) = \sqrt{3}\cos x - x\cos\frac{\pi}{6} + \frac{x^2}{\pi}$. $\quad f'(x) = -\sqrt{3}\sin x - \frac{\sqrt{3}}{2} + \frac{2x}{\pi}$.

$f'\left(\frac{\pi}{3}\right) = -\frac{3}{2} - \frac{\sqrt{3}}{2} + \frac{2}{3} = \frac{4 - 9 - 3\sqrt{3}}{6} = -\frac{5 + 3\sqrt{3}}{6}$.

28.27. а) $f(x) = 2\sqrt{x} - 5x + 3$. $\quad f'(x) = \frac{1}{\sqrt{x}} - 5 = 2$. $x = \frac{1}{49}$.

б) $f(x) = 3x - \sqrt{x} + 13$. $\quad f'(x) = 3 - \frac{1}{2\sqrt{x}} = 1$. $x = \frac{1}{16}$.

28.28. а) $y = (4x - 9)^7$. $y' = 7(4x - 9)^6 \cdot 4 = 28(4x - 9)^6$.

б) $y = \left(\frac{x}{3} + 2\right)^{12}$. $y' = 4\left(\frac{x}{3} + 2\right)^{11}$.

в) $y = (5x + 1)^9$. $y' = 45(5x + 1)^8$.

г) $y = \left(\frac{x}{4} - 2\right)^{14}$. $y' = \frac{7}{2}\left(\frac{x}{4} - 2\right)^{13}$.

28.29. а) $y = \sin(3x - 9)$. $y' = 3\cos(3x - 9)$.

б) $y = \cos\left(\frac{\pi}{3} - 4x\right)$. $y' = 4\sin\left(\frac{\pi}{3} - 4x\right)$.

в) $y = \cos(9x - 10)$. $y' = -9\sin(9x - 10)$.

г) $y = \sin(5 - 3x)$. $y' = -3\cos(5 - 3x)$.

28.30. а) $y = \sqrt{15 - 7x}$. $\quad y' = \frac{-7}{2\sqrt{15 - 7x}}$.

б) $y = \sqrt{42 + 0{,}5x}$. $\quad y' = \frac{1}{4\sqrt{42 + 0{,}5x}}$.

в) $y = \sqrt{4 + 9x}$. $\quad y' = \frac{9}{2\sqrt{4 + 9x}}$.

г) $y = \sqrt{50 - 0{,}2x}$. $\quad y' = \frac{-1}{10\sqrt{50 - 0{,}2x}}$.

28.31.

а) $y=(3x-2)^7$. $\quad x_0=3$.

$y'=21(3x-2)^6$. $\quad y'(x_0)=3\cdot 7^7$.

б) $y=\sin\left(\frac{\pi}{6}-2x\right)$. $\quad x_0=\frac{\pi}{12}$.

$y'=-2\cos\left(\frac{\pi}{6}-2x\right)$. $y'(x_0)=-2$.

в) $y=\operatorname{tg}\left(3x-\frac{\pi}{4}\right)$. $\quad x_0=\frac{\pi}{12}$.

$y'=\dfrac{3}{\cos^2\left(3x-\frac{\pi}{4}\right)}$. $\quad y'(x_0)=\dfrac{3}{\cos^2 0^\circ}=3$.

г) $y=\sqrt{25-9x}$. $\quad x_0=1$.

$y'=-\dfrac{9}{2\sqrt{25-9x}}$. $\quad y'(x_0)=-\frac{9}{8}=-1\frac{1}{8}$.

28.32. а) $y=(2x+1)^5$. $\quad x_0=-1$. $\quad y'=10(2x+1)^4$. $\quad y'(x_0)=10$.

б) $y=\sqrt{7x-3}$. $\quad x_0=1$. $\quad y'=\dfrac{7}{2\sqrt{7x-3}}$. $\quad y'(x_0)=\frac{7}{4}=1\frac{3}{4}$.

в) $y=\dfrac{4}{12x-5}$. $\quad x_0=2$. $\quad y'=\dfrac{-12\cdot 4}{(12x-5)^2}$. $\quad y'(x_0)=-\dfrac{48}{19^2}=-\dfrac{48}{361}$.

г) $y=\sqrt{11-5x}$. $\quad x_0=-1$. $\quad y'=\dfrac{-5}{2\sqrt{11-5x}}$. $\quad y'(x_0)=-\frac{5}{8}$.

28.33.

а) $y=\sin\left(3x-\frac{\pi}{4}\right)$. $\quad x_0=\frac{\pi}{4}$. $\quad y'=3\cos\left(3x-\frac{\pi}{4}\right)$. $\quad y'(x_0)=0$.

б) $y=\operatorname{tg}6x$. $\quad x_0=\frac{\pi}{24}$. $\quad y'=\dfrac{6}{\cos^2 6x}$. $\quad y'(x_0)=\dfrac{6}{\cos^2\frac{\pi}{4}}=12$.

в) $y=\cos\left(\frac{\pi}{3}-2x\right)$. $x_0=\frac{\pi}{3}$.

$y'=2\sin\left(\frac{\pi}{3}-2x\right)$. $\quad y'(x_0)=2\sin\left(-\frac{\pi}{3}\right)=-\sqrt{3}$.

г) $y=\operatorname{ctg}\frac{x}{3}$. $\quad x_0=\pi$.

$y'=\dfrac{-1}{3\sin^2\frac{x}{3}}$. $\quad y'(x_0)=-\dfrac{1}{3\sin^2\frac{\pi}{3}}=-\frac{4}{9}$.

28.34.

а) $h(x) = (0,5x + 3)^7$. $x_0 = -4$. $h'(x) = \frac{7}{2}(0,5x + 3)^6$. $h'(x_0) = \frac{7}{2} = 3,5$.

б) $h(x) = \sqrt{16x + 21}$. $x_0 = \frac{1}{4}$. $h'(x) = \frac{8}{\sqrt{16x + 21}}$. $h'(x_0) = \frac{8}{5} = 1,6$.

в) $h(x) = \frac{18}{4x + 1}$. $x_0 = \frac{1}{2}$. $h'(x) = \frac{-18 \cdot 4}{(4x + 1)^2}$. $h'(x_0) = -\frac{72}{9} = -8$.

г) $h(x) = \sqrt{6 - 2x}$. $x_0 = 1$. $h'(x) = -\frac{1}{\sqrt{6 - 2x}}$. $h'(x_0) = -\frac{1}{2}$.

28.35. а) $f(x) = \sqrt{x} - x$. $k = 1$. $f'(x) = \frac{1}{2\sqrt{x}} - 1 = 1$.

$1 - 4\sqrt{x} = 0$. $\sqrt{x} = \frac{1}{4}$. $x = \frac{1}{16}$.

б) $f(x) = \sin x \cos x$. $k = -\frac{\sqrt{2}}{2}$.

$f'(x) = \cos 2x = -\frac{\sqrt{2}}{2}$. $x = \pm\frac{3\pi}{8} + \pi n$.

в) $f(x) = \sqrt{x} + 3x$. $k = 4$. $f'(x) = \frac{1}{2\sqrt{x}} + 3 = 4$.

$1 - 2\sqrt{x} = 0$. $x = \frac{1}{4}$.

г) $f(x) = \cos^2 x$. $k = \frac{1}{2}$.

$f'(x) = -2\cos x \sin x = -\sin 2x = \frac{1}{2}$. $x = (-1)^{k+1}\frac{\pi}{12} + \frac{\pi k}{2}$.

28.36. а) $f(x) = x^3 - x^4$.
$f'(x) = 3x^2 - 4x^3 < 0$.

$x^2(3 - 4x) < 0$. $x > \frac{3}{4}$.

б) $f(x) = \frac{1}{5}x^5 - \frac{5}{3}x^3 + 6x$.

$f'(x) = x^4 - 5x^2 + 6 < 0$.

$(x^2 - 2)(x^2 - 3) < 0$ $\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) < 0$.

$x \in \left(-\sqrt{3}; -\sqrt{2}\right) \cup \left(\sqrt{2}; \sqrt{3}\right)$.

28.37. а) $f(x) = \sin 2x$.
$f'(x) = 2\cos 2x < 0$.

$2x \in \left(\frac{\pi}{2}+2\pi n; \frac{3\pi}{2}+2\pi n\right)$. $\quad x \in \left(\frac{\pi}{4}+\pi n; \frac{3\pi}{4}+\pi n\right)$.

б) $f(x) = -4\cos x + 2x$.

$f'(x) = 4\sin x + 2 < 0$.

$\sin x < -\frac{1}{2}$. $\quad x \in \left(-\frac{5\pi}{6}+2\pi n; -\frac{\pi}{6}+2\pi n\right)$.

28.38. а) $g(x) = x^3 + x^4$.

$g'(x) = 3x^2 + 4x^3 > 0$.

$x^2(3 + 4x) > 0 \quad x \in \left(-\frac{3}{4};0\right) \cup (0;+\infty)$.

б) $g(x) = \frac{4}{2-5x}$.

$g'(x) = \frac{20}{(2-5x)^2} > 0$. $x \in$ R, но $x \neq \frac{2}{5}$.

28.39. а) $g(x) = \cos^2 x - \sin^2 x = \cos 2x$.

$g'(x) = -2\sin 2x > 0$. $\quad \sin 2x < 0$.

$2x \in (-\pi+2\pi n; 2\pi n)$. $x \in (-\frac{\pi}{2}+\pi n; \pi n)$.

б) $g(x) = \sin^2 x$. $\quad g'(x) = 2\sin x\cos x = \sin 2x > 0$.

$2x \in (2\pi n; \pi + 2\pi n)$. $x \in (\pi n; \frac{\pi}{2} + \pi n)$.

28.40. а) $h(x) = x^3 - 3x^2 + 1$. $\quad h'(x) = 3x^2 - 6x > 0$.

$x(3x - 6) > 0$. $\quad x \in (-\infty;0) \cup (2;+\infty)$.

б) $h(x) = 4\sqrt{x} - x$. $\quad h'(x) = \frac{2}{\sqrt{x}} - 1 > 0$.

$\frac{2}{\sqrt{x}} > 1$. $\quad \sqrt{x} < 2$. $\quad x \in (0; 4)$.

в) $y = x^3 - x^4 - 19$.

$y'(x) = 3x^2 - 4x^3 = x^2(3 - 4x) > 0$. $\quad x \in (-\infty; 0) \cup (0; 3/4)$.

г) $h(x) = \operatorname{tg} x - 4x$.

$h'(x) = \frac{1}{\cos^2 x} - 4 > 0$.

$\cos^2 x < \frac{1}{4}$, ОДЗ $x \neq \frac{\pi}{2} + \pi n$,

$\cos x \in \left(-\frac{1}{2}, \frac{1}{2}\right) \Rightarrow x \in \left(\frac{\pi}{3}+\pi n, \frac{\pi}{2}+\pi n\right) \cup \left(\frac{\pi}{2}+\pi n, \frac{2\pi}{3}+\pi n\right)$.

28.41. а) $\varphi(x) = \sin x + 3$.

$\varphi'(x) = \cos x < 0$. $\quad x \in \left(\frac{\pi}{2}+2\pi n; \frac{3\pi}{2}+2\pi n\right)$.

б) $\varphi(x) = 0{,}2x^5 - 3\frac{1}{3}x^3 + 9x$.

$\varphi'(x) = x^4 - 10x^2 + 9 < 0$.
$(x^2 - 9)(x^2 - 1) < 0$. $(x - 3)(x + 3)(x - 1)(x + 1) < 0$.
$x \in (-3; -1) \cup (1; 3)$.

28.42. а) $f(x) = \frac{1}{3}x^3 - x^2$. $g(x) = 7{,}5x^2 - 16x$.

$f'(x) = x^2 - 2x$. $g'(x) = 15x - 16$.
$x^2 - 2x = 15x - 16$;
$x^2 - 17x + 16 = 0$.

$x = \frac{17+15}{2} = 16$; $x = 1$.

б) $f(x) = \sqrt{x}$. $g(x) = -\frac{1}{x}$.

$f'(x) = \frac{1}{2\sqrt{x}}$. $g'(x) = \frac{1}{x^2}$.

$x^2 = 2\sqrt{x}$, $\frac{1}{2\sqrt{x}} = \frac{1}{x^2}$. $x \neq 0$. $x^4 = 4x$.

$x(x^3 - 4) = 0$. $x = 0$ не подходит. $x = \sqrt[3]{4}$.

28.43. а) $g(x) = x^3 - 3x^2$. $h(x) = 1{,}5x^2 - 9$.
$g'(x) = 3x^2 - 6x$. $h'(x) = 3x$.
$3x^2 - 6x > 3x$. $x^2 - 3x > 0$. $x \in (-\infty; 0) \cup (3; +\infty)$.

б) $g(x) = \sin\left(3x - \frac{\pi}{6}\right)$. $h(x) = 6x - 12$.

$g'(x) = 3\cos\left(3x - \frac{\pi}{6}\right)$. $h'(x) = 6$.

$3\cos\left(3x - \frac{\pi}{6}\right) > 6$. $\cos\left(3x - \frac{\pi}{6}\right) > 2$.

Таких значений нет.

в) $g(x) = \operatorname{tg}x$. $h(x) = 4x - 81$.

$g'(x) = \frac{1}{\cos^2 x}$. $h'(x) = 4$.

$\frac{1}{\cos^2 x} > 4$. $\cos^2 x < \frac{1}{4}$, $\cos^2 x \neq 0$.

$\cos x \in \left(-\frac{1}{2}; \frac{1}{2}\right)$, $\cos x \neq 0$. $x \in \left(\frac{\pi}{3} + \pi n; \frac{\pi}{2} + \pi n\right) \cup \left(\frac{\pi}{2} + \pi n; \frac{2\pi}{3} + \pi n\right)$.

г) $g(x) = \cos\left(\frac{\pi}{4} - 2x\right)$. $h(x) = 3 - \sqrt{2}\,x$.

$g'(x) = 2\sin\left(\frac{\pi}{4} - 2x\right)$. $h'(x) = -\sqrt{2}$.

$2\sin\left(\frac{\pi}{4}-2x\right)>-\sqrt{2}$. $\quad \sin\left(-\frac{\pi}{4}+2x\right)<\frac{\sqrt{2}}{2}$.

$2x-\frac{\pi}{4}\in\left(-\frac{5\pi}{4}+2\pi n;\frac{\pi}{4}+2\pi n\right)$. $\quad x\in\left(-\frac{\pi}{2}+\pi n;\frac{\pi}{4}+\pi n\right)$.

28.44.

а) $f(x)=\sin(2x-3)$. $\quad g(x)=\cos(2x-3)$.

$f'(x)=2\cos(2x-3)$. $\quad g'(x)=-2\sin(2x-3)$.

$\cos(2x-3)+\sin(2x-3)=0$.

$\sin\left(2x-3+\frac{\pi}{4}\right)=0$. $\quad x=\frac{3}{2}-\frac{\pi}{8}+\frac{\pi n}{2}$.

б) $f(x)=\frac{6}{5x-9}$. $\quad g(x)=\frac{3}{7-5x}$.

$g'(x)=\frac{15}{(7-5x)^2}$. $\quad f'(x)=-\frac{30}{(5x-9)^2}$.

$\frac{2}{(5x-9)^2}=\frac{-1}{(7-5x)^2}$. $\quad (5x-9)^2+2(5x-7)^2=0$.

Решений нет.

в) $f(x)=\sqrt{3x-10}$. $\quad g(x)=\sqrt{6x+14}$.

$f'(x)=\frac{3}{2\sqrt{3x-10}}$. $\quad g'(x)=\frac{3}{\sqrt{6x+14}}$.

$\frac{1}{2\sqrt{3x-10}}=\frac{1}{\sqrt{6x+14}}$; $2\sqrt{3x-10}=\sqrt{6x+14}$;

$12x-40=6x+14$; $\quad 6x=54$; $\quad x=9$.

Проверка: $2\sqrt{17}=\sqrt{68}$.

г) $f(x)=\mathrm{ctg}x$. $\quad g(x)=2x+15$.

$f'(x)=-\frac{1}{\sin^2 x}$. $\quad g'(x)=2$.

$\sin^2x=-\frac{1}{2}$.

Решений нет.

28.45. а) $h(x)=x^2-3x+19$. $\alpha=45°$

$h'(x)=2x-3=\mathrm{tg}45$. $\quad 2x-3=1$. $x=2$.

б) $\frac{4}{x+2}=h(x)$, $\quad \alpha=135$.

$h'(x)=\frac{-4}{(x+2)^2}=\mathrm{tg}135$. $\quad 4=(x+2)^2$.

$x=0$, $\quad x=-4$.

в) $h(x) = 2\sqrt{2x-4}$. $\alpha = 60°$

$h'(x) = \dfrac{2}{\sqrt{2x-4}} = \text{tg}60$. $2 = \sqrt{3(2x-4)}$.

$4 = 6x - 12$. $6x = 16$. $x = \dfrac{8}{3} = 2\dfrac{2}{3}$.

г) $h(x) = \sin\left(4x - \dfrac{\pi}{3}\right)$. $\alpha = 0$.

$h'(x) = \cos\left(4x - \dfrac{\pi}{3}\right) = 0$.

$4x - \dfrac{\pi}{3} = \dfrac{\pi}{2} + \pi n$. $x = \dfrac{5\pi}{24} + \dfrac{\pi n}{4}$.

28.46. а) $y = 4x^2 - |a|x$. $x(4x - |a|) = 0$. $x_1 = 0$. $x_2 = \dfrac{|a|}{4}$.

$y' = 8x - |a|$

Т.к. оси параболы направлены вверх и т.к. $x_2 \ge x_1$, то

1) $y'(x_1) = -\text{tg}60°$ и $y'(x_2) = \text{tg}60°$.

2) $y'(x_1) = -\text{tg}30°$ и $y'(x_2) = \text{tg}30°$.

1) $y'(0) = -|a| = -\text{tg}60°$. $a = \pm\sqrt{3}$. $y'\left(\dfrac{|a|}{4}\right) = |a| = \text{tg}60°$. $a = \pm\sqrt{3}$.

2) $y'(0) = -|a| = -\text{tg}30°$. $a = \pm\dfrac{\sqrt{3}}{3}$.

Ответ: $a = \pm\dfrac{\sqrt{3}}{3}$, $a = \pm\sqrt{3}$.

б) $y = x^2 + |a|x$.

$y' = 2x + |a|$. $x(x - |a|) = 0$. $x_1 = 0$. $x_2 = -|a|$.

т.к. оси параболы направлены вверх и $x_1 \ge x_2$, то

1) $y'(x_2) = -\text{tg}\dfrac{3\pi}{8}$.

2) $y'(x_2) = -\text{tg}\dfrac{\pi}{8}$.

Найдем $\text{tg}\dfrac{3\pi}{8}$:

$\text{tg}\dfrac{3\pi}{4} = \dfrac{2\text{tg}\dfrac{3\pi}{8}}{1 - \text{tg}\dfrac{3\pi}{8}}$. $\text{tg}^2\dfrac{3\pi}{8} - 2\text{tg}\dfrac{3\pi}{8} - 1 = 0$.

$\text{tg}\dfrac{3\pi}{8} = 1 \pm \sqrt{2}$, но т.к. $0 < \dfrac{3\pi}{8} < \dfrac{\pi}{2}$, то $\text{tg}\dfrac{3\pi}{8} = 1 + \sqrt{2}$.

$\operatorname{tg}\frac{\pi}{4}=\frac{2\operatorname{tg}\frac{\pi}{8}}{1-\operatorname{tg}^2\frac{\pi}{8}}$. $\operatorname{tg}^2\frac{\pi}{8}+2\operatorname{tg}\frac{\pi}{8}-1=0$.

$\operatorname{tg}\frac{\pi}{8}=-1\pm\sqrt{2}$, но т.к. $0<\frac{\pi}{8}<\frac{\pi}{2}$, то $\operatorname{tg}\frac{\pi}{8}=-1+\sqrt{2}$.

$\Rightarrow$ 1) $y'(x_2)=-1-\sqrt{2}$. $-|a|=-1-\sqrt{2}$. $a=\pm(1+\sqrt{2})$.

2) $y'(x_2)=1-\sqrt{2}$. $a=\pm(\sqrt{2}-1)$.

Ответ: $a=\pm\sqrt{2}\pm1$; $\pm\sqrt{2}\mp1$.

§ 29. Уравнение касательной к графику функции

29.1. а) a: $\operatorname{tg}\alpha=0$. b: $\operatorname{tg}\alpha<0$. с: $\operatorname{tg}\alpha>0$.

б) a: $\operatorname{tg}\alpha<0$. b: $\operatorname{tg}\alpha<0$. с: $\operatorname{tg}\alpha>0$.

29.2. а) $y'=0$ при $x=0$, $x=3{,}5$; y' не существует при $x=-1$.

б) $y'=0$ при $x=-4$, $x=-1{,}5$; y' не существует при $x=4$.

в) $y'=0$ при $x=-4$; y' не существует при $x=-2$.

г) $y'\neq0$ при $x\in R$.

29.3. а) $f(x)=4+x^2$, $a=2$. $f'(x)=2x$; $f'(a)=4\Rightarrow$ острый.

б) $f(x)=1-\frac{1}{x}$, $a=3$. $f'(x)=\frac{1}{x^2}$; $f'(a)=\frac{1}{9}\Rightarrow$ острый.

в) $f(x)=(1-x)^3$, $a=-3$. $f'(x)=-3(1-x)^2$; $f'(a)=-48\Rightarrow$ тупой.

г) $f(x)=2x-x^3$, $a=1$. $f'(x)=2-3x^2$; $f'(a)=-1\Rightarrow$ тупой.

29.4. $y=1-x^2$; $y'(x)=-2x$.

а) $A(0;1)$; $y'(0)=0\Rightarrow\operatorname{tg}\alpha=0$.

б) $B(2;-3)$; $y'(2)=-4\Rightarrow\operatorname{tg}\alpha=-4$.

в) $C\left(\frac{1}{2};\frac{3}{4}\right)$; $y'\left(\frac{1}{2}\right)=-1\Rightarrow\operatorname{tg}\alpha=-1$.

г) $D(-1;0)$; $y'(-1)=2\Rightarrow\operatorname{tg}\alpha=2$.

28.5. а) $f(x)=x^3-2x^2+3$, $a=-1$. $f'(x)=3x^2-4x$; $f'(a)=7$, $\operatorname{tg}\alpha=7$.

б) $f(x)=\sqrt{4-5x}$, $a=0$. $f'(x)=-\frac{5}{2\sqrt{4-5x}}$; $f'(a)=-\frac{5}{4}$; $\operatorname{tg}\alpha=-\frac{5}{4}$.

в) $f(x)=x^4-7x^3+12x-45$, $a=0$. $f'(x)=4x^3-21x^2+12$; $f'(a)=12$, $\operatorname{tg}\alpha=12$.

г $f(x)=\sqrt{10+x}$, $a=-5$. $f'(x)=\frac{1}{2\sqrt{x+10}}$; $f'(a)=\frac{1}{2\sqrt{5}}$; $\operatorname{tg}\alpha=\frac{\sqrt{5}}{10}$.

29.6. а) $f(x)=\sin x$, $a=0$. $f'(x)=\cos x$; $f'(a)=1$; $\operatorname{tg}\alpha=1$.

б) $f(x)=\operatorname{tg}2x$, $a=\frac{\pi}{8}$. $f'(x)=\frac{2}{\cos^2 2x}$; $f'(a)=4$; $\operatorname{tg}\alpha=4$.

в) $f(x) = \cos 3x$, $a = \frac{\pi}{2}$. $f'(x) = -3\sin 3x$; $f'(a) = 3$; $\operatorname{tg}\alpha = 3$.

г) $f(x) = \sin x$ $\quad f'(x) = \cos x$, $f'(a) = \frac{1}{2}$, $\operatorname{tg}\alpha = \frac{1}{2}$.

29.7. а) $f(x) = x^2$, $a = 0{,}5$. $f'(x) = 2x$; $f'(a) = 1$; $\alpha = \frac{\pi}{4}$.

б) $f(x) = -3x^3$, $a = \frac{1}{3}$ $f'(x) = -9x^2$; $f'(a) = -1$; $\alpha = \frac{3\pi}{4}$.

в) $f(x) = 0{,}2x^5$, $a = -1$. $f'(x) = x^4$; $f'(a) = 1$; $\alpha = \frac{\pi}{4}$.

г) $f(x) = -0{,}25x^4$, $a = 0$. $f'(x) = -x^3$; $'(a) = 0$; $\alpha = 0$.

29.8. а) $f(x) = x^3 - 3x^2 + 2x - 7$, $\quad a = 1$.

$f'(x) = 3x^2 - 6x + 2$, $f'(a) = -1$, $\quad \alpha = \frac{3\pi}{4}$.

б) $f(x) = -7x^3 + 10x^2 + x - 12$, $\quad a = 0$.

$f'(x) = -21x^2 + 20x + 1$ $\quad f(a) = 1$ $\quad \alpha = \frac{\pi}{4}$.

29.9. а) $f(x) = \frac{2x-1}{3-2x}$, $a = \frac{1}{2}$. $f(x) = \frac{6-4x+4x-2}{(3-2x)^2} = \frac{4}{(3-2x)^2}$

$f'(a) = 1$ $\alpha = \frac{\pi}{4}$.

б) $f(x) = \frac{x-1}{x-2}$, $a = 1$. $f(x) = \frac{x-2-x+1}{(x-2)^2} = -\frac{1}{(x-2)^2}$

$f'(a) = -1$ $\quad \alpha = \frac{3\pi}{4}$.

29.10. а) $f(x) = \sqrt{6x+7}$, $\quad a = 3\frac{1}{3}$. $f(x) = \frac{3}{\sqrt{6x+7}}$

$f'(a) = \frac{1}{\sqrt{3}}$ $\quad \alpha = \frac{\pi}{6}$.

б) $f(x) = \sqrt{5-2x}$, $\quad a = 2$. $f(x) = -\frac{1}{\sqrt{5-2x}}$

$f'(a) = -1$ $\quad \alpha = \frac{3\pi}{4}$.

29.11. а) $f(x) = \sqrt{3}\cos\frac{x}{3}$, $\quad a = \frac{3\pi}{2}$. $f(x) = -\frac{\sqrt{3}}{3}\sin\frac{x}{3}$

$f'(a) = -\frac{\sqrt{3}}{3}$ $\quad \alpha = \frac{5\pi}{6}$.

б) $f(x)=\frac{1}{2}\sin 2x$, $a=\frac{\pi}{2}$. $f'(x)=\cos 2x$

$f'(a)=-1$ $\alpha=\frac{3\pi}{4}$.

29.12. а) $f(x)=x^2$, $a=3$, $f'(x)=2x$

$f'(a)=6$, $f(a)=9$, $y=9+6(x-3)=6x-9$.

б) $f(x)=2-x-x^3$, $a=0$, $f'(x)=-1-3x^2$

$f'(a)=-1$, $f(a)=2$, $y=2-x$.

в) $f(x)=x^3$, $a=1$, $f'(x)=3x^2$

$f'(a)=3$, $f(a)=1$, $y=1+3(x-1)=3x-2$.

г) $f(x)=x^2-3x+5$, $a=-1$, $f(a)=1+3+5=9$.

$f'(x)=2x-3$, $f'(a)=-5$, $y=9-5(x+1)=-5x+4$.

29.13. а) $f(x)=\frac{3x-2}{3-x}$, $a=2$. $f(a)=4$.

$f'(x)=\frac{9-3x+3x-2}{(3-x)^2}=\frac{7}{(x-3)^2}$ $f'(a)=7$.

$y=4+7(x-2)=7x-10$.

б) $f(x)=\frac{2x-5}{5-x}$, $a=4$. $f(a)=3$.

$f'(x)=\frac{10-2x+2x-5}{(5-x)^2}=\frac{5}{(5-x)^2}$ $f'(a)=5$.

$y=3+5(x-4)=5x-17$.

29.14.

а) $f(x)=2\sqrt{3x-5}$, $a=2$, $f(a)=2$;

$f'(x)=\frac{3}{\sqrt{3x-5}}$. $f'(a)=3$. $y=2+3(x-2)=3x-4$.

б) $f(x)=\sqrt{7-2x}$, $a=3$, $f(a)=1$;

$f'(x)=-\frac{1}{\sqrt{7-2x}}$. $f'(a)=-1$. $y=1-x+3=-x+4$.

29.15. а) $f(x)=\cos\frac{x}{3}$, $a=0$, $f(a)=1$;

$f'(x)=-\frac{1}{3}\sin\frac{x}{3}$, $f'(a)=0$. $y=1$.

в) $f(x)=\sin 2x$, $a=\frac{\pi}{4}$, $f(a)=1$;

$f'(x)=2\cos 2x$ $f'(a)=0$; $y=1$.

29.16. а) $f(x) = \operatorname{ctg}2x, a = \frac{\pi}{4}$, $f(a) = 0$;

$f'(x) = -\frac{2}{\sin^2 2x}$. $f'(a) = -2$. $y = -2\left(x - \frac{\pi}{4}\right) = \frac{\pi}{2} - 2x$.

б) $f(x) = 2\operatorname{tg}\frac{x}{3}$, $a = 0$, $f(a) = 0$;

$f'(x) = \frac{2}{3\cos^2\frac{x}{3}}$, $f'(a) = \frac{2}{3}$; $y = \frac{2}{3}x$.

29.17. $y = 9 - x^2$; $9 - x^2 = 0$, $x = \pm 3$. $y' = -2x$.
$y'(3) = -6$, $y'(-3) = 6$. $y = 6(x + 3) = 6x + 18$.
$y = -6(x - 3) = 18 - 6x$.
29.18. $y = x^2 - 3x$; $x^2 - 3x - 4 = 0$, $x = 4$, $x = -1$.
$y' = 2x - 3$, $y'(4) = 8 - 3 = 5$, $y'(-1) = -5$;
$y = 4 + 5(x - 4) = 5x - 16$;
$y = 4 - 5(x + 1) = -5x - 1$.
29.19. $y = x^3 - 3x^2 + 4x + 1$
$y' = 3x^2 - 6x + 4$, $\angle d = 45° \Rightarrow y' = \pm 1$

$$\left[\begin{array}{l} 3x^2 - 6x + 4 = 1 \\ 3x^2 - 6x + 4 = -1 \end{array}\right., \left[\begin{array}{l} (x-1)^2 = 0 \\ 3x^2 - 6x + 5 = 0 \end{array}\right.$$

$x = 1$, $y(1) = 1 - 3 + 4 + 1 = 3$
Касательная $y = 2 + x$
Ответ: $(1, 3)$, $y = x + 2$
(В ответах задачника опечатка).
29.20. $y = x^2$, $y' = 2x$.
$y = x_0^2 + 2x_0(x - x_o) = 2x_0x - x_0^2$.

а) $y = 2x + 1$, $x_0 = 1$, $y_0 = 1$;

б) $y = -\frac{1}{2}x + 5$, $x_0 = -\frac{1}{4}$, $y_0 = \frac{1}{16}$;

в) $\frac{3}{4}x - 2 = y$, $x_0 = \frac{3}{8}$, $y_0 = \frac{9}{64}$

г) $y = -x + 5$, $x_0 = -\frac{1}{2}$, $y_0 = \frac{1}{4}$.

29.21. а) $f(x) = \frac{x^3}{3} - 3x^2 + 10x - 4$, $y = x + 3$.

$f'(x) = x^2 - 6x + 10$.

$y = \frac{x_0^3}{3} - 3x_0^2 + 10x_0 - 4 + \left(x_0^2 - 6x_0 + 10\right)(x - x_0)$ — касательная.

$x_0^2-6x_0+10=1$. $x_0^2-6x_0+9=0$. $x_0=3$.

б) $f(x)=\frac{x^4}{4}-x^2+8$, $y=0$.

$f'(x)=x^3-2x$,

$y=\frac{x_0^4}{4}-x_0^2+8+\left(x_0^3-2x_0\right)(x-x_0)$;

$x_0^3-2x_0=0$, $x_0=0$, $x_0^2=2$, $x_0=\pm\sqrt{2}$;

в) $f(x)=\frac{x^3}{3}-x^2+2x-7$, $y=x-3$,

$f'(x)=x^2-2x+2$,

$y=\frac{x_0^3}{3}-x_0^2+2x_0-7+\left(x_0^2-2x_0+2\right)(x-x_0)$.

$x_0^2-2x_0+2=1$, $x_0=1$;

г) $f(x)=\frac{5x^4}{4}-x^3+6$, $y=2$,

$f'(x)=5x^3-3x^2$,

$x_0=0$, $5x_0-3=0$, $x_0=\frac{3}{5}$.

29.22.

а) $f(x)=\sin x$, $y=-x$, $f'(x)=\cos x$,
$\cos x_0=-1$, $x_0=\pi+2\pi n$;
б) $f(x)=\cos 3x$, $y=0$. $f'(x)=-3\sin 3x$.

$\sin 3x_0=0$, $x_0=\frac{\pi n}{3}$;

в) $f(x)=\operatorname{tg}x$. $y=x-3$ $f'(x)=\frac{1}{\cos^2 x}$,

$\frac{1}{\cos^2 x}=1$, $\cos x=\pm 1$. $x=\pi n$;

г) $f(x)=\sin\frac{x}{2}$, $y=-1$, $f'(x)=\frac{1}{2}\cos\frac{x}{2}$,

$\cos\frac{x}{2}=0$, $x_0=\pi+2\pi n$.

29.23. $y=\frac{x^3}{3}-2$.

$y'=x^2$. $y=\frac{x_0^3}{3}-2+x_0^2(x-x_0)=x_0^2x-\frac{2}{3}x_0^3-2$.

а) $y = x - 3,$ $\qquad x_0^2 = 1,$ $\qquad x_0 = \pm 1;$

$$y = x - \frac{2}{3} - 2 = x - \frac{8}{3}. \qquad y = x + \frac{2}{3} - 2 = x - 1\frac{1}{3}.$$

б) $y = 9x - 5.$

$x_0^2 = \pm 3.$ $\qquad y = 9x - 18 - 2 = 9x - 20.$

$y = 9x + 18 - 2 = 9x + 16.$

29.24. а) $0{,}998^5.$

$y = x^5 = f(x),$ $\qquad x = 0{,}998;$ $\qquad a = 1.$ $\qquad f(a) = 1.$

$f'(x) = 5x^4.$ $\qquad f'(a) = f'(1) = 5.$

$0{,}998^5 \approx 1 - 5 \cdot 0{,}002 = 0{,}99.$

б) $\sqrt{1{,}05} \approx 1 + \frac{1}{2} \cdot 0{,}05 = 1{,}025$

в) $1{,}03^7 \approx 1 + 7 \cdot 0{,}03 = 1{,}21.$

г) $\sqrt{3{,}99} \approx 2 - \frac{1}{2\sqrt{4}} \cdot 0{,}01 = 1{,}9975.$

29.25. а) $f(x) = \sqrt{3 - x}$. $\qquad$ В $(-2; 3),$

$$f'(x) = -\frac{1}{2\sqrt{3 - x}}.$$

$$3 = \sqrt{3 - x_0} - \frac{1}{2\sqrt{3 - x_0}}(-2 - x_0); \qquad 3 = \frac{6 - 2x_0 + 2 + x_0}{2\sqrt{3 - x_0}};$$

$6\sqrt{3 - x_0} = 8 - x_0.$ $\quad 108 - 36x_0 = 64 - 16x_0 + x_0^2.$

$x_0^2 + 20x_0 - 44 = 0.$ $\quad x_0 = -22.$ $\qquad x_0 = 2.$ $\qquad f(-22) = 5.$ $\qquad f(2) = 1.$

$$y = 5 - \frac{1}{10}(x + 22) = -\frac{x}{10} - \frac{11}{5} + 5 = -\frac{x}{10} + \frac{14}{5}.$$

$$y = 1 - \frac{1}{2}(x - 2) = -\frac{1}{2}x + 2.$$

б) $f(x) = \sqrt{3 - x}$, $\qquad$ В $(4; 0).$

$$0 = \sqrt{3 - x_0} - \frac{1}{2\sqrt{3 - x_0}}(4 - x_0); \; 6 - 2x_0 - 4 + x_0 = 0;$$

$x_0 = 2;$ $\qquad f(2) = 1;$

$$y = 1 - \frac{1}{2}(x - 2) = -\frac{1}{2}x + 2.$$

29.26. $y = \frac{1}{x^2},$ $\qquad x < 0,$ $\qquad -\frac{1}{2}xy = \frac{9}{8};$ $\qquad y = \frac{3}{x_0^2} - \frac{2x}{x_0^3}.$

При $x = 0$ $\qquad y = \frac{3}{x_0^2}.$ $\qquad\qquad$ При $y = 0$ $\qquad x = \frac{3x_0}{2}.$

$\frac{3\cdot 3}{2}\cdot\frac{1}{x_0}=-\frac{9}{4}$; $\quad \frac{1}{x_0}=-\frac{1}{2}$; $\quad x_0=-2$.

$y=\frac{3}{4}+\frac{2x}{8}=\frac{3+x}{4}$.

29.27. $y=\frac{\sqrt{3}}{6}\left(1-x^2\right)$, $\quad \alpha=120°$; $\quad y'=-\frac{\sqrt{3}}{3}x$.

$y=-\frac{\sqrt{3}}{3}x_0x+\frac{\sqrt{3}}{3}x_0^2-\frac{\sqrt{3}x_0^2}{6}+\frac{\sqrt{3}}{6}=-\frac{\sqrt{3}}{3}x_0x+\frac{\sqrt{3}}{6}x_0^2+\frac{\sqrt{3}}{6}$.

1. $y'=-\frac{\sqrt{3}}{3}x_0=\pm\mathrm{tg}60°$. $\frac{\sqrt{3}}{3}x_0=\pm\sqrt{3}$. $x_0=\pm 3$.

$y=-\sqrt{3}x+\frac{3\sqrt{3}}{2}+\frac{\sqrt{3}}{6}=-\sqrt{3}x+\frac{5\sqrt{3}}{3}$. $y=\sqrt{3}x+\frac{5\sqrt{3}}{3}$.

2. $y'=-\frac{\sqrt{3}}{3}x_0=\pm\mathrm{tg}30°$. $x_0=\pm 1$. $y=\frac{\sqrt{3}}{3}x+\frac{\sqrt{3}}{3}$. $y=-\frac{\sqrt{3}}{3}x+\frac{\sqrt{3}}{3}$.

Ответ: $y=\pm\frac{\sqrt{3}}{3}(x\pm 1)$. $y=\pm\sqrt{3}\left(x\pm\frac{5}{3}\right)$.

§ 30. Применение производной исследования функций на монотонность и экстремумы

30.1. а) $f'(a)>0$. $f'(b)>0$. $f'(c)<0$. $f'(d)>0$.
б) $f'(a)>0$. $f'(b)=0$. $f'(c)=0$. $f'(d)>0$.

30.2. а) ф–ция возрастает: $\left(-\infty;\frac{c+b}{2}\right)\cup\left[d-\frac{c-b}{2};+\infty\right)$.

убывает: $\left(\frac{c+b}{2};d-\frac{c-b}{2}\right]$

б) возрастает: $(-\infty;b]\cup[c;+\infty)$; убывает: $[b;c]$

30.3. а) убывает: $[-2;2]$; возрастает: $(+\infty;-2]\cup[2;+\infty)$

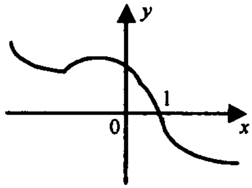
б) убывает: $[-4;0]\cup[3;+\infty)$; возрастает: $(-\infty;-4]\cup[0;3]$

в) убывает: $[-4,5;+\infty)$; возрастает: $(-\infty;-4,5]$

г) убывает: $(-\infty;-2,5]\cup[2,5;+\infty)$; возрастает: $[-2,5;2,5]$

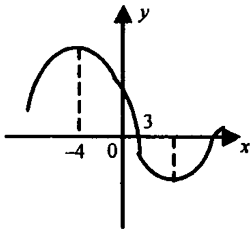
30.4. в)
30.5. для ф–ции $g(x)$.
30.6. а) $f(x)$; $\qquad$ б) $h(x)$

30.7.

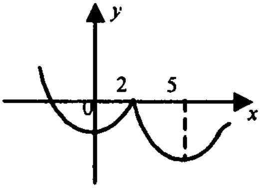


30.8.

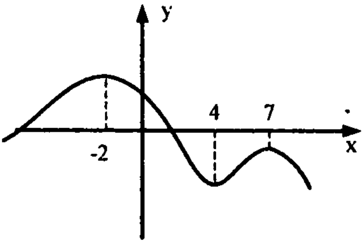
а)



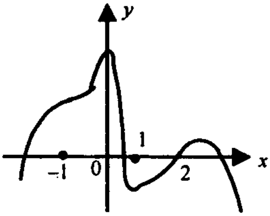
б)



в)



г)



30.9. а) $y = \cos x + 2x$.

$y' = -\sin x + 2$ $\quad$ $y' > 0$ при любых x.

б) $y = x^5 + 3x^3 + 7x + 4$.

$y' = 5x^4 + 9x^2 + 7$. $\quad$ $y' > 0$ при любых x.

в) $y = \sin x + x^3 + x$.

$y' = \cos x + 3x^2 + 1$. $\quad$ $y' > 0$ при любых x.

г) $y = x^5 + 4x^3 + 8x - 8$.

$y' = 5x^4 + 12x^2 + 8$. $\quad$ $y' > 0$ при любых x.

30.10. а) $y = \sin 2x - 3x$.

$y' = 2\cos 2x - 3$. $\quad$ $y' < 0$ при любых x.

б) $y = \cos 3x - 4x$. $\quad$ $y' = -3\sin 3x - 4$.

$y' < 0$ при любых x.

30.11. а) $y = x^5 + 6x^3 - 7$.

$y' = 5x^4 + 18x^2 \geq 0$, функция возрастает.

б) $y = \sin x - 2x - 15$. $y' = \cos x - 2 < 0$, убывает.
в) $y = x - \cos x + 8$. $y' = 1 + \sin x \geq 0$, функция возрастает.
г) $y = 11 - 5x - x^3$. $y' = -5 - 3x^2 < 0$, функция убывает.
30.12. а) $y = x^2 - 5x + 4$. $y' = 2x - 5$.

При $x \geq \frac{5}{2}$ ф–ция возрастает.

При $x \leq \frac{5}{2}$ ф–ция убывает.

б) $y = 5x^2 + 15x - 1$. $y' = 10x + 15$.

При $x \geq -\frac{3}{2}$ ф–ция возрастает.

При $x \leq -\frac{3}{2}$ ф–ция убывает.

в) $y = -x^2 + 8x - 7$. $y' = -2x + 8$.
При $x \geq 4$ ф–ция убывает.
При $x \leq 4$ ф–ция возрастает.
г) $y = x^2 - x$. $y' = 2x - 1$.

При $x \geq \frac{1}{2}$ ф–ция возрастает.

При $x \leq \frac{1}{2}$ ф–ция убывает.

30.13. а) $y = x^3 + 2x$. $y' = 3x^2 + 2$.
Возрастает при любых x.
б) $y = 60 + 45x - 3x^2 - x^3$.
$y' = 45 - 6x - 3x^2 = -3(x^2 + 2x - 15)$.
$x \in [-5; 3]$ возрастает. $x \in (-\infty; -5] \cup [3; +\infty)$ убывает.
в) $y = 2x^3 - 3x^2 - 36x + 40$. $y' = 6x^2 - 6x - 36 = 6(x^2 - x - 6)$.
$x \in [-2; 3]$ убывает. $x \in (-\infty; -2] \cup [3; +\infty)$ возрастает.
г) $y = -x^5 + 5x$. $y' = -5x^4 + 5 = -5(x^4 - 1)$.
$x \in [-1; 1]$ возрастает. $x \in (-\infty; -1] \cup [1; +\infty)$ убывает.
30.14. а) $y = x^4 - 2x^2 - 3$. $y' = 4x^3 - 4x = 4x(x^2 - 1) = 4x(x - 1)(x + 1)$.
$x \in (-\infty; -1] \cup [0; 1]$ убывает. $x \in [-1; 0] \cup [1; +\infty)$ возрастает.
б) $y = -x^5 - x$. $y' = -5x^4 - 1 = -(5x^4 + 1) < 0$. убывает при всех x.
в) $y = -3x^4 + 4x^3 - 15$. $y' = -12x^3 + 12x^2 = -12x^2(x - 1)$.
$x \in (-\infty; 1]$ возрастает. $x \geq 1$ убывает.
г) $y = 5x^5 - 1$. $y' = 25x^4$. возрастает при всех x.

30.15. а) $y = \frac{1}{x+3}$;

$y' = -\frac{1}{(x+3)^2}$. Убывает на всей ОДЗ ($x \neq -3$).

б) $y=\frac{3x-1}{3x+1}$.

$y'=\frac{9x+3-9x+3}{(3x+1)^2}=\frac{2}{(3x+1)^2}$, возрастает на всей ОДЗ ($x\neq-\frac{1}{3}$).

в) $y=\frac{2}{x}+1$,

$y'=-\frac{2}{x^2}$, убывает на всей ОДЗ ($x\neq 0$).

г) $y=\frac{1-2x}{3+2x}$,

$y'=\frac{-6-4x-2+4x}{(3+2x)^2}=-\frac{8}{(3+2x)^2}$, убывает на всей ОДЗ ($x\neq-\frac{3}{2}$).

30.16. а) $y=\sqrt{3x-1}$,

$y'=\frac{3}{2\sqrt{3x-1}}$, возрастает на всей ОДЗ ($x\geq\frac{1}{3}$).

б) $y=\sqrt{1-x}+2x$,

$y'=-\frac{1}{2\sqrt{1-x}}+2=0$, $\frac{4\sqrt{1-x}-1}{2\sqrt{1-x}}=0$, $x=\frac{15}{16}$.

$x\in\left(-\infty;\frac{15}{16}\right]$, возрастает. $x\in\left[\frac{15}{16};1\right]$ убывает.

в) $y=\sqrt{1-2x}$,

$y'=-\frac{1}{\sqrt{1-2x}}$, $x\in\left(-\infty;\frac{1}{2}\right]$ убывает.

г) $y=\sqrt{2x-1}-x$,

$y'=\frac{1}{\sqrt{2x-1}}-1=0$, $\frac{1-\sqrt{2x-1}}{\sqrt{2x-1}}=0$;

$x\in\left[\frac{1}{2};1\right]$ возрастает. $x\in[1;+\infty)$ убывает.

30.17. а) $f'(b)=f'(d)=0$.

б) $f'(c)=0$.

в) $f'(a)=f'(0)=0$.

г) нет точек, в которых производная равна нулю.

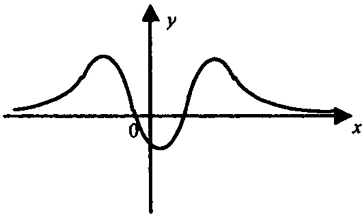
30.18. а) e б) a, b в) b, c г) a, b, c, d, e

30.19. а) 1 б) 2 в) 2 г) 2

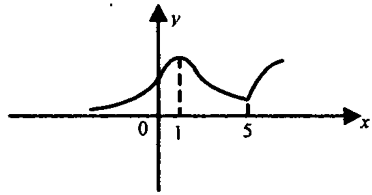
30.20. а) 2 б) 1 в) 2 г) 2

30.21. а) $(-\infty;-5]\cup[-2;+\infty)$ б) $[-5;-2]$ в) -5. г) -2.

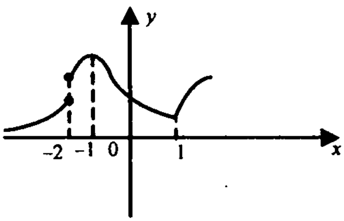
30.22. а)



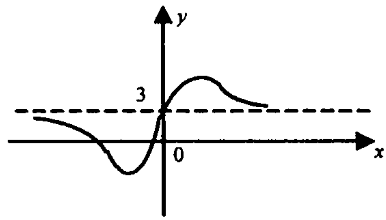
б)



в)



г)



30.23. а) да; б) нет; в) нет; г) нет.

30.24. а) $y = x^3 - 3ax^2 + 27x - 5$;

$y' = 3x^2 - 6ax + 27$;

$x^2 - 2ax + 9 = 0$; $\dfrac{D}{4} = a^2 - 9 = 0$; $a = \pm 3$.

б) $y = x^3 - 3ax^2 + 75x - 10$;

$y' = 3x^2 - 6ax + 75$;

$x^2 - 2ax + 25 = 0$; $\dfrac{D}{4} = a^2 - 25 = 0$; $a = \pm 5$.

30.25. а) Да. б) Да. в) Да. г) Да.

30.26. а) $y = 7 + 12x - x^3$;

$y' = 12 - 3x^2$; $3x^2 = 12$; $x = \pm 2$.

б) $y = 8 + 2x^2 - x^4$; $y' = 4x - 4x^3$;

$4x(1 - x^2) = 0$; $x = 0, x = \pm 1$.

в) $y = 3x^3 + 2x^2 - 7$; $y' = 9x^2 + 4x$;

$x(9x + 4) = 0$; $x = 0,$ $x = -\dfrac{4}{9}$.

г) $y = x^4 - 8x^2$; $y' = 4x^3 - 16x$;

$4x(x^2 - 4) = 0$; $x = 0, x = \pm 2$.

30.27.

а) $y=2x+\frac{8}{x}$; $y'=2-\frac{8}{x^2}=0;\ x=\pm2$

б) $\sqrt{2x-1}=y$;

$y'=\frac{1}{\sqrt{2x-1}}$ $x=\frac{1}{2}$ – критическая

в) $y=\frac{x}{5}+\frac{5}{x}$; $y'=\frac{1}{5}-\frac{5}{x^2}=0;\ x^2-25=0;\ x=\pm5$.

г) $y=(x-3)^4$; $y'=4(x-3)^3=0\ \ x=3$.

30.28. а) $y=\frac{x^3}{3}-\frac{5}{2}x^2+6x-1$;

$y'=x^2-5x+6=0;\ x=2,\ x=3.\ \ x=2-\max,\ \ x=3-\min$.

б) $y=x^3-27x+26$;

$y'=3x^2-27=0;\ x=\pm3;\ x=-3-\max,\ \ x=3-\min$.

в) $y=x^3-7x^2-5x+11$;

$y'=3x^2-14x-5=0;\ x=\frac{7+8}{3}=5;\ \ x=-\frac{1}{3};\ x=5-\min$,

$x=-\frac{1}{3}-\max$.

г) $y=-2x^3+21x^2+19$;

$y'=-6x^2+42x=0;\ \ x^2-7x=0;\ x(x-7)=0$;

$x=0,\qquad x=7$;

$x=7-\max,\ x=0-\min$.

30.29. а) $y=-5x^5+3x^3$;

$y'=-25x^4+9x^2$; $x^2\left(9-25x^2\right)=0$

$x=0,\ x^2=\frac{9}{25},\ x=\pm\frac{3}{5};\ \ x=\frac{3}{5}-\max,\ \ x=-\frac{3}{5}-\min$.

б) $y=x^4-4x^3-8x^2+13$;

$y'=4x^3-12x^2-16x$; $4x\left(x^2-3x-4\right)=0$;

$x=0,\qquad x=4,\qquad x=-1;\ x=4,\ \ x=-1-\min,\ \ x=0-\max$.

в) $y=x^4-50x^2$;

$y'=4x^3-100x\quad 4x\left(x^2-25\right)=0\quad x=0\qquad x=\pm5$

$x=-5, x=5-\min,\ \ x=0-\max$.

г) $y = 2x^5 + 5x^4 - 10x^3 + 3$;

$y' = 10x^4 + 20x^3 - 30x^2$; $10x^2\left(x^2 + 2x - 3\right) = 0$;

$x = 0$, $x = -3$; $x = 1$,

$x = -3 - \max$, $x = 1 - \min$.

30.30. а) $y = x + \frac{4}{x}$;

$y' = 1 - \frac{4}{x^2} = 0$; $x = \pm 2$, $x = 2 - \min$, $x = -2 - \max$.

б) $y = \frac{x^2 + 9}{x} = x + \frac{9}{x}$;

$y' = 1 - \frac{9}{x^2} = 0$; $x = \pm 3$, $x = 3 - \min$, $x = -3 - \max$.

30.31. а) $y = x - 2\sqrt{x-2}$;

$y' = 1 - \frac{1}{\sqrt{x-2}} = 0$; $\sqrt{x-2} = 1$; $x = 3$, $x = 3 - \min$.

б) $y = 4\sqrt{2x-1} - x$;

$y' = \frac{4}{\sqrt{2x-1}} - 1 = 0$; $\sqrt{2x-1} = 4$; $x = 8{,}5 - \max$.

30.32. а) $y = x - 2\cos x$, $x \in [-\pi; \pi]$;

$y' = 1 + 2\sin x$; $\sin x = -\frac{1}{2}$; $x = (-1)^{k+1}\frac{\pi}{6} + \pi k$;

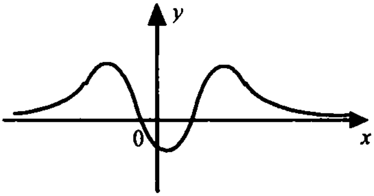
$x = -\frac{5\pi}{6} - \max$; $x = -\frac{\pi}{6} - \min$.

б) $y = 2\sin x - x$, $x \in [\pi; 3\pi]$; $y' = 2\cos x - 1$;

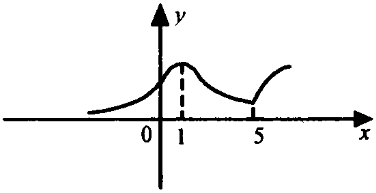
$\cos x = \frac{1}{2}$; $x = \pm\frac{\pi}{3} + 2\pi n$; $x = \frac{5\pi}{3} - \min$; $x = \frac{7\pi}{3} - \max$.

§ 31. Построение графиков функций

31.1. а)

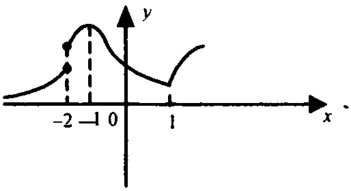


б)

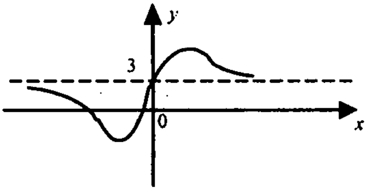


31.2.

а)



б)



31.3.

а) $y = 3x^2 - 4x + 5$;

$y' = 6x - 4$;

$x = \frac{2}{3} - \min$,

$y\left(\frac{2}{3}\right) = \frac{11}{3}$

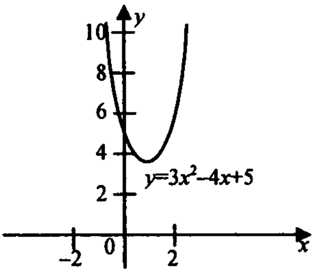
при $x \geq \frac{2}{3}$ функция возрастает,

при $x \leq \frac{2}{3}$ функция убывает

пересечение с Oy:(0;5)

с Ox: нет

$y > 0$ при $x \in R$



б) $y = 3 + 2x - x^2$;

$y' = 2 - 2x$;

$x = 1 - \min$,

$y(1) = 4$

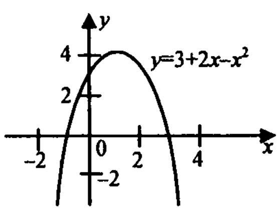
при $x \geq 1$ функция убывает,

при $x \leq 1$ функция возрастает

пересечение с Oy:(0;3)

с Ox: (3;0), (-1;0)

$y > 0$ при $x \in (-1; 3)$, $y < 0$ при $x < -1$, $x > 3$

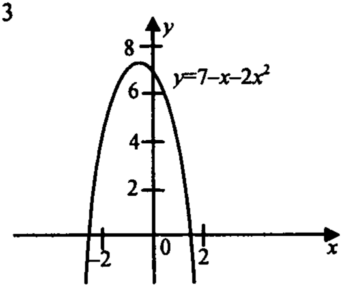


в) $y = 7 - x - 2x^2$; $y' = -1 - 4x$;

$x = -\frac{1}{4} - \max$, $y\left(-\frac{1}{4}\right) = \frac{57}{8}$

при $x \geq -\frac{1}{4}$ функция убывает

при $x \leq -\frac{1}{4}$ функция возрастает



пересечение с Oy: $(0;7)$

с Ox: $\left(\dfrac{-1-\sqrt{57}}{4};0\right)$, $\left(\dfrac{-1+\sqrt{57}}{4};0\right)$;

$y>0$, при $x\in\left(\dfrac{-1-\sqrt{57}}{4};\dfrac{-1+\sqrt{57}}{4}\right)$

$y<0$ при $x\in\left(-\infty;\dfrac{-1-\sqrt{57}}{4}\right)$, $\left(\dfrac{-1+\sqrt{57}}{4};+\infty\right)$

г) $y=5x^2-15x-4$; $y'=10x-15$;

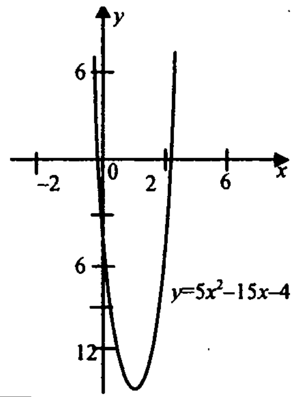
$x=\dfrac{3}{2}-\min$

при $x\geq\dfrac{3}{2}$ функция возрастает,

при $x\leq\dfrac{3}{2}$ функция убывает

пересечение с Oy:$(0;-4)$

с Ox: $\left(\dfrac{15-\sqrt{305}}{10};0\right)$,$\left(\dfrac{15+\sqrt{305}}{10};0\right)$



функция $y>0$ при $x<\dfrac{15-\sqrt{305}}{10}$, $x>\dfrac{15+\sqrt{305}}{10}$

$y<0$ при $x\in\left(\dfrac{15-\sqrt{305}}{10};\dfrac{15+\sqrt{305}}{10}\right)$.

31.4. а) $y=3x^2-x^3$;

$y'=6x-3x^2$;

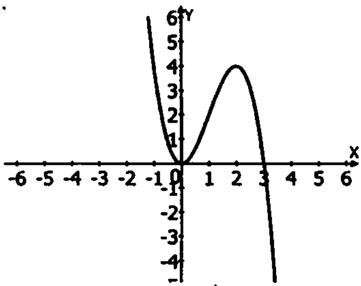
$x=0,\ x=2$

$y(0)=0\qquad y(2)=12-8=4$,

$x=0-\min$, $x=2-\max$,

при $x\in[0;2]$ функция возрастает,

при $x\leq 0, x\geq 2$ функция убывает.

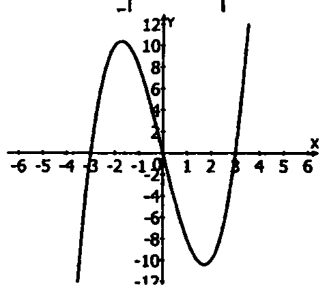


б) $y=-9x+x^3$;

$y'=-9+3x^2$;

$x^2=3$; $x=\pm\sqrt{3}$;

$y(\sqrt{3})=-9\sqrt{3}+3\sqrt{3}$; $y(-\sqrt{3})=9\sqrt{3}-3\surd$



$x = \sqrt{3} - \min$, $x = -\sqrt{3} - \max$,

при $x \le -\sqrt{3}, x \ge \sqrt{3}$ функция возрастает,

при $x \in [-\sqrt{3}; \sqrt{3}]$ функция убывает.

в) $y = x^3 + 3x^2$;

$y' = 3x^2 + 6x$;

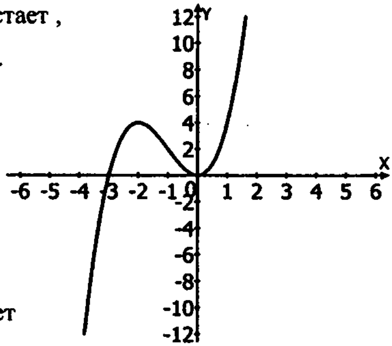
$x = 0, x = -2,$

$y(0) = 0;\ \ y(-2) = -8 + 12 = 4$

$x = 0 - \min$, $x = -2 - \max$;

при $x \in [-2; 0]$ функция убывает,

при $x \le -2, x \ge 0$ функция возрастает



г) $y = 3x - x^3$; $y' = 3 - 3x^2$;

$x = \pm 1$,

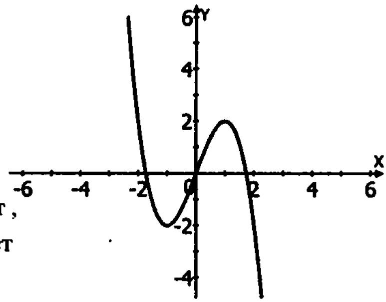
$y(1) = 3 - 1 = 2$

$y(-1) = -3 + 1 = -2$

$x = 1 - \max$, $x = -1 - \min$;

при $x \in [-1; 1]$ функция возрастает,

при $x \le -1, x \ge 1$ функция убывает



31.5. а) $y = x^3 - 3x^2 + 2$;

$y' = 3x^2 - 6x$

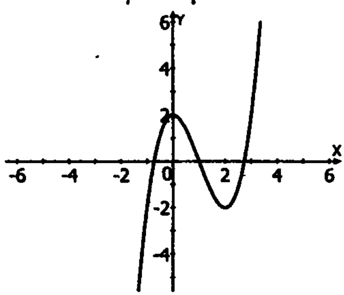
$x = 0, x = 2;$

$y(0) = 2;\ \ y(2) = 8 - 12 + 2 = -2$

$x = 0 - \max$, $x = 2 - \min$,

при $x \in [0; 2]$ функция убывает,

при $x \le 0, x \ge 2$ функция возрастает



б) $y = -x^3 + 3x - 2$;

$y' = -3x^2 + 3$;

$x = \pm 1$

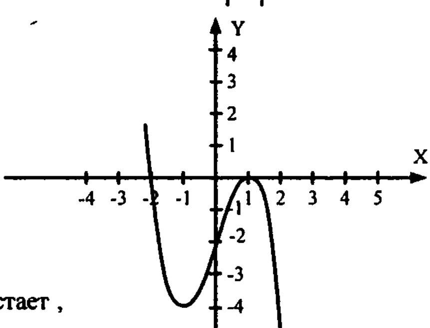
$y(1) = -1 + 3 - 2 = 0$

$y(-1) = 1 - 3 - 2 = -4$

$x = 1 - \max$, $x = -1 - \min$

при $x \in [-1; 1]$ функция возрастает,

при $x \le -1$, $x \ge 1$ функция убывает



в) $y=-x^3+6x^2-5$;

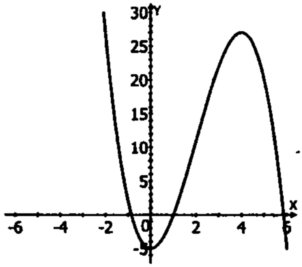
$y'=-3x^2+12x=-x^2+4x,$

$x=0,\quad x=4,$

$x=0-\min,\ x=4-\max,$

при $x\in[0;4]$ функция возрастает,

при $x\le 0, x\ge 4$ функция убывает



г) $y=x^3-3x+2$;

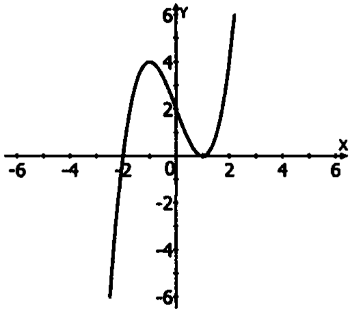
$y'=3x^2-3$;

$x=\pm 1$;

$x=1-\min,\quad x=-1-\max$

при $x\in[-1;1]$ функция убывает,

при $x\le -1, x\ge 1$ функция возрастает



31.6.

а) $y=2x^3+x^2-8x-7$;

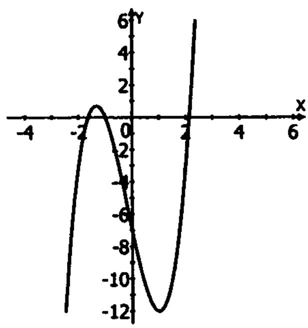
$y'=6x^2+2x-8$;

$x=\dfrac{-1-7}{6}=-\dfrac{4}{3},\ x=1,$

$x=-\dfrac{4}{3}-\max,\ x=1-\min,$

при $x\in\left[-\dfrac{4}{3};1\right]$ функция убывает,

при $x\le -\dfrac{4}{3},\quad x\ge 1$ функция возрастает.



б) $y=-\dfrac{x^3}{3}+x^2+3x-\dfrac{11}{3}$;

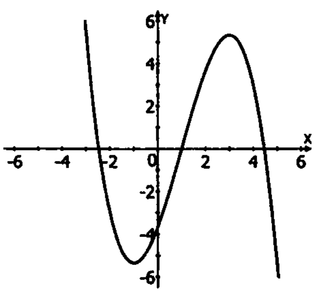
$y'=-x^2+2x+3$;

$x=3,\ x=-1;$

$x=-1-\min,\quad x=3-\max$

при $x\in[-1;3]$ функция возрастает,

при $x\le -1, x\ge 3$ функция убывает.



в) $y = x^3 + x^2 - x - 1$;

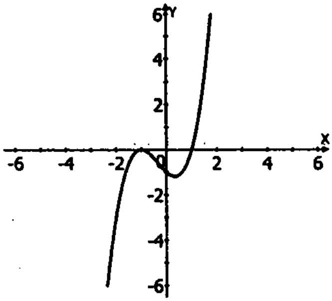
$y' = 3x^2 + 2x - 1$;

$x = \dfrac{-1-2}{3} = -1$, $x = \dfrac{1}{3}$;

$x = \dfrac{1}{3} - \min$, $x = -1 - \max$;

при $x \in \left[-1; \dfrac{1}{3}\right]$ функция убывает,

при $x \le -1$, $x \ge \dfrac{1}{3}$ функция возрастает.



г) $y = \dfrac{x^3}{3} + x^2 - 3x + \dfrac{5}{3}$,

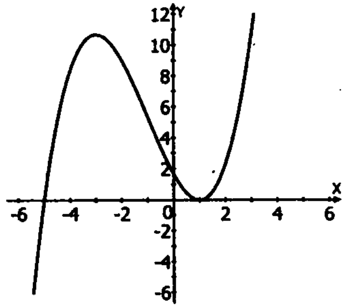
$y' = x^2 + 2x - 3$,

$x = -3, x = 1$

$x = -1 - \min$, $x = -3 - \max$

при $x \in [-3;1]$ функция убывает,

при $x \le -3, x \ge 1$ функция возрастает.



31.7.

а) $y = -x^4 + 5x^2 - 4$,

$y' = -4x^3 + 10x$,

$y = (x-1)^2(x+2)$,

$x = 0, x = \sqrt{\dfrac{5}{2}}$, $x = -\sqrt{\dfrac{5}{2}}$;

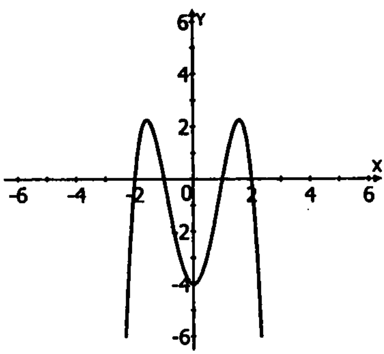
$x = 0 - \min$, $x = \pm\sqrt{\dfrac{5}{2}} - \max$

при $x \in \left(-\infty; -\dfrac{\sqrt{5}}{\sqrt{2}}\right] \cup \left[0; \dfrac{\sqrt{5}}{\sqrt{2}}\right]$

функция возрастает.

при $x \in \left[-\dfrac{\sqrt{5}}{\sqrt{2}}; 0\right] \cup \left[\dfrac{\sqrt{5}}{\sqrt{2}}; +\infty\right)$

функция убывает.



б) $y = x^5 - 5x$;

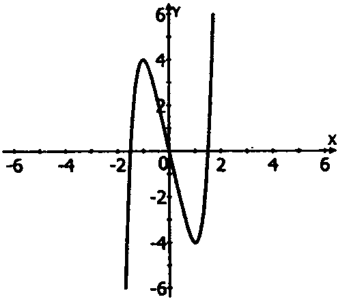
$y' = 5x^4 - 5$;

$x^4 - 1 = 0$; $x = \pm 1$;

$x = 1 - \min$, $x = -1 - \max$;

при $x \in [-1;1]$ функция убывает,

при $x \le -1, x \ge 1$ функция возрастает.



в) $y = 2x^4 - 9x^2 + 7$;

$y' = 8x^3 - 18x$;

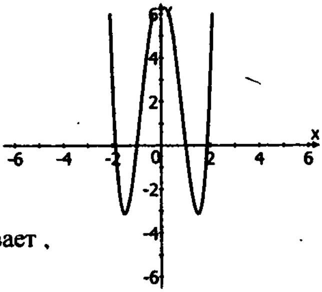
$2x(4x^2 - 9) = 0$;

$x = 0, x = \pm\frac{3}{2}$;

$x = 0 - \max$, $x = \pm\frac{3}{2} - \min$;

при $x \in \left(-\infty; -\frac{3}{2}\right] \cup \left[0; \frac{3}{2}\right]$ функция убывает,

при $x \in \left[-\frac{3}{2}; 0\right] \cup \left[\frac{3}{2}; +\infty\right)$ функция возрастает.



г) $y = 5x^3 - 3x^5$,

$y' = 15x^2 - 15x^4$,

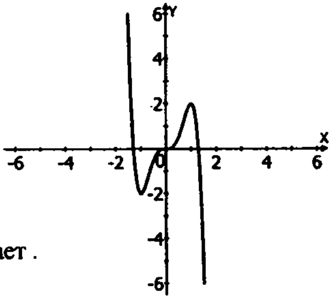
$15x^2(1 - x^2) = 0$,

$x = 0, x = \pm 1$,

$x = 1 - \max$, $x = -1 - \min$,

при $x \in [-1;1]$ функция возрастает,

при $x \in (-\infty;-1] \cup [1;+\infty)$ функция убывает.



31.8.

а) $y = (x-1)^2(x+2)$,

$y' = 2(x-1)(x+2) + (x-1)^2 =$

$= (x-1)(2x + 4 + x - 1) =$

$= (x-1)(3x+3) = 0$

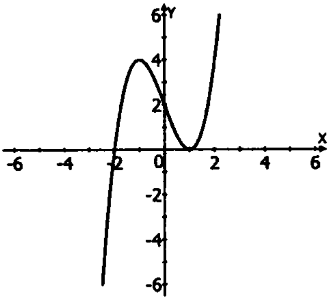
$(x-1)(x+1) = 0$,

$x = 1$, $x = -1$,

$x = -1 - \max$, $x = 1 - \min$,

при $x \in [-1; 1]$ функция убывает,

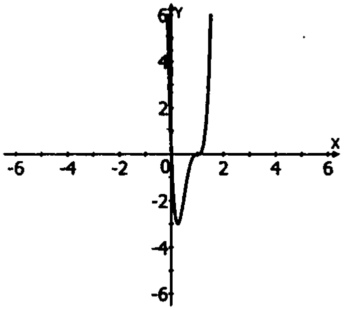
при $x \le -1, x \ge 1$ функция возрастает



б) $y = \frac{256}{9}x(x-1)^3$

$y' = \frac{256}{9}\left((x-1)^3 + 3(x-1)^2 x\right) =$

$= \frac{256}{9}(x-1)^2(4x-1)$;



$\frac{256}{9}(x-1)^2(4x-1) = 0$;

$x = 1,\quad x = \frac{1}{4}$,

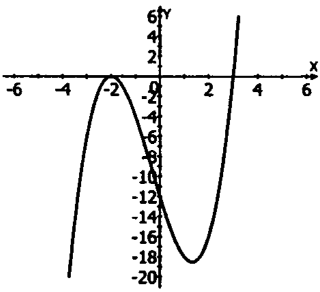
$x = \frac{1}{4} - \min$,

при $x \ge \frac{1}{4}$ функция возрастает,

при $x \le \frac{1}{4}$ функция убывает

в) $y = (x+2)^2(x-3)$,

$y' = 2(x+2)(x-3) + (x+2)^2 =$
$= (x+2)(3x-4)$,



$(x+2)(3x-4) = 0,\quad x = -2,\quad x = \frac{4}{3}$,

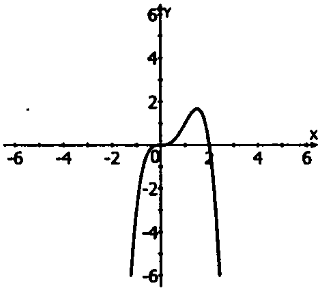
$x = -2 - \max,\quad x = \frac{4}{3} - \min$,

при $x \in \left[-2; \frac{4}{3}\right]$ функция убывает,

при $x \le -2,\ x \ge \frac{4}{3}$ функция возрастает.

г) $y = x^3(2-x)$,

$y' = \left(3x^2(2-x) - x^3\right) = x^2(6-4x)$,



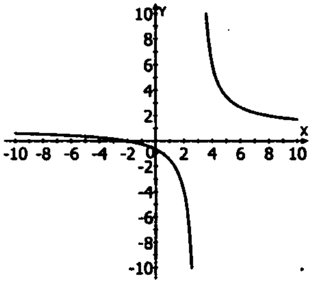
$x^2(6-4x) = 0$,

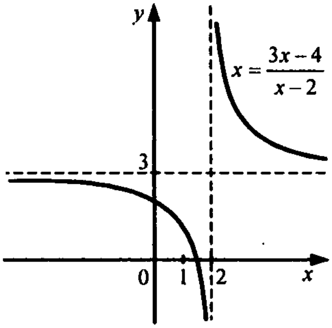
$x = 0,\ x = \frac{3}{2}$,

$x = \frac{3}{2} - \max$,

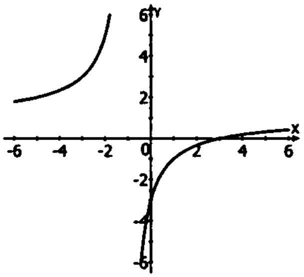
при $x \ge \frac{3}{2}$ функция убывает, при $x \le \frac{3}{2}$ функция возрастает.

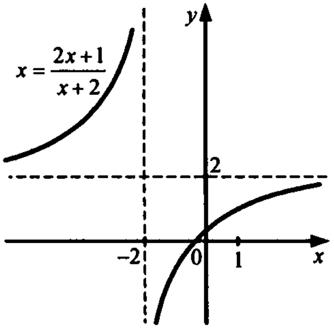
31.9. а) б) $y=\frac{3x-4}{x-2}=3+\frac{2}{x-2}$





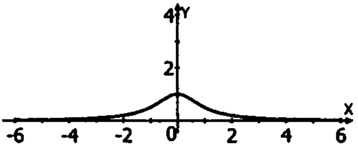
в) г) $y=\frac{2x+1}{x+2}=2-\frac{3}{x+2}$





В задачах 31.10–31.12 для иследования функции на возрастание и убывание будем исследовать ее производную. На промежутках, где $y' \geq 0$, функция возрастает, а где $y' \leq 0$ функция убывает.

31.10. а)

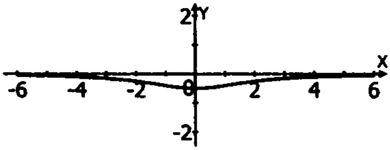


$y=\frac{1}{x^2+1}$, $y'=-\frac{2x}{(x^2+1)^2}$,

$y'<0$ при $x>0$, $y'>0$ при $x<0$, асимптота – $y=0$, $x=0$ – max

$x \in (-\infty; 0]$ — возрастает $x \in [0, +\infty)$ — убывает.

б)

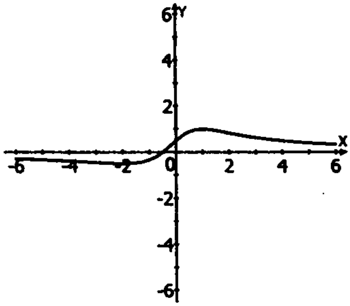


$y = -\dfrac{2}{x^2+4}$, $\quad y' = \dfrac{4x}{(x^2+4)^2}$,

$y' < 0$ при $x < 0$, $\quad y' > 0$ при $x > 0$, асимптота – $y = 0$, $\quad x = 0$ – min.

$x \in (-\infty; 0]$ — убывает $x \in [0, +\infty)$ — возрастает.

31.11. а)



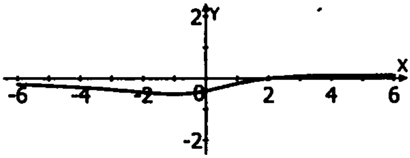
$y = \dfrac{2x+1}{x^2+2}$, $\quad y' = \dfrac{2x^2+4-4x^2-2x}{(x^2+2)^2} = \dfrac{-2x^2-2x+4}{(x^2+2)^2}$,

$y' > 0$ при $x^2+x-2<0$ $(x+2)(x-1) < 0$

$x \in [-2; 1]$ — возрастает $x \in (-\infty; -2] \cup [1; +\infty)$ — убывает

асимптота – $y = 0$.

б)



$y = \dfrac{x-2}{x^2+5}$, $\quad y' = \dfrac{x^2+5-2x^2+4x}{(x^2+5)^2} = \dfrac{-x^2+4x+5}{(x^2+5)^2}$,

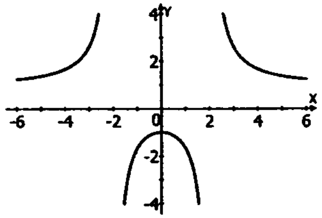
$y' > 0$ при $x^2 - 4x - 5 < 0$; $\qquad -1 < x < 5$

при $x \in [-1; 5]$ – возрастает, $\quad$ при $x \le -1, x \ge 5$ – убывает

$x = -1$ – min, $\quad x = 5$ – max, асимптота – $y = 0$.

31.12.

а)



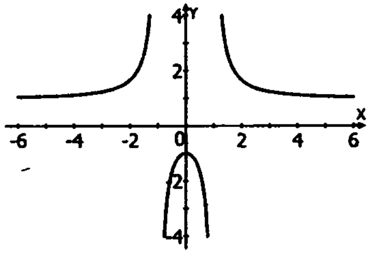
$$y = \frac{x^2+4}{x^2-4}, \quad y' = \frac{2x^3-8x-2x^3-8x}{(x^2-4)^2} = \frac{-16x}{(x^2-4)^2},$$

$x \in [0;2) \cup (2;+\infty)$ — убывает

$x \in (-\infty;-2) \cup (-2;0]$ — возрастает

$x = 0 - \max$, асимптоты: $y = 0$; $x = \pm 2$.

б)



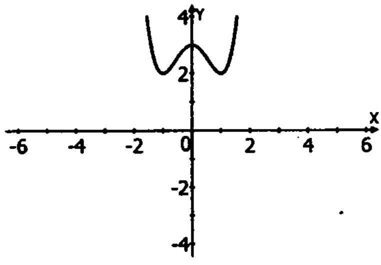
$$y = \frac{x^2+1}{x^2-1}, \quad y' = \frac{2x^3-2x-2x^3-2x}{(x^2-1)^2} = \frac{-4x}{(x^2-1)^2},$$

$x \in [0;1) \cup (1;+\infty)$ - убывает

$x \in (-\infty;-1) \cup (-1;0]$ - возрастает

$x = 0 - \max$, асимптоты: $y = 0$; $x = \pm 1$.

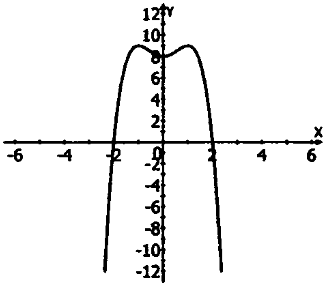
31.13. а)



б) Количество корней в данном уравнении – это количество пересечений графиков $y = x^4 - 2x^2 + 3$ и $y = a$.

Из рисунка видно, что такой случай имеет место, когда прямая $y = a$ касается графика функции в точке $(0;\ y(0))$ $y(0) = 3$, следовательно, $a = 3$.

31.14. а)



б) Чтобы уравнение не имело корней, необходимо, чтобы прямая лежала выше графика функции $y = -x^4 + 2x^2 + 8$.

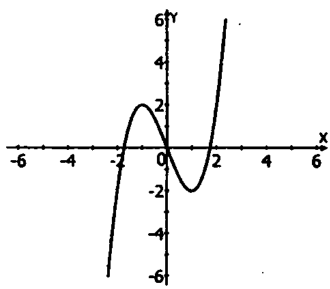
Найдем точки максимума: $y' = -4x^3 + 4x = 4x(1 - x^2) = 0$,

$x = 0$ – точка минимума, $x = \pm 1$ – точки максимума,

$y(1) = y(-1) = -1 + 2 + 8 = 9$.

Следовательно, $a > 9$.

31.15.

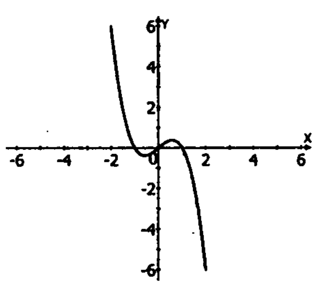


а)

$y' = 3x^2 - 3$ $x = 1$ – min,

$x = -1$ – max, $y(1) = -2$ $y(-1) = 2$

при $|a| > 2$



б)

$y' = 3 - 3x^2$ $x = 1$ — max

$x = -1$ – min, $y(1) = 2$, $y(-1) = -2$

$a = -2$ и $a = 2$.

§ 32. Применение производной для нахождения наибольших и наименьших значений

32.1. а) $y = 3x - 6$, $x \in [-1;4]$

$y' = 3 \Rightarrow$ функция растет на

$x_{\min} = -1$, $y(-1) = -9$, $y_{\min} = -9$;

$x_{\max} = 4$, $y(4) = 6$, $y_{\max} = 6$.

б) $y = -\dfrac{8}{x}$, $x \in \left[\dfrac{1}{4};8\right]$

$y' = \dfrac{8}{x^2} > 0 \Rightarrow$ функция растет при $x \in \left[\dfrac{1}{4};8\right]$

$x_{\min} = \dfrac{1}{4}$ $y(\dfrac{1}{4}) = -32$ $y_{\min} = -32$

$x_{\max} = 8$, $y(8) = -1$, $y_{\max} = -1$.

в) $y = -\dfrac{1}{2}x + 4$, $x \in [-2;6]$

$y' = -\dfrac{1}{2}$ $\Rightarrow$ функция убывает на R

$x_{\min} = 6$, $y(6) = 1$, $y_{\min} = 1$;

$x_{\max} = -2$ $y(-2) = 5$ $y_{\max} = 5$

г) $y = \dfrac{3}{x}$, $x \in [0,3;\ 2]$

$y' = -\dfrac{3}{x^2} < 0 \Rightarrow$ функция убывает при $[0,3;\ 2]$

$x_{\max} = 0,3$, $y(0,3) = 10$, $y_{\max} = 10$;

$x_{\min} = 2$, $y(2) = \dfrac{3}{2}$, $y_{\min} = \dfrac{3}{2}$.

32.2. а) $y = 2\sin x$, $x \in \left[-\dfrac{\pi}{2};\ \pi\right]$; $y' = 2\cos x$;

$x = -\dfrac{\pi}{2}$, $x = \dfrac{\pi}{2}$; $y_{\max} = 2$, $y_{\min} = -2$.

б) $y = -2\cos x$, $x \in \left[-2\pi;\ -\dfrac{\pi}{2}\right]$ $y_{\max} = 2$, $y_{\min} = -2$.

в) $y = 6\cos x$, $x \in \left[-\dfrac{\pi}{2};\ 0\right]$; $y_{\max} = 6$, $y_{\min} = 0$.

г) $y=-\frac{1}{2}\sin x,\quad x\in\left[-\frac{\pi}{2};\ \frac{\pi}{2}\right];\qquad y_{max}=\frac{1}{2},\ y_{min}=-\frac{1}{2}.$

32.3. а) $y=\operatorname{tg}x,\ x\in\left[-\frac{\pi}{3};\ -\frac{\pi}{6}\right];\qquad y_{max}=-\frac{\sqrt{3}}{3},\ y_{min}=-\sqrt{3}.$

б) $y=-3\operatorname{tg}x,\ x\in\left[\pi;\frac{4\pi}{3}\right];\qquad y_{min}=-3\sqrt{3},\ y_{max}=0.$

в) $y=-2\operatorname{tg}x,\ x\in\left[0;\frac{\pi}{6}\right];\qquad y_{max}=0,\ y_{min}=-\frac{2\sqrt{3}}{3}.$

г) $y=\frac{1}{2}\operatorname{tg}x,\ x\in\left[-\pi;-\frac{3\pi}{4}\right];\qquad y_{max}=\frac{1}{2}\quad y_{min}=0.$

32.4. а) $y=\sqrt{x},\ x\in[0;9];\qquad y_{max}=3,\ y_{min}=0.$

б) $y=\sqrt{-x},\ x\in[-4;0];\qquad y_{max}=2,\ y_{min}=0.$

в) $y=-\sqrt{x},x\in[4;16];\qquad y_{max}=-2,\ y_{min}=-4.$

г) $y=-\sqrt{-x},x\in[-9;-4];\qquad y_{max}=-2,\ y_{min}=-3.$

32.5. а) $y=12x^4,\ x\in[-1;2];\qquad y_{max}=192,\ y_{min}=0.$

б) $y=-6x^5,\ x\in[0,1;2]\qquad y_{max}=-\frac{6}{100000}=-\frac{3}{50000},\ y_{min}=-192.$

в) $y=-3x^7,\ x\in[0;1];\qquad y_{max}=0,\ y_{min}=-3.$

г) $y=\frac{x^4}{9},\ x\in[-1;3];\qquad y_{max}=9,\ y_{min}=0.$

32.6. а) $y=x^2-8x+19,\qquad x\in[-1;5]$

$y'=2x-8,\qquad x=4$ – точка минимума;

$y(4)=16-32+19=3;\quad y(-1)=1+8+19=28;$

$y(5)=25-40+19=4;$

$y_{max}=28,\qquad y_{min}=3.$

б) $y=x^2+4x-3,\qquad x\in[0;2]$

$y'=2x+4,\qquad x=-2,\qquad -2\notin[0;2]$

$y(0)=-3,\quad y(2)=4+8-3=9;$

$y_{max}=9\qquad y_{min}=-7$

в) $y=2x^2-8x+6\qquad x\in[-1;4]$

$y'=4x-8,\qquad x=2;\quad y(2)=8-16+6=-2;$

$y(-1)=2+8+6=16$; $y(4)=32-32+6=6$;

$y_{max}=16,\quad y_{min}=-2.$

г) $y=-3x^2+6x-10,\quad x\in[-2;9];\quad y'=-6x+6,\quad x=1;$

$y(1)=-3+6-10=-7;\quad y(-2)=-12-12-10=-34$

$y(9)=-243+54-10=-199;$

$y_{max}=-7,\quad y_{min}=-199.$

32.7. $y=\sin x$;

а) $\left[0;\frac{2\pi}{3}\right];\quad y_{max}=1,\ y_{min}=0.$

б) $\left[2\pi;\frac{8\pi}{3}\right];\quad y_{max}=1,\ y_{min}=0.$

в) $\left[-2\pi;-\frac{4\pi}{3}\right];\ y_{max}=1,\ y_{min}=0.$

г) $\left[6\pi;\frac{26\pi}{3}\right];\ y_{max}=1,\ y_{min}=-1.$

32.8.

$y=x^3-9x^2+24x-1;\quad y'=3x^2-18x+24=0;$

$x^2-6x+8=0;\quad x=4,\quad x=2;$

$y_{max}=y(2)=8-36+48-1=19;$

$y_{min}=y(4)=64-144+96-1=15.$

а) $[-1;3]$;

$y_{max}=y(2)=19;\quad y_{min}=y(-1)=-1-9-24-1=-35.$

б) $[3;6]$;

$y(3)=27-81+72-1=17;\quad y(6)=35;\quad y_{max}=35;\quad y_{min}=15.$

в) $[-2;3]$;

$y_{max}=y(2)=19,\quad y_{min}=y(-2)=-8-36-48-1=-93.$

г) $[3;5]$;

$y_{max}=y(5)=125-225+120-1=19,\quad y_{min}=y(4)=15.$

32.9. $y=x^3+3x^2-45x-2;\quad y'=3x^2+6x-45=0;$

$x^2+2x-15=0;\quad x=-5,\quad x=3;$

$y_{max}=y(-5)=-125+75+225-2=173,$

$y_{min}=y(3)=27+27-135-2=-79.$

а) $[-6;0];\quad y_{max}=173;\quad y_{min}=-2.$

б) $[1;2]$; $y_{max} = -43$, $y_{min} = -72$.

в) $[-6;-1]$; $y_{max} = 173$, $y_{min} = 45$.

г) $[0;2]$; $y_{max} = -2$, $y_{min} = -72$.

32.10. $y = x^3 - 9x^2 + 15x - 3$; $y' = 3x^2 - 18x + 15 = 0$;

$x^2 - 6x + 5 = 0$; $x = 5,$ $x = 1$;

$y_{max} = y(1) = 1 - 9 + 15 - 3 = 4$,

$y_{min} = y(5) = 125 - 225 + 75 - 3 = -28$.

а) $[0;2]$; $y_{max} = 4$, $y_{min} = -3$.

б) $[3;6]$; $y_{max} = -12$, $y_{min} = -28$.

в) $[-1;3]$; $y_{max} = 4$, $y_{min} = -28$.

г) $[2;7]$; $y_{max} = 4$, $y_{min} = -28$.

32.11. $y = x^4 - 8x^3 + 10x^2 + 1$;

$y' = 4x^3 - 24x^2 + 20x = 0$;

$4x(x^2 - 6x + 5) = 0$; $x = 0,$ $x = 5,$ $x = 1$; $y(0) = 1$,

$y_{max} = y(1) = 1 - 8 + 10 + 1 = 4$,

$y_{min} = y(5) = 625 - 1000 + 250 + 1 = -124$.

а) $[-1;2]$; $y_{max} = 20$, $y_{min} = -7$.

б) $[1;6]$; $y_{max} = 4$, $y_{min} = -124$.

в) $[-2;3]$; $y_{max} = 121$, $y_{min} = -44$.

г) $[-1;7]$; $y_{max} = 148$, $y_{min} = -124$.

32.12. $y = x + \dfrac{4}{x-1}$;

$y' = 1 - \dfrac{4}{(x-1)^2} = 0$; $(x-1)^2 = 4$; $x = -1,$ $x = 3$;

$y_{min} = y(3) = 3 + 2 = 5$, $y_{max} = y(-1) = -1 + \dfrac{4}{-2} = -1 - 2 = -3$.

а) $[2;4]$; $y_{max} = 6$, $y_{min} = 5$.

б) $[-2;0]$; $y_{max} = -3$, $y_{min} = -4$.

32.13. а) $y = \operatorname{ctg} x + x$ $x \in \left[\dfrac{\pi}{4}; \dfrac{3\pi}{4}\right]$;

$y' = -\dfrac{1}{\sin^2 x} + 1 \le 0$;

$y_{max}=1+\frac{\pi}{4}$, $y_{min}=-1+\frac{3\pi}{4}$.

$y\in[y_{min}, y_{max}]$

б) $y=2\sin x-x \quad x\in[0;\pi]$;

$y'=2\cos x-1$;

$x=\pm\frac{\pi}{3}+\pi n$, $x=\frac{\pi}{3}$; $y_{max}=y\left(\frac{\pi}{3}\right)=\sqrt{3}-\frac{\pi}{3}$; $y_{min}=-\pi$.

$y\in[y_{min}, y_{max}]$

в) $y=2\cos x+x, \quad x\in\left[-\frac{\pi}{2};\frac{\pi}{2}\right]$;

$y'=-2\sin x+1$; $\sin x=\frac{1}{2}$; $x=\frac{\pi}{6}$;

$y_{max}=y\left(\frac{\pi}{6}\right)=\sqrt{3}+\frac{\pi}{6}$; $y_{min}=-\frac{\pi}{2}$.

$y\in[y_{min}, y_{max}]$

г) $y=\operatorname{tg}x-x \quad x\in\left[0;\frac{\pi}{3}\right]$; $y'=\frac{1}{\cos^2 x}-1$;

$x=\pi n$, $x=0$; $y_{max}=y\left(\frac{\pi}{3}\right)=\sqrt{3}-\frac{\pi}{3}$; $y_{min}=y(0)=0$.

$y\in[y_{min}, y_{max}]$

32.14. а) $y=x^3-2x^2+1$, $[0{,}5;+\infty)$;

$y'=3x^2-4x=0$; $x(3x-4)=0$;

$x=0$, $x=\frac{4}{3}$;

y_{max} не существует; $y_{min}=y(\frac{4}{3})=\frac{64}{27}-\frac{32}{9}+1=-\frac{5}{27}$.

б) $y=x-2\sqrt{x}$, $[0;+\infty)$; $y'=1-\frac{1}{\sqrt{x}}$; $x=1$;

y_{max} не существует, $y_{min}=y(1)=1-2=-1$.

в) $y=\frac{1}{5}x^5-x^2$, $(-\infty;1]$; $y'=x^4-2x=0$; $x(x^3-2)=0$;

$x=0$, $x=\sqrt[3]{2}$;

$y_{max}=y(0)=0$; y_{min} не существует;

г) $y=\frac{x^4}{x^4+1}=1-\frac{1}{x^4+1}$, $\quad x\in R$;

$y' = \frac{4x^3}{(x^4+1)^2} = 0$,

y_{max} не существует..; $y_{min} = y(0) = 0$.

32.15. а) $y = x + \frac{1}{x}$, $(-\infty;0)$; $y' = 1 - \frac{1}{x^2}$; $x = \pm 1$;

$y_{max} = y(-1) = -1 - 1 = -2$; y_{min} не существует;

б) $y = \frac{3x}{x^2+3}$, $[0;+\infty)$; $y' = \frac{3x^2+9-6x^2}{(x^2+3)^2} = -3 \cdot \frac{x^2-3}{x^2+3}$; $x = \pm\sqrt{3}$.

$y_{max} = y(\sqrt{3}) = \frac{3\sqrt{3}}{3+3} = \frac{\sqrt{3}}{2}$; $y_{min} = y(0) = 0$.

в) $y = -2x - \frac{1}{2x}$, $(0;+\infty)$; $y' = -2 + \frac{1}{2x^2} = 0$; $4x^2 = 1$; $x = \pm\frac{1}{2}$;

$y_{max} = y\left(\frac{1}{2}\right) = -1 - 1 = -2$; y_{min} не существует..

г) $y = \sqrt{2x+6} - x$, $[-3;+\infty)$; $y' = \frac{1}{\sqrt{2x+6}} - 1 = 0$; $\sqrt{2x+6} = 1$;

$x = -\frac{5}{2}$; $y_{max} = y\left(-\frac{5}{2}\right) = 1 + \frac{5}{2} = 3{,}5$, y_{min} не существует.

32.16. а) $y = x^2 - 4x + 5 + |1 - x|$, $[0;4]$;

1) $x \ge 1$; $y = x^2 - 3x + 4$; $y' = 2x - 3 = 0$; $x = \frac{3}{2}$;

$y(1) = 1 - 4 + 5 + (1 - 1) = 2$;

$y\left(\frac{3}{2}\right) = \frac{9}{4} - 6 + 5 + \left(1 - \frac{3}{2}\right) = \frac{7}{4}$;

$y(4) = 8$;

2) $x \le 1$; $y = x^2 - 5x + 6$;

$y' = 2x - 5 = 0$; $x = \frac{5}{2}$ — не подходит; $y(0) = 6$;

$y_{min} = \frac{7}{4}$; $y_{max} = 8$.

б) $y = |x^3 - 1| - 3x$, $[-1;3]$;

1) $x \ge 1$; $y = x^3 - 3x - 1$; $y' = 3x^2 - 3$; $x = 1$; $y(1) = -3$; $y(3) = 17$;

2) $x \le 1$; $y = 1 - 3x - x^3$; $y' = -3 - 3x^2$; $y(1) = -3 = y_{min}$; $y(-1) = 5$;

$y_{min} = -3$; $y_{max} = 17$.

32.17. а) $y = 2x - \sqrt{16x-4}$, $x \in \left[\frac{1}{4}; \frac{17}{4}\right]$;

$y' = 2 - \frac{8}{\sqrt{16x-4}}$; $x = \frac{5}{4}$; $y\left(\frac{5}{4}\right) = \frac{5}{2} - 4 = -\frac{3}{2}$;

$y\left(\frac{1}{4}\right) = \frac{1}{2}$; $y\left(\frac{17}{4}\right) = \frac{17}{2} - 8 = \frac{1}{2}$; $y \in \left[-\frac{3}{2}; \frac{1}{2}\right]$.

б) $y = 2\sqrt{x-1} - \frac{1}{2}x$, $x \in [1;10]$;

$y' = \frac{1}{\sqrt{x-1}} - \frac{1}{2}$; $x = 5$;

$y(5) = 4 - \frac{5}{2} = \frac{3}{2}$; $y(1) = -\frac{1}{2}$; $y(10) = 6 - 5 = 1$; $y \in \left[-\frac{1}{2}; \frac{3}{2}\right]$.

32.18. а) $y = x\sqrt{x+2}$;

$y' = \sqrt{x+2} + \frac{x}{2\sqrt{x+2}} = \frac{2x+4+x}{2\sqrt{x+2}} = \frac{3x+4}{2\sqrt{x+2}}$;

$x = -\frac{4}{3}$; $y\left(-\frac{4}{3}\right) = -\frac{4}{3} \cdot \sqrt{-\frac{4}{3}+2} = -\frac{4\sqrt{6}}{9}$; $y \in \left[-\frac{4}{9}\sqrt{6}; +\infty\right)$.

б) $y = x\sqrt{1-2x}$;

$y' = \sqrt{1-2x} - \frac{x}{\sqrt{1-2x}} = \frac{1-2x-x}{\sqrt{1-2x}} = \frac{1-3x}{\sqrt{1-2x}}$; $x = \frac{1}{3}$;

$y\left(\frac{1}{3}\right) = \frac{1}{3} \cdot \sqrt{1-\frac{2}{3}} = \frac{1}{3\sqrt{3}}$; $y \in \left(-\infty; \frac{\sqrt{3}}{9}\right]$.

32.19.

$y = x^3 - 3x^2 - 9x + \sqrt{16-x^4} + \left|\sqrt{16-x^4} - 5\right|$

а) $16 - x^4 \le 16 \Rightarrow \sqrt{16-x^4} \le 4 \Rightarrow 5 - \sqrt{16-x^4} \ge 1$

$\Rightarrow y = x^3 - 3x^2 - 9x + \sqrt{16-x^4} + 5 - \sqrt{16-x^4}$

$y = x^3 - 3x^2 - 9x + 5$

б) $16 - x^4 \ge 0 \Rightarrow x^2 \le 4 \Rightarrow |x| \le 2$

$y' = 3x^2 - 6x - 9$

$x^2 - 2x - 3 = 0 \Rightarrow x = -1, x = 3$

$y(-2) = 3$, $y(-1) = 10$, $y(2) = -17$

$\Rightarrow y \in [-17; 10]$

32.20. $\begin{cases} a+b=24 \\ ab=\max \end{cases}$; $\begin{cases} a=24-b \\ 24b-b^2=y \end{cases}$;

$y'=24-2b$;

$b=12,\qquad y(12)=144$;

$b=12,\qquad a=12$.

32.21. $ab=484$; $a=\dfrac{484}{b}$; $\dfrac{484}{b}+b=y$; $y'=1-\dfrac{484}{b^2}$;

$b=22,\qquad a=22$.

32.22. $a-b=98$; $a=98+b$; $b^2+98b=y$; $y'=2b+98$;

$b=-49,\qquad a=49$.

32.23. $\begin{cases} a+b=3 \\ y=3a+b^3 \end{cases}$; $\begin{cases} a=3-b \\ y=3a+b^3 \end{cases}$;

$y=9-3b+b^3$; $y'=3b^2-3$;

$b=\pm1$, но т.к. по условию $b>0$, то

$b=1,\qquad a=2$.

32.24.

$\begin{cases} a+b=5 \\ y=ab^3 \end{cases}$; $\begin{cases} a=5-b \\ y=ab^3 \end{cases}$;

$y=5b^3-b^4$;

$y'=15b^2-4b^3=b^2(15-4b)$;

$b=0,\qquad b=\dfrac{15}{4}$, но т.к. по условию $b>0$, то $b=\dfrac{15}{4}$, $a=\dfrac{5}{4}$.

32.25.

$\begin{cases} 2a+2b=56 \\ ab=y \end{cases}$; $\begin{cases} a=28-b \\ 28b-b^2=y \end{cases}$; $y'=28-2b$; $2b=28$;

$b=14,\qquad a=14$.

32.26.

$\begin{cases} a+b=100 \\ y=ab \end{cases}$; $\begin{cases} a=100-b \\ y=ab \end{cases}$; $y=100b-b^2$; $y'=100-2b$;

$b=50,\qquad a=50$.

32.27.

а) $\begin{cases} ab=16 \\ 2a+2b=y \end{cases}$; $\begin{cases} a=\dfrac{16}{b} \\ 2a+2b=y \end{cases}$; $y=\dfrac{32}{b}+2b$; $y'=2-\dfrac{32}{b^2}$;

$b=4,\qquad a=4$.

32.28.

$\begin{cases} ab = 2500 \\ 2a + 2b = y \end{cases}$; $\begin{cases} a = \dfrac{2500}{b} \\ 2a + 2b = y \end{cases}$; $y = \dfrac{5000}{b} + 2b$; $y' = 2 - \dfrac{5000}{b^2}$;

$b = 50$, $a = 50$.

32.29. $KD = DM = x$; $BH_1 = \dfrac{3\sqrt{2}}{2}$;

$DH_2 = \dfrac{x\sqrt{2}}{2}$ (H_1 и H_2 – точки пересечения BD с PE и KM соответственно)

$S = \dfrac{1}{2}(PE + KM) \cdot H_1H_2 = \dfrac{1}{2}(x\sqrt{2} + 3\sqrt{2})(8\sqrt{2} - \dfrac{3\sqrt{2}}{2} - \dfrac{x\sqrt{2}}{2}) =$

$= \dfrac{1}{2}(39 + 10x - x^2)$;

$S' = \dfrac{1}{2}(10 - 2x) = 0; x = 5$; $S = \dfrac{1}{2}(39 + 50 - 25) = 32$.

32.30. $y = x^2$, $A(0;1{,}5)$;

$\sqrt{(0-x)^2 + (1{,}5-y)^2} = f(x)$;

$f(x) = \sqrt{x^2 + (1{,}5 - x^2)^2} = = \sqrt{x^2 + \dfrac{9}{4} - 3x^2 + x^4} = \sqrt{x^4 - 2x^2 + \dfrac{9}{4}}$

$f'(x) = \dfrac{4x^3 - 4x}{2\sqrt{x^4 - 2x^2 + \dfrac{9}{4}}} = 2 \cdot \dfrac{x^3 - x}{\sqrt{x^4 - 2x^2 + \dfrac{9}{4}}}$;

$x = 0$, $x = \pm 1$;

$f(0) = \dfrac{3}{2}$; $f(1) = \sqrt{1 + \dfrac{1}{4}} = \dfrac{\sqrt{5}}{2} = f(-1)$; $(1;1), (-1;1)$

32.31. $y = \sqrt{x}$, $A(4{,}5;0)$;

$f(x) = \sqrt{(4{,}5 - x)^2 + y^2} = \sqrt{\dfrac{81}{4} - 9x + x^2 + x} = \sqrt{\dfrac{81}{4} - 8x + x^2}$;

$f'(x) = \dfrac{2x - 8}{2\sqrt{\dfrac{81}{4} - 8x + x^2}}$; $x = 4$; $y(4) = 2$; $(4;2)$.

32.32. $V = x^2y = 32$ (y – высота бака, x – длина стороны его основания);

$S = x^2 + 4xy = x^2 + \dfrac{128}{x}$; $S'(x) = 2x - \dfrac{128}{x^2} = 0$;

$x^3 = 64$; $x_0 = 4$; $y_0 = 2$.

Ответ: 4 дм; 4 дм; 2 дм.

32.33. $V = x^2y = 343$ (y – высота бака, x – длина стороны его основания);

$S = 2x^2 + 4xy = 2x^2 + \frac{1372}{x}$; $S' = 4x - \frac{1372}{x^2} = 0$; $x_0 = 7$; $y_0 = 7$.

Ответ: 7 м; 7 м; 7 м.

32.34. $V = 6x^2y = 576$ (y – высота короба, $2x$ и $3x$ – длины сторон его основания);

$S = 12x^2 + 6xy + 4xy = 12x^2 + \frac{960}{x}$; $S'(x) = 24x - \frac{960}{x^2} = 0$;

$x^3 = 40$; $x_0 = 2\sqrt[3]{5}$; $y_0 = \frac{576}{6} \cdot \frac{1}{4\sqrt[3]{25}} = 24\frac{\sqrt[3]{5}}{5}$;

Ответ: $4\sqrt[3]{5}$м; $6\sqrt[3]{5}$м; $\frac{24\sqrt[3]{5}}{5}$м.

32.35. $V = \sqrt{(d^2 - x^2)}x$ (x – длина бокового ребра призмы);

$V' = d^2 - 3x^2 = 0$; $x = \frac{d}{\sqrt{3}} = \frac{\sqrt{3}}{3}d$.

32.36. $S = \frac{1}{2}\left(15 + 2\sqrt{225 - h^2} + 15\right)h =$

$= \left(15 + \sqrt{225 - h^2}\right)h$ (здесь h – высота трапеции);

$S' = 15 + \sqrt{225 - h^2} - \frac{h^2}{\sqrt{225 - h^2}}$; $15 \cdot \sqrt{225 - h^2} + 225 - 2h^2 = 0$;

$50625 - 225h^2 = 50625 - 900h^2 + 4h^4$;

$4h^4 - 675h^2 = 0$; $h^2(4h^2 - 675) = 0$;

$h^2 = \frac{675}{4}$; $a = 15 + 2\sqrt{225 - \frac{675}{4}} = 15 + 2 \cdot \frac{15}{2} = 30$.

32.37. а) Пусть α - угол между основанием и боковой стороной x - сторона прямоугольника, которая совпадает с высотой, y – его другая сторона.

Тогда $tg\alpha = 5 = \frac{x}{80 - y}$; $80 - y = \frac{x}{5}$; $y = 80 - \frac{x}{5}$; $S(x) = 80x - \frac{x^2}{5}$;

$S' = 80 - \frac{2x}{5}$;

$x = 200$, но $x \in (0;100] \Rightarrow$ $x = 100$ $y = 60$; $S = 6000$.

б) $a = 24$, $b = 8$, $h = 12$.

Пусть α - угол между большим основанием трапеции и ее боковой стороной, x –сторона прямоугольника, которая совпадает с высотой, y – его другая сторона.

Тогда $tg\alpha = \frac{3}{4} = \frac{x}{24-y}$; $24 - y = \frac{4x}{3}$; $y = 24 - \frac{4x}{3}$; $S = 24x - \frac{4x^2}{3}$;

$S' = 24 - \frac{8x}{3}$; $x = 9$, $y = 12$; $S = 108$.

32.38. а) Пусть x – сторона прямоугольника, лежащая на AB, y – его другая сторона, $\alpha = \angle DCB$.

Тогда $\mathrm{tg}\alpha = \frac{7-5}{9-3} = \frac{2}{6} = \frac{1}{3} = \frac{x}{9-y}$; $x = 3 - \frac{1}{3}y$; $y = -3x + 9$;

$S = -3x^2 + 9x$; $S' = -6x + 9$; $x = \frac{3}{2}$; $y = -\frac{9}{2} + 9 = \frac{9}{2}$

но $AE \cdot AB = 21 \Rightarrow S_{max} = 21$.

б) $a = 7$, $b = 18$, $c = 3$, $m = 1$;

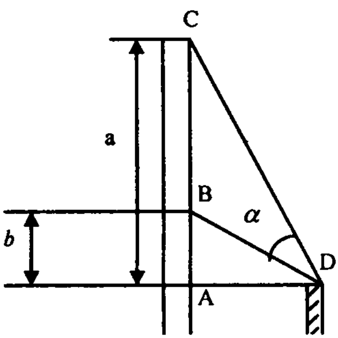
Пусть x – сторона прямоугольника, лежащая на AB, y – его другая сторона, $\alpha = \angle DCB$.

$AE \cdot AB$ Тогда $\mathrm{tg}\alpha = \frac{7-1}{18-3} = \frac{2}{5} = \frac{x}{18-y}$; $18 - y = \frac{5}{2}x$; $y = 18 - \frac{5}{2}x$;

$S = 18x - \frac{5}{2}x^2$; $S' = 18 - 5x$; $x = \frac{18}{5}$; $y = 9$; $S = 32,4$;

$AE \cdot AB = 21 \Rightarrow S_{max} = 32,4$.

32.39.



$AC = a$; $AB = b$;

$CB = AC - AB = a - b$; $AD = x$;

$BD = \sqrt{x^2 + b^2}$; $CD = \sqrt{x^2 + a^2}$.

По теореме косинусов

$$(a-b)^2 = x^2 + b^2 + x^2 + a^2 - 2\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}\cos\alpha\ ;$$

$$2x^2 + 2ab = 2\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}\cos\alpha\ ;$$

$$\cos\alpha = \frac{x^2+ab}{\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}}.$$

$$f(x) = \frac{x^2+ab}{\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}} = \frac{x^2+ab}{\sqrt{x^4+x^2\left(a^2+b^2\right)+b^2a^2}};$$

$$f'(x) = \frac{2x\sqrt{x^4+x^2\left(a^2+b^2\right)+b^2a^2} - \dfrac{(x^2+ab)(4x^3+2x(a^2+b^2))}{2\sqrt{x^4+x^2\left(a^2+b^2\right)+b^2a^2}}}{\left(x^2+b^2\right)\left(x^2+a^2\right)} = 0;$$

$$2x\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)} = x(x^2+ab)\frac{2x^2+a^2+b^2}{\sqrt{\left(x^2+b^2\right)\left(x^2+a^2\right)}};$$

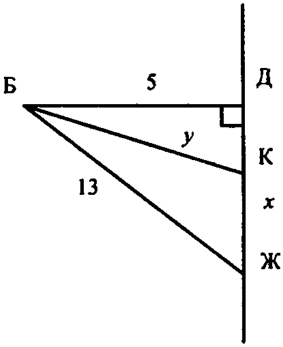
$$2(a^2b^2+x^2a^2+x^2b^2+x^4) = 2x^4+2x^2ab+x^2a^2+x^2b^2+a^3b+ab^3;$$

$$2ab(ab-x^2)+a^2(x^2-ab)+b^2(x^2-ab)=0\ ;$$

$$(x^2-ab)(a-b)^2=0\ ;\quad x=\sqrt{ab}\ ;$$

α_{max} при $x=\sqrt{ab}$.

32.40.



$ЖД = 12$;

x – расстояние, которое пешеход пройдет по дороге;

y – расстояние, которое пешеход пройдет по лесу;

Суммарное время: $t=\frac{x}{5}+\frac{y}{3}$;

$DK=12-x$; $y=\sqrt{25+(12-x)^2}$;

$$t'=\frac{1}{5}+\frac{2x-24}{2\cdot 3\sqrt{x^2-24x+169}}=\frac{1}{5}+\frac{x-12}{3\sqrt{x^2-24x+169}}=0;$$

$\frac{3}{5}\sqrt{x^2-24x+169}=12-x$;

$9x^2-9\cdot 24x+9\cdot 169=25\left(144-24x+x^2\right)$;

$16x^2-384x+2079=0$;

$x_1=8{,}25$; $x_2=15{,}75$ – не подходит;

$x=\frac{33}{4}$; $y=\sqrt{25+3{,}75^2}=\frac{25}{4}$;

$t=\frac{x}{5}+\frac{4}{5}=\frac{33}{20}+\frac{25}{12}=\frac{56}{15}\approx$ 3 часа 44 минуты.

Глава 6. Степени и корни. Степенные функции

§ 33. Понятие корня n-й степени действительного числа

33.1. а) 3; 4 б) 5; 7 в) 11; 2 г) 37; 15

33.2. а) $\sqrt{361}=19$; $19^2=361$. б) $\sqrt[6]{\frac{1}{64}}=\frac{1}{2}$; $\left(\frac{1}{2}\right)^6=\frac{1}{64}$.

в) $\sqrt[3]{343}=7$; $7^3=343$. г) $\sqrt[5]{\frac{32}{243}}=\frac{2}{3}$; $\left(\frac{2}{3}\right)^5=\frac{32}{243}$

33.3. а) $\sqrt{25}=-5$; $\sqrt{25}=5$. б) $\sqrt[6]{-64}=-2$; $(-2)^6\neq-64$.

в) $-\sqrt[3]{-8}=-2$; $\sqrt[3]{-8}=2$;$-8\neq2^3$. г) $\sqrt[4]{625}=-25$; $(-25)^4=625^2$.

33.4. а) $\sqrt{7-4\sqrt{3}}=2-\sqrt{3}$; $7-4\sqrt{3}=4+3-4\sqrt{3}$. Верно.

б) $\sqrt{9-4\sqrt{5}}=2-\sqrt{5}$; $2-\sqrt{5}<0\Rightarrow$ *Неверно.*

в) $\sqrt{7-4\sqrt{3}}=\sqrt{3}-2$; $\sqrt{3}-2<0\Rightarrow$ *Неверно.*

г) $\sqrt{9-4\sqrt{5}}=\sqrt{5}-2$; $9-4\sqrt{5}=5+4-4\sqrt{5}\Rightarrow$ Верно.

33.5. а) $\sqrt[4]{16}=2$; б) $\sqrt[5]{32}=2$; в) $\sqrt[4]{81}=3$; г) $\sqrt[3]{64}=4$.

33.6. а) $\sqrt[3]{0,125}=0,5$; б) $\sqrt[4]{0,0625}=0,5$;

в) $\sqrt[4]{0,0081}=0,3$; г) $\sqrt[3]{0,027}=0,3$.

33.7. а) $\sqrt[4]{\frac{16}{625}}=\frac{2}{5}$ б) $\sqrt[3]{3\frac{3}{8}}=\sqrt[3]{\frac{27}{8}}=\frac{3}{2}$

в) $\sqrt{\frac{100}{121}}=\frac{10}{11}$ г) $\sqrt[5]{7\frac{19}{32}}=\sqrt[5]{\frac{243}{32}}=\frac{3}{2}$

33.8. а) $\sqrt[7]{-128}=-2$; б) $\sqrt[3]{-\frac{1}{8}}=-\frac{1}{2}$;

в) $\sqrt[3]{-64}=-4$; г) $\sqrt[5]{-\frac{1}{32}}=-\frac{1}{2}$.

33.9. а) $\sqrt[5]{32}+\sqrt[3]{-8}=2-2=0$; б) $\sqrt[4]{625}-\sqrt[3]{-125}=5+5=10$;

в) $3\sqrt[4]{16}-4\sqrt[3]{27}=6-12=-6$; г) $12-6\sqrt[3]{0,125}=12-3=9$.

33.10. а) $2^2<5<3^2\Rightarrow\sqrt{5}\in[2,3]$; б) $2^3<19<3^3\Rightarrow\sqrt[3]{19}\in[2,3]$;

в) $2^4<52<3^4\Rightarrow\sqrt[4]{52}\in[2,3]$; г) $3^3<63<4^3\Rightarrow\sqrt[3]{63}\in[3,4]$.

33.11. а) $x^3=125$; $x=\sqrt[3]{125}$; $x=5$; б) $x^7=\frac{1}{128}$; $x=\frac{1}{2}$;

в) $x^5=32$; $x=2$. г) $x^9=1$; $x=1$.

33.12. а) $x^4=17$; $x=\pm\sqrt[4]{17}$. б) $x^4=-16$ — решений нет.

в) $x^6=11$; $x=\pm\sqrt[6]{11}$. г) $x^8=-3$ — решений нет.

33.13. а) $0{,}02x^6-1{,}28=0$; $x^6=64$; $x=\pm 2$.

б) $-\frac{3}{4}x^8+18\frac{3}{4}=0$; $x^8=25$; $x=\pm\sqrt[4]{5}$.

в) $0{,}3x^9-2{,}4=0$; $x^9=8$; $x=\sqrt[3]{2}$.

г) $\frac{1}{8}x^4-2=0$; $x^4=16$; $x=\pm 2$.

33.14. а) $\sqrt[3]{x-5}=-3$; $x-5=-27$; $x=-22$.

б) $\sqrt[4]{4-5x}=-2$ — решений нет.

в) $\sqrt[5]{2x+8}=-1$; $2x+8=-1$; $x=-\frac{9}{2}$.

г) $\sqrt[3]{7-4x}=4$; $7-4x=64$; $x=-\frac{57}{4}$.

33.15. а) $\sqrt[3]{x^2-9x-19}=-3$; $x^2-9x-19=-27$; $x^2-9x+8=0$; $x=1$, $x=8$.

б) $\sqrt[4]{x^2-10x+25}=2$; $x^2-10x+25=16$; $x^2-10x+9=0$; $x=9$, $x=1$.

в) $\sqrt[7]{2x^2+6x-57}=-1$; $2x^2+6x-56=0$; $x^2+3x-28=0$;

$x=\frac{-3+11}{2}=4$; $x=\frac{-3-11}{2}=-7$.

г) $\sqrt[6]{x^2+7x+13}=1$; $x^2+7x+12=0$; $x=-4$, $x=-3$.

33.16. а) $\sqrt[3]{5}$; 2; $\sqrt[4]{17}$. б) $\sqrt[5]{100}$; 4; $\sqrt[3]{75}$.

в) $\sqrt[3]{7}$; $\sqrt[5]{40}$; 3. г) $\sqrt[6]{60}$; 2; $\sqrt[4]{20}$.

33.17. а) $\sqrt[4]{0{,}1}$; -1; $\sqrt[3]{-5}$. б) 0; $\sqrt[3]{-0{,}25}$; $\sqrt[5]{-29}$.

в) $\sqrt[5]{-1{,}5}$; -2; $\sqrt[3]{-9}$. г) $\sqrt[3]{2}$; 1; $\sqrt[3]{-2}$.

33.18. а) $\sqrt[3]{15}-\sqrt[4]{90}\vee 0$; $\sqrt[3]{15}<\sqrt[3]{27}=3$, $\sqrt[4]{90}>\sqrt[4]{64}=4\Rightarrow\sqrt[3]{15}-\sqrt[3]{90}<0$

Пункты б), в), г) решаются аналогично.

б) $3-\sqrt[7]{150}>0$. в) $\sqrt[5]{40}-\sqrt[3]{50}<0$. г) $\sqrt[4]{300}-5<0$.

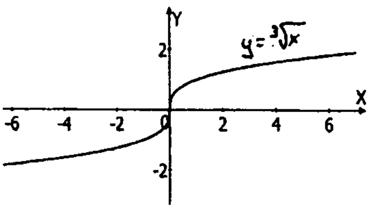
33.19. а) $\sqrt[5]{-12}$; $\frac{\pi}{2}$; 2; $\sqrt[6]{70}$. б) $\sqrt[5]{-\pi}$; $\frac{3}{\pi}$; 1; $\sqrt[7]{\pi}$.

в) $\sqrt[3]{-2}$; $\frac{\pi}{3}$; $2{,}5$; $\sqrt{2\pi}$. г) $\sqrt[5]{-\frac{1}{2}}$; 0; $\sqrt[3]{200}$; 2π.

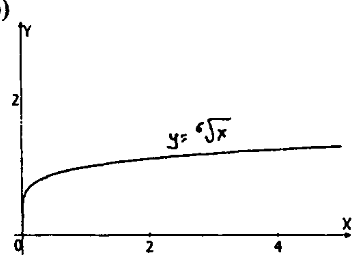
§ 34. Функции $y = \sqrt[n]{x}$, их свойства и графики

34.1.

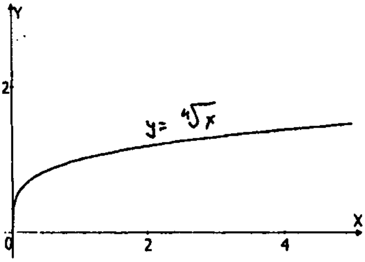
а)



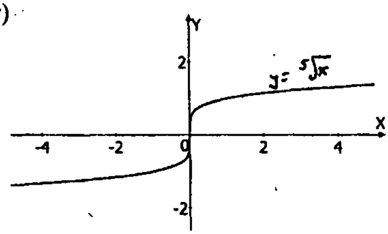
б)



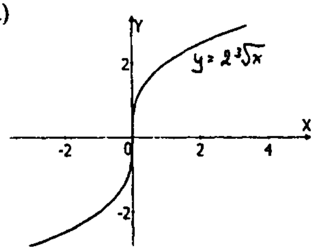
в)



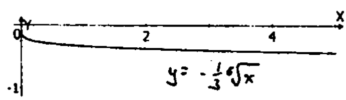
г)



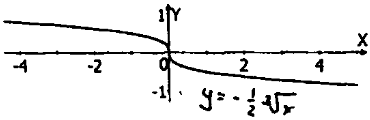
34.2. а)



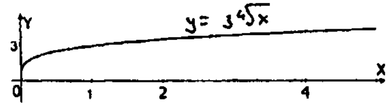
б)



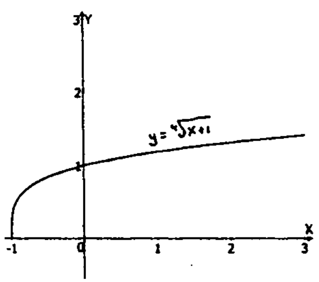
в)



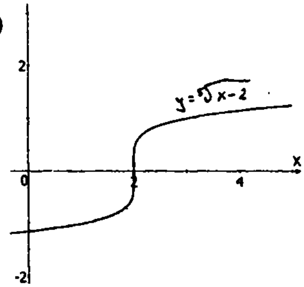
г)



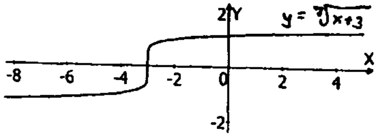
34.3. а)



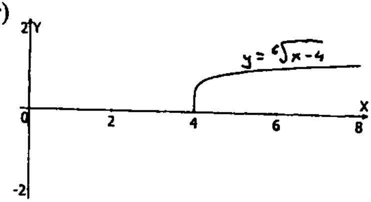
б)



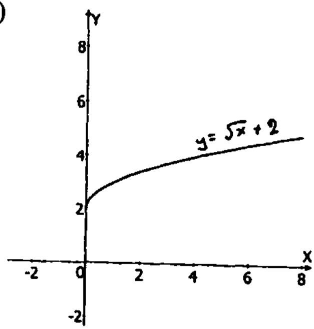
в)



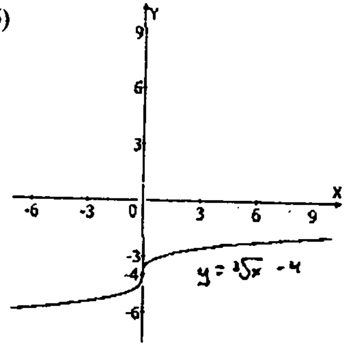
г)



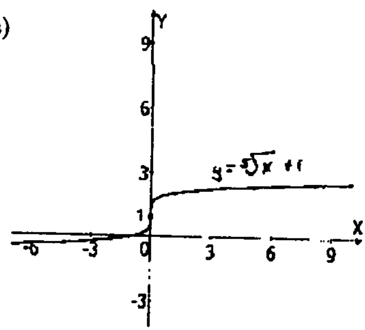
34.4. а)



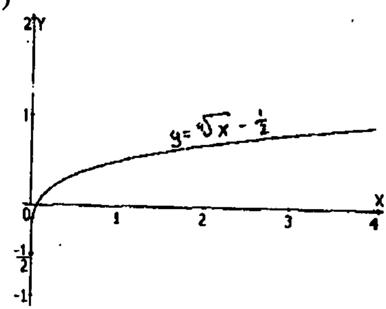
б)



в)

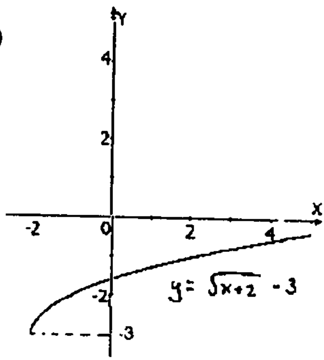


г)

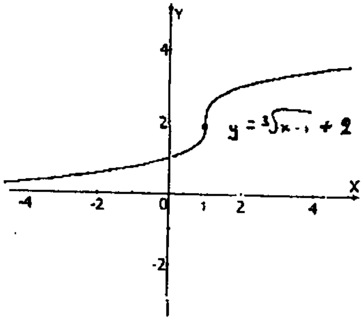


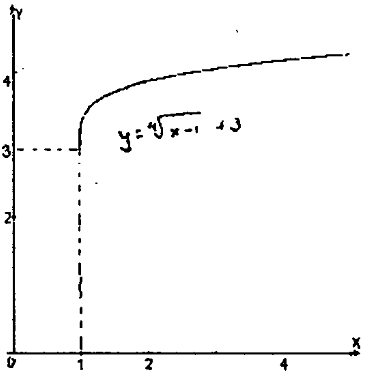
34.5.

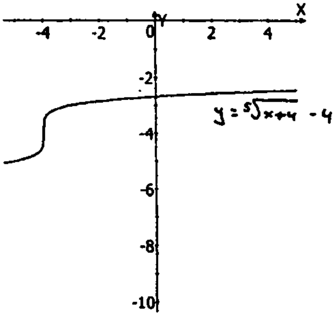
а)



б)



в) 

г) 

34.6. $y=\sqrt[4]{x}$; а) $x\in[0;1]$, $\min y=0$, $\max y=1$;

б) $x\in[1;3)$, $\min y=1$, $\max y$ не существует;

в) $x\in[5;16]$, $\min y=\sqrt[4]{5}$, $\max y=2$;

г) $x\in[16;+\infty)$, $\min y=2$, $\max y$ не существует;

34.7. $y=\sqrt[5]{x}$; а) $x\in[-1;1]$, $\min y=-1$, $\max y=1$;

б) $x\in(-\infty;1]$, $\min y$ не существует, $\max y=1$;

в) $x\in[-32;32]$, $\min y=-2$, $\max y=2$;

г) $x\in[2;+\infty)$, $\min y=\sqrt[5]{2}$, $\max y$ не существует.

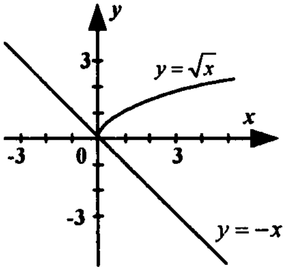
34.8. а) $y=\sqrt[4]{x}$; $y=x^2$; $\sqrt[4]{x}=x^2$; $x=x^8$; $x=1$, $x=0$; $(0;0)$, $(1;1)$.

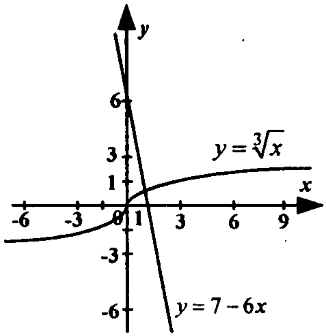
б) $y=\sqrt[3]{x}$; $y=|x|$; $\sqrt[3]{x}=|x|$; $x=1$, $x=0$; $(0;0)$, $(1;1)$.

в) $y=\sqrt[6]{x}$; $y=x$; $\sqrt[6]{x}=x$; $x=1$, $x=0$; $(0;\ 0)$; $(1;\ 1)$.

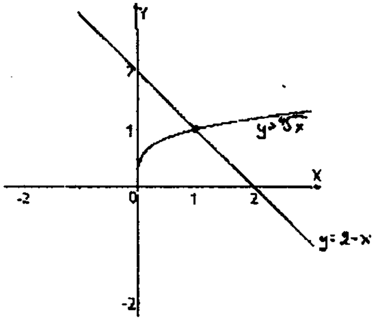
г) $y=\sqrt[5]{x}$; $y=-x-2$; $\sqrt[5]{x}=-x-2$; $x=1$; $(-1;-1)$.

34.9. а) $x=0$ б) $x=1$

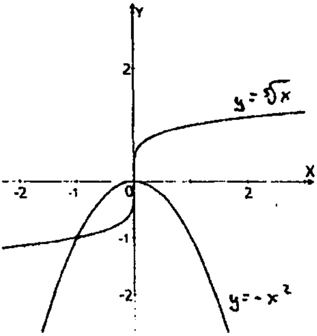




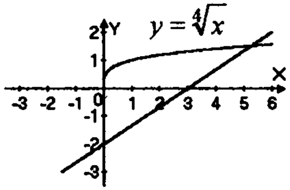
в) $x = 1$



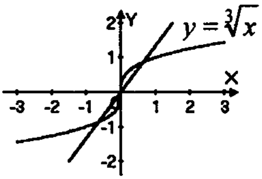
г) $x = 0, \quad x = -1$



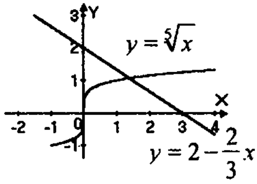
34.10. а) $\begin{cases} y = \sqrt[4]{x} \\ 2x - 3y = 6 \end{cases}$; $\begin{cases} y = \sqrt[4]{x} \\ y = \dfrac{2x}{3} - 2 \end{cases}$ — одно решение.



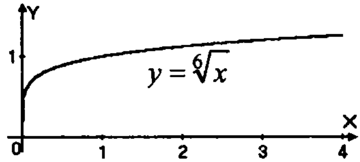
б) $\begin{cases} y = \sqrt[3]{x} \\ 3y - 4x = 0 \end{cases}$; $\begin{cases} y = \dfrac{4}{3}x \\ y = \sqrt[3]{x} \end{cases}$ — три решения (в ответе задачника опечатка).



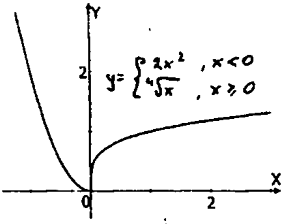
в) $\begin{cases} y = \sqrt[5]{x} \\ 6 - 2x - 3y = 0 \end{cases}$; $\begin{cases} y = \sqrt[5]{x} \\ y = 2 - \dfrac{2}{3}x \end{cases}$ — одно решение.



г) $\begin{cases} y=\sqrt[6]{x} \\ 5+x-2y=0 \end{cases}$; $\begin{cases} y=\sqrt[6]{x} \\ y=\dfrac{5}{2}+\dfrac{x}{2} \end{cases}$ — нет решений.

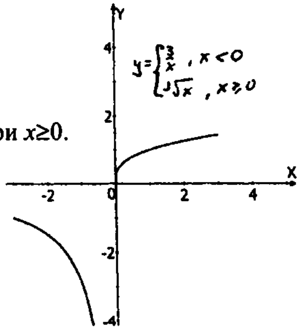


34.11. $y=\begin{cases} 2x^2, & x<0 \\ \sqrt[4]{x}, & x\ge 0 \end{cases}$



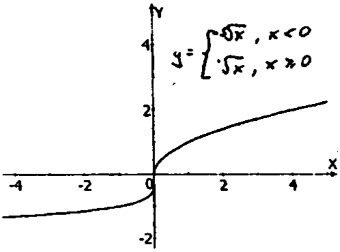
1) $y(x)$ убывает при $x\le 0$, возрастает при $x\ge 0$.
2) $x_{min}=0$, $y_{min}\le 0$.
3) $y=0$ при $x=0$.

34.12. $y=\begin{cases} \dfrac{3}{x}, & x<0 \\ \sqrt[3]{x}, & x\ge 0 \end{cases}$



1) $y(x)$ убывает при $x<0$, возрастает при $x\ge 0$.
2) Экстремумов нет.
3) $y=0$ при $x=0$.

34.13. $y=\begin{cases} \sqrt[5]{x}, & x<0 \\ \sqrt{x}, & x\ge 0 \end{cases}$



1) $y(x)$ убывает на всей числовой прямой.
2) Экстремумов нет.
3) $y=0$ при $x=0$.

34.14. а) $y=\sqrt[4]{2x-4}$; $2x-4\ge 0$; $x\ge 2$;

б) $y=\sqrt[6]{3x-9}$; $3x-9\ge 0$; $x\ge 3$.

в) $y=\sqrt[8]{2-3x}$; $2-3x\ge 0$; $x\le\dfrac{2}{3}$.

г) $y=\sqrt[12]{1-5x}$; $1-5x\ge 0$; $x\le\dfrac{1}{5}$.

34.15. а) $y=\sqrt[3]{x^2+5}$; $x\in R$. б) $y=\sqrt[7]{x^3-1}$; $x\in R$.

в) $y=\sqrt[9]{6x-7}$; $x\in R$. г) $y=\sqrt[5]{2x+1}$; $x\in R$.

34.16. а) $y=\sqrt{5x+8}+\sqrt[4]{2x-4}$;

$\begin{cases} x\ge 2 \\ x\ge -\dfrac{8}{5} \end{cases} \Rightarrow x\ge 2$; $x\in [2, +\infty)$.

б) $y=\sqrt[6]{2x+1}-\sqrt[8]{5-10x}$; $\begin{cases} 2x+1\ge 0 \\ 5-10x\ge 0 \end{cases}$;

$\begin{cases} x\ge -\dfrac{1}{2} \\ x\le \dfrac{1}{2} \end{cases} \Rightarrow x\in\left[-\dfrac{1}{2};\dfrac{1}{2}\right]$.

в) $y=\sqrt[10]{3x-12}-\sqrt[4]{2x-1}$; $\begin{cases} 3x-12\ge 0 \\ 2x-1\ge 0 \end{cases}$;

$\begin{cases} x\ge 4 \\ x\ge \dfrac{1}{2} \end{cases} \Rightarrow x\ge 4$; $x\in [4, +\infty)$.

г) $y=\sqrt{8-16x}+\sqrt[12]{10x+20}$; $\begin{cases} 8-16x\ge 0 \\ 10x+20\ge 0 \end{cases}$;

$\begin{cases} x\le \dfrac{1}{2} \\ x\ge -2 \end{cases} \Rightarrow x\in\left[-2, \dfrac{1}{2}\right]$.

34.17. а) $y=\sqrt{x^2+4x-12}$; $x^2+4x-12\ge 0$; корни: $x_1=-6$; $x_2=2$;
$x\in(-\infty;-6]\cup[2;+\infty)$,

б) $y=\sqrt[12]{15-x^2+2x}$; $-x^2+2x+15\ge 0$; $x^2-2x-15\le 0$;
корни: $x_1=-3$; $x_2=5$; $x\in[-3;5]$.

в) $y=\sqrt{x^2-8x+12}$; $x^2-8x+12\ge 0$; корни: $x_1=2$; $x_2=6$;
$x\le 2$, $x\ge 6$; $x\in(-\infty, 2]\cup[6, +\infty)$.

г) $y=\sqrt[6]{4-x^2-3x}$; $4-x^2-3x\ge 0$; $x^2+3x-4\le 0$; $x\in[-4;1]$.

34.18. а) $y=\sqrt[4]{\dfrac{x-8}{3x+5}}$; $\dfrac{x-8}{3x+5}\ge 0$; $x\ge 8$, $x<-\dfrac{5}{3}$, $x\in\left(-\infty;-\dfrac{5}{3}\right)\cup(8;+\infty)$.

б) $y=\sqrt[5]{\dfrac{1+9x}{4+3x}}$; $x\in R$ кроме $x=-\dfrac{4}{3}$.

в) $y=\sqrt[3]{\dfrac{12-5x}{7-2x}}$; $x\in R$ кроме $x=\dfrac{7}{2}$.

г) $y=\sqrt{\dfrac{3-7x}{2x+9}}$; $\dfrac{3-7x}{2x+9}\geq 0$; $\dfrac{7x-3}{2x+9}\leq 0$; $x\in\left(-4{,}5;\dfrac{3}{7}\right]$.

34.19. а) $y=\sqrt[4]{x+1}$; $y\in[0;+\infty)$. б) $y=\sqrt[5]{x-2}$; $y\in R$.

в) $y=\sqrt[7]{x+3}$; $y\in R$. г) $y=\sqrt[6]{x-4}$; $y\in[0;+\infty)$.

34.20. а) $y=2+\sqrt[4]{x}$; $y\in[2;+\infty)$. б) $y=\sqrt[5]{x}-3$; $y\in R$.

в) $y=\sqrt[6]{x}-3$; $y\in[-3;+\infty)$. г) $y=2+\sqrt[3]{x}$; $y\in R$

34.21. а) $y=\sqrt[4]{x^2-6x+8}$

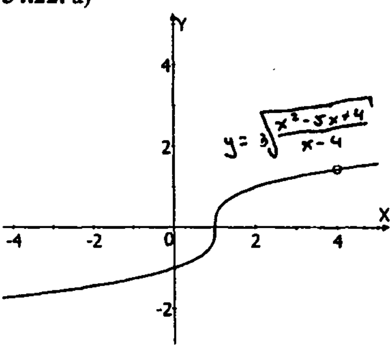
Y принимает минимальное значение, когда x^2-6x+8 минимальное неотрицательное.

$x_в=\dfrac{-6}{2}=-3$; $\Rightarrow$ $\min(x^2+6x+13)=x_в^2+6x_в+13=4$ $\Rightarrow$ $y_{\min}=\sqrt[3]{2}$.

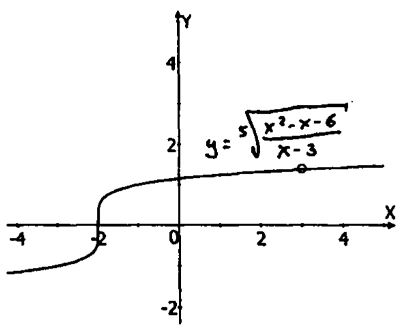
б) $y=\sqrt[6]{x^2+6x+10}$

$$y'=\frac{2x+6}{6(x^2+6x+10)^{\frac{5}{6}}}\Rightarrow x=-3\Rightarrow y_{\min}=y(-3)=1$$

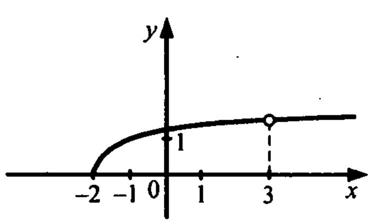
34.22. а)



б)



б) $y=\sqrt[4]{\dfrac{x^2-x-6}{x-3}}=\sqrt[4]{x+2},\ x\neq 3$



§ 35. Свойства корня n-й степени

35.1. а) $\sqrt[3]{8\cdot 27}=2\cdot 3=6$; б) $\sqrt[4]{16\cdot 0{,}0001}=2\cdot 0{,}1=0{,}2$;

в) $\sqrt[4]{625\cdot 16}=5\cdot 2=10$; г) $\sqrt[5]{0{,}00032\cdot 243}=0{,}2\cdot 3=\frac{3}{5}$.

35.2. а) $\sqrt[5]{243\cdot\frac{1}{32}}=3\cdot\frac{1}{2}=\frac{3}{2}$; б) $\sqrt[3]{\frac{8}{125}}=\frac{2}{5}$;

в) $\sqrt[5]{7\cdot\frac{19}{32}}=\sqrt[5]{\frac{243}{32}}=\frac{3}{2}$; г) $\sqrt[6]{64\cdot\frac{1}{729}}=2\cdot\frac{1}{3}=\frac{2}{3}$.

35.3. а) $\sqrt[3]{24\cdot 9}=\sqrt[3]{8\cdot 27}=2\cdot 3=6$;

б) $\sqrt[5]{48\cdot 162}=\sqrt[5]{16\cdot 3\cdot 2\cdot 81}=\sqrt[5]{2^5\cdot 3^5}=2\cdot 3=6$;

в) $\sqrt[3]{75\cdot 45}=3\sqrt[3]{75\cdot\frac{5}{3}}=3\cdot 5=15$;

г) $\sqrt[4]{54\cdot 24}=\sqrt[4]{9\cdot 6\cdot 3\cdot 8}=3\cdot 2=6$.

35.4. а) $\sqrt[4]{\frac{125}{0{,}2}}=\sqrt[4]{625}=5$; б) $\sqrt[4]{\frac{16}{0{,}0625}}=\frac{2}{0{,}5}=4$;

в) $\sqrt[3]{\frac{27}{0{,}125}}=\frac{3}{0{,}5}=6$; г) $\sqrt[6]{\frac{16}{0{,}25}}=\sqrt[6]{64}=2$.

35.5. а) $\sqrt[3]{5^6\cdot 2^9}=5^2\cdot 2^3=25\cdot 8=200$; в) $\sqrt[3]{0{,}2^3\cdot 5^6}=0{,}2\cdot 5^2=5$;

б) $\sqrt[5]{0{,}2^{10}\cdot 10^{10}}=0{,}2^2\cdot 10^2=2^2=4$; г) $\sqrt[6]{36^3\cdot 2^6}=6\cdot 2=12$.

35.6. а) $\sqrt[4]{\frac{7^8}{3^4}}=\frac{7^2}{3}=\frac{49}{3}$; б) $\sqrt[3]{\frac{5^6}{3^9}}=\frac{5^2}{3^3}=\frac{25}{27}$;

в) $\sqrt[4]{\frac{3^{12}}{2^8}}=\frac{3^3}{2^2}=\frac{27}{4}$; г) $\sqrt[5]{\frac{5^5}{13^{10}}}=\frac{5}{169}$.

35.7. а) $\sqrt[4]{x^2}=\sqrt{x}$; б) $\sqrt[6]{y^4}=\sqrt[3]{y^2}$;

в) $\sqrt[10]{a^5}=\sqrt{a}$; г) $\sqrt[24]{n^{16}}=\sqrt[3]{n^2}$.

35.8. а) $\sqrt[4]{b^8}=b^2$; б) $\sqrt{l^6}=l^3$; в) $\sqrt[5]{d^{15}}=d^3$; г) $\sqrt[3]{t^{12}}=t^4$.

35.9. а) $\sqrt{a^2b^4}=ab^2$; б) $\sqrt[3]{a^3b^6}=ab^2$;

в) $\sqrt[4]{a^4b^8}=ab^2$; г) $\sqrt[5]{a^5b^{15}}=ab^3$.

35.10. а) $\sqrt{\frac{49a^4}{169b^2}}=\frac{7a^2}{13b}=\frac{7}{13}\frac{a^2}{b}$; б) $\sqrt[4]{\frac{16a^4b^8}{c^{12}}}=\frac{2ab^2}{c^3}$;

в) $\sqrt[3]{\frac{27a^6}{64b^3}}=\frac{3a^2}{4b}$; г) $\sqrt[5]{\frac{32a^{40}b^{10}}{243c^{15}}}=\frac{2\cdot a^8b^2}{3c^3}$.

35.11. а) $\sqrt[4]{4}\cdot\sqrt[4]{4}=\sqrt[4]{16}=2$; б) $\sqrt[3]{135}\cdot\sqrt[3]{25}=\sqrt[3]{27\cdot5\cdot25}=15$;
в) $\sqrt{20}\cdot\sqrt{5}=\sqrt{100}=10$; г) $\sqrt[5]{16}\cdot\sqrt[5]{486}=\sqrt[5]{16\cdot2\cdot243}=6$.

35.12. а) $\frac{\sqrt[3]{54}}{\sqrt[3]{2}}=\sqrt[3]{27}=3$; б) $\frac{\sqrt[5]{3}}{\sqrt[5]{96}}=\sqrt[5]{\frac{1}{32}}=\frac{1}{2}$;
в) $\frac{\sqrt[7]{256}}{\sqrt[7]{2}}=\sqrt[7]{128}=2$; г) $\frac{\sqrt[4]{256}}{\sqrt[4]{4}}=\sqrt[4]{64}=2\sqrt{2}$.

35.13. а) $\sqrt[4]{32\cdot3}\cdot\sqrt[4]{8\cdot27}=\sqrt[4]{256}\sqrt[4]{81}=4\cdot3=12$;
б) $\sqrt[5]{2^57^2}\cdot\sqrt[5]{7^3}=2\cdot7=14$.

35.14. а) $\sqrt[3]{2}$ и $\sqrt[6]{3}$, $\sqrt[6]{4}$ и $\sqrt[6]{3}$; б) $\sqrt[4]{5}$ и $\sqrt[3]{9}$, $\sqrt[12]{125}$ и $\sqrt[12]{6561}$;
в) $\sqrt{7}$ и $\sqrt[12]{8}$, $\sqrt[4]{49}$ и $\sqrt[4]{2}$; г) $\sqrt[3]{3}$ и $\sqrt[5]{2}$, $\sqrt[15]{243}$ и $\sqrt[15]{8}$.

35.15. а) $\sqrt{3}$, $\sqrt[3]{4}$ и $\sqrt[6]{7}$, $\sqrt[6]{27}$, $\sqrt[6]{16}$ и $\sqrt[6]{7}$;
б) $\sqrt{2}$, $\sqrt[3]{3}$ и $\sqrt[4]{4}$, $\sqrt[6]{8}$, $\sqrt[6]{9}$ и $\sqrt[6]{8}$;
в) $\sqrt{6}$, $\sqrt[4]{17}$ и $\sqrt[8]{40}$, $\sqrt[8]{1296}$, $\sqrt[8]{289}$ и $\sqrt[8]{40}$;
г) $\sqrt[5]{3}$, $\sqrt[3]{2}$ и $\sqrt[15]{100}$, $\sqrt[15]{27}$, $\sqrt[15]{32}$ и $\sqrt[15]{100}$.

35.16/ а) $\sqrt[4]{26}\vee\sqrt{5}$, $\sqrt[4]{26}>\sqrt[4]{25}$;
б) $\sqrt[3]{5}\vee\sqrt{3}$, $\sqrt[6]{25}<\sqrt[6]{27}$;
в) $\sqrt[3]{7}\vee\sqrt[6]{47}$, $\sqrt[6]{49}>\sqrt[6]{47}$;
г) $-\sqrt[4]{4}\vee-\sqrt[3]{3}$, $-\sqrt[6]{8}>-\sqrt[6]{9}$.

35.17. а) $\sqrt{2}\sqrt[4]{2}=\sqrt[4]{4}\sqrt[4]{2}=\sqrt[4]{8}$; б) $\sqrt[3]{3}\sqrt[6]{3}=\sqrt[6]{9}\sqrt[6]{3}=\sqrt[6]{27}$;
в) $\sqrt{2}\sqrt[3]{3}=\sqrt[6]{8\cdot9}=\sqrt[6]{72}$; г) $\sqrt[4]{2}\sqrt[6]{3}=\sqrt[12]{8\cdot9}=\sqrt[12]{72}$.

35.18. а) $\sqrt[4]{3b^3}\sqrt{3b}=\sqrt[4]{3b^3}\sqrt[4]{9b^2}=\sqrt[4]{27b^5}$;
б) $\sqrt{2a}\sqrt[6]{4a^5}=\sqrt[6]{8a^3}\sqrt[6]{4a^5}=\sqrt[6]{32a^8}$;
в) $\sqrt{a}\sqrt[6]{a^5}=\sqrt[6]{a^3}\sqrt[6]{a^5}=\sqrt[6]{a^8}$; г) $\sqrt[3]{y}\sqrt[6]{3y^3}=\sqrt[6]{y^2}\sqrt[6]{y^3}=\sqrt[6]{3y^5}$.

35.19. а) $\sqrt[3]{ab}\sqrt[6]{4ab}=\sqrt[6]{a^2b^2}\sqrt[6]{4a^3b^3}=\sqrt[6]{4a^3b^3}$;
б) $\sqrt[5]{a^4b^3}\cdot\sqrt[10]{a^5b^2}=\sqrt[10]{a^8b^6}\sqrt[10]{a^5b^2}=\sqrt[10]{a^{13}b^8}$;
в) $\sqrt[6]{5ab^2}\cdot\sqrt[3]{5a^3b^4}=\sqrt[6]{5ab^2}\sqrt[6]{25a^6b^8}=\sqrt[6]{125a^7b^{10}}$;
г) $\sqrt[8]{6xz}\cdot\sqrt[6]{xz^5}=\sqrt[24]{216x^3z^3}\sqrt[24]{x^4z^{20}}=\sqrt[24]{216x^7z^{23}}$.

35.20. а) $\sqrt[4]{a^3}:\sqrt{a}=\sqrt[4]{a}$; б) $\sqrt[12]{a^2b^3}:\sqrt[6]{ab^4}=\sqrt[12]{b^{-5}}$;
в) $\sqrt[6]{a^5}:\sqrt[4]{a}=\sqrt[12]{a^7}$; г) $\sqrt[4]{a^3b^5}:\sqrt[5]{ab}=\sqrt[20]{a^{11}b^{21}}$.

35.21. а) $\left(\sqrt{3}\right)^2=3$; б) $\left(\sqrt[n]{a}\right)^n=a$; в) $\left(\sqrt[5]{7}\right)^5=7$; г) $\left(\sqrt[p]{b}\right)^p=b$.

35.22. а) $\left(2\sqrt{5}\right)^4=16\cdot 25=400$; б) $\left(b\sqrt[n]{\frac{1}{b}}\right)^{2n}=b^{2n}\cdot\frac{1}{b^2}=b^{2n-2}$;

в) $\left(3\cdot\sqrt[n]{\frac{1}{2}}\right)^{2n}=\frac{243}{2}$; г) $\left(\frac{1}{b}\sqrt[p]{b}\right)^{2p}=\frac{1}{b^{2p}}\cdot b^2=b^{2-2p}$.

35.23. а) $\left(\sqrt[3]{3a}\right)^9=27a^3$; б) $\left(5a\sqrt[3]{a}\right)^2=25a^2\cdot\sqrt[3]{a^2}=25\sqrt[3]{a^8}$;

в) $\left(-5\sqrt[3]{a^2}\right)^2=25\sqrt[3]{a^4}$; г) $\left(2\sqrt[3]{-3a^2}\right)^5=32\sqrt[3]{-243a^{10}}$.

35.24. а) $\sqrt{\sqrt[3]{x}}=\sqrt[6]{x}$; б) $\sqrt[3]{\sqrt{a^3}}=\sqrt{a}$;

в) $\sqrt[5]{\sqrt[3]{a^{10}}}=\sqrt[15]{a^{10}}=\sqrt[3]{a^2}$; г) $\sqrt{\sqrt[3]{ab}}=\sqrt[6]{ab}$.

35.25. а) $\frac{1}{2}\sqrt[3]{5x}+13+\frac{\sqrt[3]{5x}}{5}=2\sqrt[3]{5x}$; $\sqrt[3]{5x}\left(2-\frac{1}{5}-\frac{1}{2}\right)=13$;

$\sqrt[3]{5x}\frac{13}{10}=13$; $\sqrt[3]{5x}=10$; $x=200$.

б) $\sqrt[4]{2x}+\sqrt[4]{32x}+\sqrt[4]{162x}=6$; $\sqrt[4]{x}\left(\sqrt[4]{2}+2\sqrt[4]{2}+3\sqrt[4]{2}\right)=6$;

$\sqrt[4]{x}\cdot 6\sqrt[4]{2}=6$; $\sqrt[4]{x}=\frac{1}{\sqrt[4]{2}}$; $x=\frac{1}{2}$.

35.26. а) $\sqrt[4]{6+2\sqrt{5}}\sqrt[4]{6-2\sqrt{5}}=\sqrt[4]{36-20}=2$;

б) $\sqrt[5]{6-2\sqrt{17}}\sqrt[5]{6+2\sqrt{17}}=\sqrt[5]{-32}=-2$;

в) $\sqrt[3]{8-\sqrt{37}}\sqrt[3]{8+\sqrt{37}}=\sqrt[3]{64-37}=3$;

г) $\sqrt[3]{\sqrt{17}+3}\sqrt[3]{\sqrt{17}-3}=\sqrt[3]{17-9}=2$.

35.27. а) $\sqrt[3]{x}-2\sqrt[6]{x}=0$; $\sqrt[6]{x}\left(\sqrt[6]{x}-2\right)=0$; $x_1=0,\ x_2=64$;

б) $\sqrt{x}-5\sqrt[4]{x}+6=0$ — это уравнение относительно $\sqrt[4]{x}$:

$(\sqrt[4]{x})^2-5\sqrt[4]{x}+6$; $\sqrt[4]{x}=2$; $x=16$; $\sqrt[4]{x}=3$; $x=81$.

в) $\sqrt[6]{x}+2\sqrt[3]{x}-1=0$; $\sqrt[6]{x}=\frac{-1+3}{4}=\frac{1}{2}$; $x=\frac{1}{64}$;

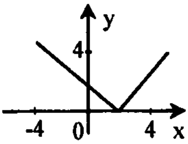
$\sqrt[6]{x}=-1$ – решений нет.

г) $\sqrt[4]{x}+2\sqrt[8]{x}-3=0$; $\sqrt[8]{x}=-3$ решений нет; $\sqrt[8]{x}=1$; $x=1$.

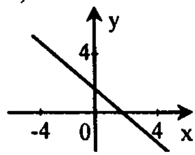
35.28. $f(x)=\sqrt[7]{x}$; $2f(x)=4\sqrt[7]{x}$; $f(128x)=2\cdot\sqrt[7]{128x}=4\sqrt[7]{x}$.

35.29. $f(x)=2\sqrt[5]{x}$; $2f(x)=4\sqrt[5]{x}$; $f(32x)=2\cdot\sqrt[5]{32x}=4\sqrt[5]{x}$.

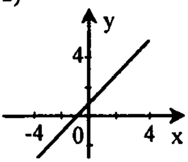
35.30. а)



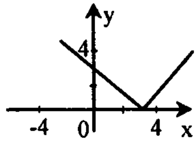
б)



в)



г)



§ 36. Преобразование выражений, содержащих радикалы

36.1. а) $\sqrt{20}=2\sqrt{5}$; б) $\sqrt{147}=7\sqrt{3}$;
в) $\sqrt{108}=2\cdot3\sqrt{3}=6\sqrt{3}$; г) $\sqrt{245}=7\sqrt{5}$.

36.2. а) $\sqrt[3]{24}=2\sqrt[3]{3}$; б) $\sqrt[4]{160}=2\sqrt[4]{10}$;
в) $\sqrt[3]{512}=8$; г) $\sqrt[4]{486}=3\sqrt[4]{6}$.

36.3. а) $\sqrt{x^3}=x\sqrt{x}$; б) $\sqrt[3]{a^4}=a\sqrt[3]{a}$;
в) $\sqrt[5]{m^7}=m\sqrt[5]{m^2}$; г) $\sqrt[4]{n^{13}}=n^3\cdot\sqrt[4]{n}$.

36.4. а) $\sqrt{25a^3}=5a\sqrt{a}$; б) $\sqrt[4]{405a^5}=3a\sqrt[4]{5a}$;
в) $\sqrt[3]{24x^3}=2x\sqrt[3]{3}$; г) $\sqrt[5]{160m^{10}}=2m^2\cdot\sqrt[5]{5}$.

36.5. а) $\sqrt{75t^4r^3}=5t^2r\sqrt{3r}$;

б) $\dfrac{x^2}{b}\sqrt[3]{\dfrac{72a^4b^3}{343x^3}}=\dfrac{x^2}{b}\cdot\dfrac{2ab}{7x}\sqrt[3]{9a}=\dfrac{2}{7}xa\sqrt[3]{9a}$;

в) $\sqrt[3]{250x^4y^7}=5xy^2\sqrt[3]{2xy}$;

г) $3mn\sqrt[4]{\dfrac{80x^3}{243m^5n^9}}=\dfrac{3mn\cdot2}{3mn^2}\sqrt[4]{\dfrac{5x^3}{3mn}}=\dfrac{2}{n}\sqrt[4]{\dfrac{5x^3}{3mn}}$.

36.6. а) $\sqrt{a^2b}=|a|\sqrt{b}$; б) $\sqrt[3]{a^3b}=a\sqrt[3]{b}$;
в) $\sqrt[4]{a^4b}=|a|\sqrt[4]{b}$; г) $\sqrt{a^5b}=a^2\sqrt{ab}$.

36.7. а) $2\sqrt{5}=\sqrt{20}$; б) $6\sqrt[3]{1\frac{1}{9}}=\sqrt[3]{240}$;

в) $5\sqrt{3}=\sqrt{75}$; г) $3\sqrt[4]{2\frac{5}{27}}=\sqrt{177}$.

36.8. а) $\frac{2}{3}\sqrt{3}=\sqrt{\frac{4}{3}}$; б) $\frac{1}{2}\sqrt[3]{12}=\sqrt[3]{\frac{3}{2}}$;

в) $\frac{7}{5}\sqrt{3\frac{4}{7}}=\sqrt{\frac{49}{25}\cdot\frac{25}{7}}=\sqrt{7}$; г) $0,2\sqrt[3]{25}=\sqrt[3]{\frac{1}{5}}$.

36.9. а) $7a^2\sqrt{ab}=\sqrt{49a^5b}$; б) $5ab^2\sqrt[3]{a^2b}=\sqrt[3]{125a^5b^7}$;

в) $5x\sqrt{2x}=\sqrt{50x^3}$; г) $2m\sqrt[3]{3m^2}=\sqrt[3]{24m^5}$.

36.10. а) $\sqrt[3]{24}-\sqrt[3]{3}=\sqrt[3]{3}(2-1)=\sqrt[3]{3}$; б) $2\sqrt[7]{3}+\sqrt[7]{384}=2\sqrt[7]{3}+2\sqrt[7]{3}=4\sqrt[7]{3}$;

в) $2\sqrt[5]{64}+\sqrt[5]{486}=4\sqrt[5]{2}+3\sqrt[5]{2}=7\sqrt[5]{2}$; г) $\sqrt[4]{512}-\sqrt[4]{2}=\sqrt[4]{2}(4-1)=3\sqrt[4]{2}$.

36.11. а) $\sqrt[3]{4}$; $\sqrt[6]{18}$; $\sqrt{3}$; б) $\sqrt[3]{2}$; $\sqrt[15]{40}$; $\sqrt[5]{4}$;

в) $\sqrt[5]{3}$; $\sqrt[15]{30}$; $\sqrt[3]{2}$; г) $\sqrt[6]{3}$; $\sqrt[3]{2}$; $\sqrt[4]{4}$.

36.12. а) $\left(\sqrt[3]{m}-2\sqrt[3]{n}\right)\left(\sqrt[3]{m}+2\sqrt[3]{n}\right)=\sqrt[3]{m^2}-4\sqrt[3]{n^2}$;

б) $\left(\sqrt[3]{5}-\sqrt{3}\right)\left(\sqrt{3}+\sqrt[3]{5}\right)=\sqrt[3]{25}-3$;

в) $\left(a-\sqrt{b}\right)\left(a+\sqrt{b}\right)=a^2-b$;

г) $\left(\sqrt[3]{4}+2\sqrt{2}\right)\left(2\sqrt{2}-\sqrt[3]{4}\right)=8-\sqrt[3]{16}=8-2\sqrt[3]{2}$.

36.13. а) $\left(\sqrt{x}+\sqrt{y}\right)\left(x-\sqrt{xy}+y\right)=\left(\sqrt{x}\right)^3+\left(\sqrt{y}\right)^3=\sqrt{x^3}+\sqrt{y^3}$;

б) $\left(3+\sqrt[4]{a}\right)\left(9-3\sqrt{a}+\sqrt{a}\right)=3^3+\left(\sqrt[4]{a}\right)^3=27+\sqrt[4]{a^3}$;

в) $\left(2\sqrt{p}+\sqrt{q}\right)\left(4p-2\sqrt{pq}+q\right)=\left(2\sqrt{p}\right)^3+\left(\sqrt{q}\right)^3=8\sqrt{p^3}+\sqrt{q^3}$;

г) $\left(\sqrt[3]{a}+\sqrt[6]{ab}+\sqrt[3]{b}\right)\left(\sqrt[6]{a}-\sqrt[6]{b}\right)=\left(\sqrt[6]{a}\right)^3-\left(\sqrt[6]{b}\right)^3=\sqrt{a}-\sqrt{b}$.

36.14. а) $\left(\sqrt[3]{m}-2\sqrt[3]{n}\right)^2=\sqrt[3]{m^2}-4\sqrt[3]{mn}+4\sqrt[3]{n^2}$;

б) $\left(\sqrt[3]{5}-\sqrt{3}\right)^2=\sqrt[3]{25}-2\sqrt{3}\sqrt[3]{5}+3$;

в) $\left(a^2-\sqrt{a}\right)^2=a^4+a-2a^2\sqrt{a}$;

г) $\left(\sqrt[3]{4}+2\sqrt{2}\right)^2=2\sqrt[3]{2}+8+4\sqrt{2}\sqrt[3]{4}$.

36.15. а) $(a-b):\left(\sqrt{a}-\sqrt{b}\right)=\frac{(a-b)\left(\sqrt{a}+\sqrt{b}\right)}{a-b}=\sqrt{a}+\sqrt{b}$;

б) $\frac{k+l}{\sqrt[3]{k}+\sqrt[3]{l}}=\frac{k+l}{(k+l)}\left(\sqrt[3]{k^2}-\sqrt[3]{kl}+\sqrt[3]{l^2}\right)=\sqrt[3]{k^2}+\sqrt[3]{l^2}-\sqrt[3]{kl}$;

в) $\dfrac{m-n}{\sqrt[3]{m}-\sqrt[3]{n}}=\sqrt[3]{m^2}+\sqrt[3]{n^2}+\sqrt[3]{mn}$;

г) $\dfrac{x-4y}{\sqrt{x}+2\sqrt{y}}=\sqrt{x}-2\sqrt{y}$;

36.16. а) $\dfrac{\sqrt{10b}-\sqrt{15}}{\sqrt{15b}-\sqrt{5}}=\dfrac{\sqrt{2b}-\sqrt{3}}{\sqrt{3b}-1}$; б) $\dfrac{\sqrt[3]{x^2}-\sqrt[3]{xy}}{\sqrt[3]{x}-\sqrt[3]{xy}}=\dfrac{\sqrt[3]{x}-\sqrt[3]{y}}{1-\sqrt[3]{y}}$;

в) $\dfrac{\sqrt[4]{14}+\sqrt[4]{21k}}{\sqrt[4]{7k}-\sqrt[4]{14}}=\dfrac{\sqrt[4]{2}+\sqrt[4]{3k}}{\sqrt[4]{k}-\sqrt[4]{2}}$; г) $\dfrac{\sqrt[4]{a^2}-\sqrt[4]{ad}}{\sqrt[4]{3a}-\sqrt[4]{a^2d}}=\dfrac{\sqrt[4]{a}-\sqrt[4]{d}}{\sqrt[4]{3}-\sqrt[4]{ad}}$.

36.17. а) $\dfrac{\sqrt{a}-2\sqrt[4]{a}\sqrt[3]{b}+\sqrt[3]{b^2}}{\sqrt[4]{a}-\sqrt[3]{b}}=\dfrac{(\sqrt[4]{a}-\sqrt[3]{b})^2}{\sqrt[4]{a}-\sqrt[3]{b}}=\sqrt[4]{a}-\sqrt[3]{b}$;

б) $\dfrac{\sqrt[3]{m}+2\sqrt[3]{n}}{4\sqrt[3]{n^2}+4\sqrt[3]{mn}+\sqrt[3]{m^2}}=\dfrac{1}{2\sqrt[3]{n}+\sqrt[3]{m}}$;

в) $\dfrac{\sqrt[4]{a}+\sqrt{b}}{\sqrt{a}+2\sqrt[4]{a^2b}+b}=\dfrac{1}{\sqrt[4]{a}+\sqrt{b}}$;

г) $\dfrac{\sqrt{b}+2a\sqrt[4]{a^2b}+a^3}{a\sqrt{a}+\sqrt[4]{b}}=\dfrac{(a\sqrt{a}+\sqrt[4]{b})^2}{a\sqrt{a}+\sqrt[4]{b}}=a\sqrt{a}+\sqrt[4]{b}$,

36.18. а) $\dfrac{\sqrt{a}-\sqrt[3]{b^2}}{\sqrt[4]{a}-\sqrt[3]{b}}=\sqrt[4]{a}+\sqrt[3]{b}$; б) $\dfrac{\sqrt[5]{x^9}-1}{\sqrt[5]{x^3}-1}=x\sqrt[5]{x}+\sqrt[5]{x^3}+1$;

в) $\dfrac{\sqrt{b}-a^3}{a\sqrt{a}+\sqrt[4]{b}}=\sqrt[4]{b}-a\sqrt{a}$; г) $\dfrac{\sqrt{a}-b\sqrt{b}}{\sqrt[6]{a}-\sqrt{b}}=\sqrt[3]{a}+\sqrt[6]{ab^3}+b$.

36.19. а) $\sqrt[4]{2\sqrt[3]{2m^4n^8}}=\sqrt[4]{\sqrt[3]{2^4m^4n^8}}=\sqrt[3]{2mn^2}$;

б) $\sqrt{y\sqrt[5]{9x^4y^2}}=\sqrt{\sqrt[5]{9x^4y^7}}=\sqrt[10]{9x^4y^7}$;

в) $\sqrt[5]{4\sqrt[3]{k^2l^5}}=\sqrt[5]{\sqrt[3]{64k^2l^5}}=\sqrt[15]{64k^2l^5}$;

г) $\sqrt[7]{q\sqrt[5]{2p^3q}}=\sqrt[7]{\sqrt[5]{2p^3q^6}}=\sqrt[35]{2p^3q^6}$.

36.20. а) $\sqrt[5]{2\sqrt[3]{2\sqrt{2}}}=\sqrt[5]{\sqrt[3]{16\sqrt{2}}}=\sqrt[5]{\sqrt[3]{\sqrt{512}}}=\sqrt[10]{8}$;

б) $\sqrt[4]{\dfrac{4}{3}\sqrt[3]{\dfrac{3}{4}\sqrt{\dfrac{4}{3}}}}=\sqrt[4]{\sqrt[3]{\left(\dfrac{4}{3}\right)^2\sqrt{\dfrac{4}{3}}}}=\sqrt[4]{\sqrt[3]{\sqrt{\left(\dfrac{4}{3}\right)^5}}}=\sqrt[24]{\dfrac{1024}{243}}$;

в) $\sqrt[3]{\dfrac{2}{3}\sqrt[3]{\dfrac{3}{2}\sqrt{\dfrac{2}{3}}}}=\sqrt[3]{\sqrt[3]{\left(\dfrac{2}{3}\right)^2\sqrt{\dfrac{2}{3}}}}=\sqrt[3]{\sqrt[3]{\sqrt{\left(\dfrac{2}{3}\right)^5}}}=\sqrt[18]{\dfrac{32}{243}}$;

г) $\sqrt{3\sqrt[4]{3\sqrt[3]{3}}}=\sqrt{\sqrt[4]{3^5\sqrt[3]{3}}}=\sqrt{\sqrt[4]{\sqrt[3]{3^{16}}}}=\sqrt[3]{9}$.

36.21. а) $\sqrt{50}-\sqrt[3]{3}-6\sqrt{2}+\sqrt[3]{24}+\sqrt{8}=5\sqrt{2}-\sqrt[3]{3}-6\sqrt{2}+2\sqrt[3]{3}+2\sqrt{2}=\sqrt{2}+\sqrt[3]{3}$;

б) $6\sqrt[4]{x}+\sqrt{xy}-\sqrt{9xy}-\sqrt[8]{x^2}+\frac{7}{x}\sqrt{x^3y}=6\sqrt[4]{x}+\sqrt{xy}-3\sqrt{xy}-\sqrt[4]{x}+7\sqrt{xy}=$

$=8\sqrt{xy}-3\sqrt{xy}+5\sqrt[4]{x}=5\sqrt{xy}+5\sqrt[4]{x}$.

36.22. а) $-\sqrt[5]{2\sqrt[4]{10}}\vee-\sqrt[4]{\sqrt[5]{99}}$; $-\sqrt[20]{160}<-\sqrt[20]{99}$.

б) $\sqrt{2\sqrt[3]{3}}\vee\sqrt[3]{5}$; $\sqrt[6]{24}\vee\sqrt[3]{5}$; $\sqrt[6]{24}<\sqrt[6]{25}$.

в) $\sqrt[4]{3}\vee\sqrt[8]{6\sqrt{2}}$; $\sqrt[16]{81}>\sqrt[16]{72}$;

г) $-\sqrt{2\sqrt[3]{6}}\vee-\sqrt[3]{5\sqrt{2}}$; $-\sqrt[6]{48}>-\sqrt[6]{50}$.

36.23. а) $\sqrt[3]{5\sqrt{3}}$; $\sqrt[6]{100}$; $\sqrt{3\sqrt[3]{4}}$;

б) $\sqrt[5]{3\sqrt[5]{3}}$; $\sqrt[5]{4}$; $\sqrt[10]{25}$;

в) $\sqrt[3]{2}$; $\sqrt[3]{2\sqrt[5]{2}}$; $\sqrt[5]{3\sqrt{4}}$;

г) $\sqrt[48]{7\sqrt{7}}$; $\sqrt[4]{2\sqrt{1{,}25}}$; $\sqrt[16]{64}$;

36.24. а) $\dfrac{4-3\sqrt{2}}{\left(\sqrt[4]{2}-\sqrt[4]{8}\right)^2}=\dfrac{4-3\sqrt{2}}{\sqrt{2}+\sqrt{8}-4}=\dfrac{4-\sqrt{2}-\sqrt{8}}{\sqrt{2}+\sqrt{8}-4}=-1$;

б) $\dfrac{\left(\sqrt[4]{24}+\sqrt[4]{6}\right)^2}{4\sqrt{3}+3\sqrt{6}}=\dfrac{\sqrt{24}+\sqrt{6}+2\sqrt{12}}{\sqrt{24}+\sqrt{6}+2\sqrt{12}}=1$;

в) $\dfrac{\left(\sqrt[3]{9}+\sqrt{3}\right)^2}{\sqrt[3]{3}+2\sqrt[6]{3}+1}=\dfrac{\left(\sqrt[3]{9}+\sqrt{3}\right)^2}{\left(\sqrt[6]{3}+1\right)^2}=3$;

г) $\dfrac{1-2\sqrt[4]{5}+\sqrt{5}}{\left(\sqrt{3}-\sqrt[4]{45}\right)^2}=\dfrac{\left(1-\sqrt[4]{5}\right)^2}{\left(\sqrt{3}-\sqrt[4]{45}\right)^2}=\dfrac{1}{3}$.

36.25. а) $\left(1+\sqrt{a}\right)\left(1+\sqrt[4]{a}\right)\left(1-\sqrt[4]{a}\right)=(1+\sqrt{a})(1-\sqrt{a})=1-a$;

б) $\left(\sqrt{m}+\sqrt{n}\right)\left(\sqrt[4]{m}-\sqrt[4]{n}\right)\left(\sqrt[4]{m}+\sqrt[4]{n}\right)=(\sqrt{m}+\sqrt{n})(\sqrt{m}-\sqrt{n})=m-n$.

36.26. а) $\dfrac{\left(\sqrt[3]{9a^2x}-2\sqrt[3]{3abx}+\sqrt[3]{b^2x}\right)}{\sqrt[3]{3a}-\sqrt[3]{b}}=\dfrac{\sqrt[3]{x}\left(\sqrt[3]{3a}-\sqrt[3]{b}\right)^2}{\sqrt[3]{3a}-\sqrt[3]{b}}=\sqrt[3]{x}\left(\sqrt[3]{3a}-\sqrt[3]{b}\right)$;

б) $\dfrac{\sqrt[3]{16x^2}-\sqrt[3]{25y^2}}{\sqrt[3]{4x}-\sqrt[3]{5y}}=\sqrt[3]{4x}+\sqrt[3]{5y}$.

36.27. а) $\sqrt{2x}-\sqrt{3y}+\sqrt{2y}-\sqrt{3x}=\sqrt{x}(\sqrt{2}-\sqrt{3})+\sqrt{y}(\sqrt{2}-\sqrt{3})=$
$=(\sqrt{2}-\sqrt{3})(\sqrt{x}+\sqrt{y})$;

б) $\sqrt[3]{4x^2}+\sqrt[4]{2}\sqrt[3]{x^2}-\sqrt[3]{4}\sqrt[4]{y^3}-\sqrt[4]{2y^3}=\sqrt[3]{x^2}(\sqrt[3]{4}+\sqrt[4]{2})-\sqrt[4]{y^3}(\sqrt[3]{4}+\sqrt[4]{2})=$
$=(\sqrt[3]{x^2}-\sqrt[4]{y^3})(\sqrt[3]{4}+\sqrt[4]{2})$;

в) $\sqrt[3]{a^4}+\sqrt[3]{ab^3}-\sqrt[3]{a^3b}-\sqrt[3]{b^4}=\sqrt[3]{a}(a+b)-\sqrt[3]{b}(a+b)=(a+b)(\sqrt[3]{a}-\sqrt[3]{b})$;

г) $b\sqrt{a}-ab+\sqrt{ab}-ab\sqrt{b}=b\sqrt{a}(1-\sqrt{ab})+\sqrt{ab}(1-\sqrt{ab})=$
$=(1-\sqrt{ab})(b\sqrt{a}+\sqrt{ab})=\sqrt{ab}\left(1-\sqrt{ab}\right)\left(1+\sqrt{b}\right)$.

36.28. Рассматриваем данные выражения как квадратные трехчлены и находим их корни:

а) $\sqrt[4]{m}-\sqrt[8]{m}-6=\left(\sqrt[8]{m}-3\right)\left(\sqrt[8]{m}+2\right)$; б) $\sqrt{m}+5\sqrt[4]{m}+6=\left(\sqrt[4]{m}+2\right)\left(\sqrt[4]{m}+3\right)$;

в) $\sqrt[5]{a}+7\sqrt[10]{a}+12=\left(\sqrt[10]{a}+4\right)\left(\sqrt[10]{a}+3\right)$;

г) $2\sqrt[3]{x}-\sqrt[6]{x}-1$; $\sqrt[6]{x}=\dfrac{1\pm\sqrt{1-4\cdot2(-1)}}{4}=\dfrac{1\pm3}{4}$; $\sqrt[6]{x}=1$; $\sqrt[6]{x}=-\dfrac{1}{2}\Rightarrow$

$$2\sqrt[3]{x}-\sqrt[6]{x}-1=2\left(\sqrt[6]{x}-1\right)\left(\sqrt[6]{x}+\frac{1}{2}\right)=\left(\sqrt[6]{x}-1\right)\left(2\sqrt[6]{x}+1\right).$$

36.29. а) $\dfrac{6\sqrt[3]{x^2}+\sqrt[3]{x}-1}{2\sqrt[3]{x^2}+\sqrt[3]{x}}=\dfrac{\left(2\sqrt[3]{x}+1\right)\left(3\sqrt[3]{x}-1\right)}{\sqrt[3]{x}\left(2\sqrt[3]{x}+1\right)}=3-\dfrac{1}{\sqrt[3]{x}}$;

б) $\dfrac{3\sqrt{x}-5\sqrt[4]{x}-2}{9\sqrt{x}-1}=\dfrac{\left(\sqrt[4]{x}-2\right)\left(3\sqrt[4]{x}+1\right)}{\left(3\sqrt[4]{x}-1\right)\left(3\sqrt[4]{x}+1\right)}=\dfrac{\sqrt[4]{x}-2}{3\sqrt[4]{x}-1}$.

36.30. а) $\dfrac{\sqrt{ab}\sqrt[4]{a}}{(a+b)\sqrt[4]{\dfrac{b^2}{a}}}-\dfrac{a^2+b^2}{\left(a^2-b^2\right)}=\dfrac{\sqrt{ab}\cdot\sqrt[4]{a}\cdot\sqrt[4]{a}}{(a+b)\sqrt{b}}-\dfrac{a^2+b^2}{(a+b)(a-b)}=$

$=-\dfrac{b(a+b)}{(a+b)(a-b)}=-\dfrac{b}{a-b}$;

б) $\dfrac{\left(\sqrt[4]{m}+\sqrt[4]{n}\right)^2+\left(\sqrt[4]{m}-\sqrt[4]{n}\right)^2}{2(m-n)}:\dfrac{1}{\sqrt{m^3}-\sqrt{n^3}}-3\sqrt{mn}=$

$=\dfrac{2\left(\sqrt{m}+\sqrt{n}\right)}{2\left(\sqrt{m}-\sqrt{n}\right)\left(\sqrt{m}+\sqrt{n}\right)}\cdot\left(\sqrt{m}-\sqrt{n}\right)\left(m+n+\sqrt{mn}\right)-\left(3\sqrt{mn}\right)=$

$=m+n-2\sqrt{mn}=\left(\sqrt{m}-\sqrt{n}\right)^2$;

36.31. а) $\dfrac{x\sqrt[3]{x}-1}{\sqrt[3]{x^2}-1}-\dfrac{\sqrt[3]{x^2}-1}{\sqrt[3]{x}+1}=4$;

$\dfrac{\sqrt[3]{x^4}-1}{\sqrt[3]{x^2-1}}-\dfrac{(\sqrt[3]{x}-1)(\sqrt[3]{x}+1)}{\sqrt[3]{x}+1}=\dfrac{(\sqrt[3]{x^2}-1)(\sqrt[3]{x^2}+1)}{\sqrt[3]{x^2}-1}-(\sqrt[3]{x}-1)=\sqrt[3]{x^2}+1-\sqrt[3]{x}+1$;

$\sqrt[3]{x^2}+1-\sqrt[3]{x}+1=4$; $\sqrt[3]{x^2}-\sqrt[3]{x}-2=0$; $\sqrt[3]{x}=2,\ x=8$; $\sqrt[3]{x}=-1,\ x=-1$.

$x=-1$ не подходит, т.к. знаменатель обращается в 0, т.е. $x=-1$ не входит в ОДЗ.

Ответ: $x = 8$.

б) $\dfrac{x+8}{\sqrt[3]{x}+2}+\dfrac{\sqrt[3]{x^2}-25}{\sqrt[3]{x}+5}=5$; $\dfrac{(\sqrt[3]{x}+2)(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)}{\sqrt[3]{x}+2}+\dfrac{(\sqrt[3]{x}-5)(\sqrt[3]{x}+5)}{\sqrt[3]{x}+5}$;

$\sqrt[3]{x^2}-2\sqrt[3]{x}+4+\sqrt[3]{x}-5=5$; $\sqrt[3]{x^2}-\sqrt[3]{x}-6=0$; $\sqrt[3]{x}=3,\ x=27$;

$\sqrt[3]{x}=-2,\ x=-8$ – не входит в OD3.

Ответ: $x = 27$.

§ 37. Обобщение понятия о показателе степени

37.1. а) $c^{\frac{3}{4}}=\sqrt[4]{c^3}$; б) $p^{5\frac{1}{2}}=\sqrt{p^{11}}$; в) $x^{\frac{3}{4}}=\sqrt[4]{x^3}$; г) $y^{2\frac{2}{3}}=\sqrt[3]{y^8}$.

37.2. а) $0,2^{0,5}=\sqrt{\dfrac{1}{5}}$; б) $t^{0,8}=\sqrt[5]{t^4}$; в) $b^{1,5}=\sqrt{b^3}$; г) $8,5^{0,6}=\sqrt[5]{8,5^3}$.

37.3. а) $\sqrt{1,3}=1,3^{\frac{1}{2}}$; б) $\sqrt[7]{\dfrac{3}{5}}=0,6^{\frac{1}{7}}$; в) $\sqrt[4]{\dfrac{2}{3}}=\left(\dfrac{2}{3}\right)^{\frac{1}{4}}$; г) $\sqrt[3]{4,3}=4,3^{\frac{1}{3}}$.

37.4. а) $\sqrt[5]{b^4}=b^{\frac{4}{5}}$; б) $\sqrt[3]{a^2}=a^{\frac{2}{3}}$; в) $\sqrt[11]{c^2}=c^{\frac{2}{11}}$; г) $\sqrt[5]{a}=a^{\frac{1}{5}}$.

37.5. а) $49^{\frac{1}{2}}=7$; б) $1000^{\frac{1}{3}}=10$; в) $27^{\frac{1}{3}}=3$; г) $25^{\frac{1}{2}}=5$.

37.6. а) $9^{2\frac{1}{2}}=3^5=243$; б) $0,16^{1\frac{1}{2}}=0,064$;

в) $\left(3\dfrac{3}{8}\right)^{\frac{4}{3}}=\left(\dfrac{3}{2}\right)^4=\dfrac{81}{16}$; г) $0,001^{\frac{2}{3}}=0,01$.

37.7. а) $\dfrac{a^5a^{-8}}{a^{-2}}=a^{-1}$; $a=6,\ a^{-1}=\dfrac{1}{6}$; б) $\dfrac{b^{-9}}{\left(b^2\right)^{-3}}=b^{-3}$; $b=\dfrac{1}{2},\ b^{-3}=8$;

в) $\dfrac{p^{-9}}{p^{-2}p^{-5}}=p^{-2}$; $p=\dfrac{1}{2},\ p^{-2}=4$; г) $\left(t^{-3}\right)^2\dfrac{1}{t^{-5}}=t^{-1}$; $t=0,1,\ t^{-1}=10$;

37.8. а) $\left(27\cdot3^{-4}\right)^2=\frac{1}{9}$; б) $16\cdot\left(2^{-3}\right)^2=\frac{1}{4}$.

37.9. а) $\frac{6^{-4}\cdot6^{-9}}{6^{-12}}=6^{-1}=\frac{1}{6}$; б) $\frac{7^{-7}\cdot7^{-8}}{7^{-13}}=7^{-2}=\frac{1}{49}$.

37.10. а) $\frac{5^4\cdot49^{-3}}{7^{-7}\cdot25^3}=5^{-2}\cdot7^1=\frac{7}{25}$; б) $\frac{81^{12}\cdot10^{-7}}{10^{-5}\cdot27^{17}}=3^{-3}\cdot10^{-2}=\frac{1}{2700}$.

37.11. а) $\sqrt{b^{-1}}=b^{-\frac{1}{2}}$; б) $\sqrt[12]{b^{-5}}=b^{-\frac{5}{12}}$; в) $\frac{1}{\sqrt[4]{x^{-3}}}=x^{\frac{3}{4}}$; г) $\frac{1}{\sqrt[3]{a^{-2}}}=a^{\frac{2}{3}}$.

37.12. а) $4^{-\frac{1}{2}}=\frac{1}{2}$; б) $8^{-\frac{1}{3}}=\frac{1}{2}$; в) $32^{-\frac{1}{5}}=\frac{1}{2}$; г) $16^{-\frac{1}{4}}=\frac{1}{2}$.

37.13. а) $5^{-\frac{4}{3}}$ Да. б) $(-16)^{\frac{2}{3}}$ Нет.

в) $23^{-\frac{1}{5}}$ Да. г) $(-25)^{-\frac{1}{2}}$ Нет.

37.14. а) $2^{\frac{1}{2}}<3^{\frac{1}{2}}$; б) $0{,}3^{\frac{1}{2}}<0{,}5^{\frac{1}{2}}$; в) $5^{\frac{1}{2}}>5^{\frac{1}{3}}$; г) $7^{\frac{1}{3}}=7^{\frac{2}{6}}$.

37.15. а) $c^{\frac{1}{2}}c^{\frac{1}{3}}=c^{\frac{5}{6}}$; б) $b^{-\frac{1}{3}}b^{\frac{1}{2}}=b^{\frac{1}{6}}$; в) $a^{\frac{2}{3}}a^{-\frac{1}{6}}=a^{\frac{1}{2}}$; г) $d^5d^{\frac{1}{2}}=d^{\frac{11}{2}}$.

37.16. а) $x^{\frac{1}{2}}:x^{\frac{3}{2}}=\frac{1}{x}$; б) $y^{-\frac{5}{6}}:y^{\frac{1}{3}}=y^{-\frac{7}{6}}$;

в) $z^{\frac{1}{5}}:z^{-\frac{1}{2}}=z^{\frac{7}{10}}$; г) $m^{\frac{1}{3}}:m^2=m^{-\frac{5}{3}}$.

37.17. а) $\left(b^{1/2}\right)^{\frac{1}{3}}=b^{\frac{1}{6}}$; б) $\left(c^{-1/2}\right)^{\frac{1}{2}}=c^{-\frac{1}{4}}$;

в) $\left(a^{\frac{3}{2}}\right)^{\frac{4}{3}}=a^2$; г) $\left(p^{-\frac{3}{4}}\right)^{-\frac{2}{9}}=p^{\frac{1}{6}}$.

37.18. а) $x^{\frac{1}{2}}\sqrt{x}=x$; б) $y^{\frac{7}{3}}\sqrt[3]{y^2}=y^3$; в) $z^{\frac{3}{4}}z^{\frac{1}{4}}=z$; г) $\sqrt[4]{c^3}c^{\frac{1}{4}}=c$.

37.19. а) $\left(a^{0,4}\right)^{\frac{1}{2}}a^{0,8}=a^{\frac{1}{5}}a^{\frac{4}{5}}=a$; б) $\sqrt[10]{c}\left(c^{-1,2}\right)^{\frac{3}{4}}=c^{\frac{1}{10}}c^{-\frac{9}{10}}=c^{-\frac{4}{5}}$;

в) $\left(x^{\frac{3}{4}}\right)^{\frac{5}{4}}\left(\sqrt[4]{x}\right)^{\frac{17}{4}}=x^{\frac{15}{16}}x^{\frac{17}{16}}=x^2$; г) $\left(b^{0,8}\right)^{-\frac{3}{4}}\left(b^{-\frac{2}{5}}\right)^{-1,5}=b^{-\frac{3}{5}}b^{\frac{3}{5}}=b^0=1$.

37.20. а) $10^{\frac{2}{5}}\cdot10^{\frac{1}{2}}\cdot10^{0,1}=10$; б) $2^{1,3}\cdot2^{-0,7}\cdot4^{0,7}=4$;

в) $49^{-\frac{2}{3}}\cdot 7^{\frac{1}{12}}\cdot 7^{-\frac{3}{4}}=7^{-\frac{16}{12}+\frac{1}{12}-\frac{9}{12}}=\frac{1}{49}$;

г) $25^{0,3}\cdot 5^{1,4}\cdot 625^{0,25}=25\cdot 5=125$.

37.21. а) $4^{0,6}\cdot 2^{0,2}:2^{-0,6}=2^{1,2}\cdot 2^{0,2}\cdot 2^{0,6}=4$

б) $3\cdot 9^{0,4}:\sqrt[5]{3^{-1}}=3^{1+0,8+\frac{1}{5}}=9$;

в) $4^{\frac{1}{3}}\cdot 2^{1\frac{2}{3}}:4^{-\frac{1}{3}}=4^{\frac{1}{3}+\frac{5}{6}+\frac{1}{3}}=4^{\frac{3}{2}}=8$;

г) $8^{-\frac{1}{3}}\cdot 16^{\frac{1}{3}}:\sqrt[3]{2}=2^{-1+\frac{4}{3}-\frac{1}{3}}=1$.

37.22. а) $(27\cdot 64)^{1/3}=3\cdot 4=12$; б) $\left(\frac{1}{16}\cdot 81^{-1}\right)^{-1/4}=2\cdot 3=6$;

в) $\left(\frac{1}{36}\cdot 0,04\right)^{-1/2}=6\cdot 5=30$; г) $\left(5^{-3}\cdot\frac{1}{64}\right)^{-1/3}=5\cdot 4=20$;

37.23. а) $\left(m^{-3}\right)^{1/3}=\frac{1}{m}$; б) $\left(8x^{-1\frac{1}{2}}\right)^{2/3}=4x^{-1}=\frac{4}{x}$;

в) $\left(x^{-\frac{3}{4}}\right)^{-(2/3)}=\sqrt{x}$; г) $\left(81x^{-4}\right)^{-\frac{3}{4}}=\frac{x^3}{27}$.

37.24. а) $\frac{x^{-\frac{2}{3}}\cdot x^{\frac{5}{3}}}{x^{\frac{3}{5}}}=x^{\frac{2}{5}}$; б) $\frac{y^{\frac{6}{7}}\cdot\left(y^{-\frac{1}{2}}\right)^2}{(y^{\frac{4}{7}})^{-2}}=y^{\frac{6}{7}}y^{-1}y^{\frac{8}{7}}=y$;

в) $\frac{(c^{-\frac{2}{3}})^{-4}}{c^{\frac{1}{6}}\cdot c^{\frac{1}{2}}}=c^{\frac{8}{3}-\frac{1}{6}-\frac{1}{2}}=c^2$; г) $\left(\frac{a^{\frac{1}{2}}\cdot b^{\frac{3}{5}}}{a^{\frac{1}{4}}b^{\frac{2}{5}}}\right)^{20}=a^5b^4$.

37.25. а) $\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right)x^{\frac{1}{2}}y^{\frac{1}{2}}=xy^{\frac{1}{2}}-yx^{\frac{1}{2}}$; б) $a^{\frac{2}{3}}b^{\frac{2}{3}}\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\right)=ab^{\frac{2}{3}}+ba^{\frac{2}{3}}$;

в) $b^{\frac{1}{3}}c^{\frac{1}{4}}\left(b^{\frac{2}{3}}+c^{\frac{3}{4}}\right)=bc^{\frac{1}{4}}+cb^{\frac{1}{3}}$; г) $x^{\frac{1}{2}}y^{\frac{1}{2}}\left(x^{\frac{1}{2}}-y^{\frac{3}{2}}\right)=xy^{\frac{1}{2}}-y^2x^{\frac{1}{2}}$.

37.26. а) $\left(m^{\frac{1}{2}}+n^{\frac{1}{2}}\right)^2=m+n+2\sqrt{mn}$ б) $\left(1-c^{\frac{1}{3}}\right)^2=1+c^{\frac{2}{3}}-2c^{\frac{1}{3}}$

в) $\left(1+b^{\frac{1}{2}}\right)^2=1+b+2b^{\frac{1}{2}}$ г) $\left(a^{\frac{1}{2}}-2b^{\frac{1}{2}}\right)^2=a+4b-4\sqrt{ab}$

37.27. а) $\left(x^{\frac{1}{3}}+3\right)\left(x^{\frac{1}{3}}-3\right)=x^{\frac{2}{3}}-9$; б) $\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)\left(a-a^{\frac{1}{2}}b^{\frac{1}{2}}+b\right)=a^{1,5}+b^{1,5}$;

в) $\left(d^{\frac{1}{2}}-1\right)\left(d^{\frac{1}{2}}+1\right)=d-1$;

г) $\left(p^{\frac{1}{3}}-q^{\frac{1}{3}}\right)\left(p^{\frac{2}{3}}+(qp)^{\frac{1}{3}}+q^{\frac{2}{3}}\right)=p-q$.

37.28. а) $\dfrac{4\cdot 3^{\frac{1}{2}}}{3^{\frac{1}{2}}-3}=\dfrac{4}{1-3^{\frac{1}{2}}}$; б) $\dfrac{a^{\frac{1}{2}}-b^{\frac{1}{2}}}{a-b}=\dfrac{1}{a^{\frac{1}{2}}+b^{\frac{1}{2}}}$;

в) $\dfrac{x+x^{\frac{1}{2}}}{2x}=\dfrac{x^{\frac{1}{2}}+1}{2x^{\frac{1}{2}}}$; г) $\dfrac{p^{\frac{1}{2}}-5}{p-25}=\dfrac{1}{p^{\frac{1}{2}}+5}$.

37.29. а) $\dfrac{c+c^{\frac{1}{2}}d^{\frac{1}{2}}+d}{c^{\frac{3}{2}}-d^{\frac{3}{2}}}=\dfrac{1}{c^{\frac{1}{2}}-d^{\frac{1}{2}}}$; б) $\dfrac{m+n}{m^{\frac{2}{3}}-(mn)^{\frac{1}{3}}+n^{\frac{2}{3}}}=m^{\frac{1}{3}}+n^{\frac{1}{3}}$.

37.30. а) $(1+c^{\frac{1}{2}})^2-2c^{\frac{1}{2}}=1+c+2c^{\frac{1}{2}}-2c^{\frac{1}{2}}=1+c$;

б) $\left(m^{\frac{1}{4}}-m^{\frac{1}{3}}\right)^2+2m^{\frac{7}{12}}=m^{\frac{1}{2}}+m^{\frac{2}{3}}-2m^{\frac{7}{12}}-2m^{\frac{7}{12}}=m^{\frac{1}{2}}+m^{\frac{2}{3}}$;

в) $\left(x^{\frac{1}{2}}-y^{\frac{1}{2}}\right)^2+2x^{\frac{1}{2}}y^{\frac{1}{2}}=x+y$;

г) $\sqrt{b}+\sqrt{c}-\left(b^{\frac{1}{4}}+c^{\frac{1}{4}}\right)^2=\sqrt{b}+\sqrt{c}-\sqrt{b}-\sqrt{c}-2\sqrt[4]{bc}=-2\sqrt[4]{bc}$.

37.31. а) $\left(a^{\frac{1}{3}}+b^{\frac{1}{3}}\right)^2-\left(a^{\frac{1}{3}}-b^{\frac{1}{3}}\right)^2=a^{\frac{2}{3}}+b^{\frac{2}{3}}+2\sqrt[3]{ab}-a^{\frac{2}{3}}-b^{\frac{2}{3}}+2\sqrt[3]{ab}=4\sqrt[3]{ab}$;

б) $\left(a^{\frac{3}{2}}+5a^{\frac{1}{2}}\right)^2-10a^2=a^3+25a$.

37.32. а) $\left(x^{\frac{1}{4}}+1\right)\left(x^{\frac{1}{4}}-1\right)\left(x^{\frac{1}{2}}+1\right)=(x^{\frac{1}{2}}-1)(x^{\frac{1}{2}}+1)=x-1$;

б) $\left(k^{\frac{1}{4}}+l^{\frac{1}{4}}\right)\left(k^{\frac{1}{8}}+l^{\frac{1}{8}}\right)\left(k^{\frac{1}{8}}-l^{\frac{1}{8}}\right)=\left(k^{\frac{1}{4}}+l^{\frac{1}{4}}\right)\left(k^{\frac{1}{4}}-l^{\frac{1}{4}}\right)=k^{\frac{1}{2}}-l^{\frac{1}{2}}$.

37.33. а) $\dfrac{a-b}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}-\dfrac{a^{\frac{3}{2}}-b^{\frac{3}{2}}}{a-b}=$

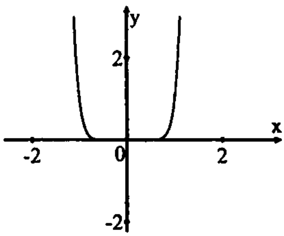
$$=\frac{\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}-\frac{\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)\left(a+a^{\frac{1}{2}}b^{\frac{1}{2}}+b^{\frac{1}{2}}\right)}{\left(a^{\frac{1}{2}}-b^{\frac{1}{2}}\right)\left(a^{\frac{1}{2}}+b^{\frac{1}{2}}\right)}=$$

$$= a^{\frac{1}{2}} + b^{\frac{1}{2}} - \frac{a+b+(ab)^{\frac{1}{2}}}{a^{\frac{1}{2}} + b^{\frac{1}{2}}} = \frac{a+b+2a^{\frac{1}{2}}b^{\frac{1}{2}} - a - b - a^{\frac{1}{2}}b^{\frac{1}{2}}}{a^{\frac{1}{2}} + b^{\frac{1}{2}}} = \frac{\sqrt{ab}}{\sqrt{a}+\sqrt{b}};$$

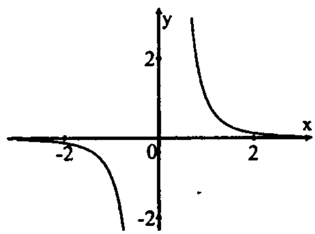
б) $$\frac{\sqrt{x}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}} + \frac{\sqrt{y}}{x^{\frac{1}{2}} - y^{\frac{1}{2}}} = \frac{x-(xy)^{\frac{1}{2}} + (xy)^{\frac{1}{2}} + y}{x-y} = \frac{x+y}{x-y}.$$

§ 38. Степенные функции, их свойства и графики

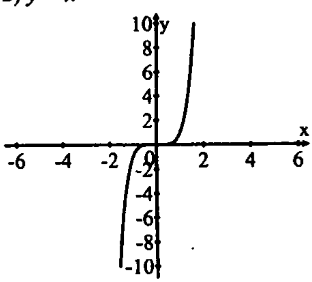
38.1. а) $y = x^{10}$



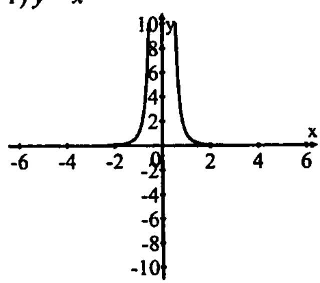
б) $y = x^{-\frac{1}{3}}$



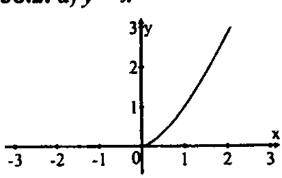
в) $y = x^5$



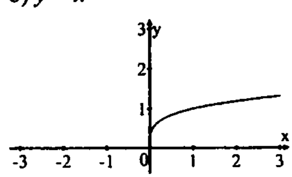
г) $y = x^{-4}$



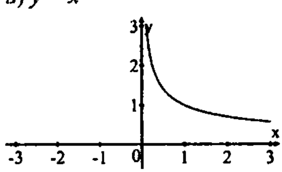
38.2. а) $y = x^{3/2}$



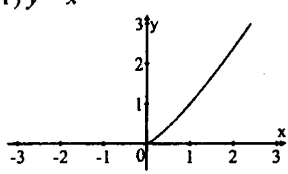
б) $y = x^{1/4}$



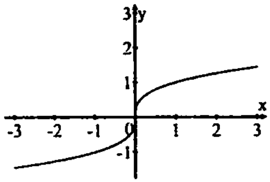
в) $y = x^{-(1/2)}$

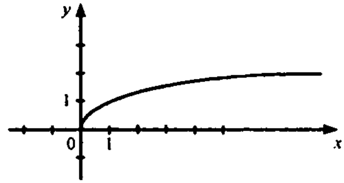


г) $y = x^{5/4}$

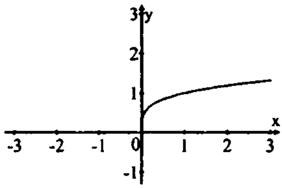


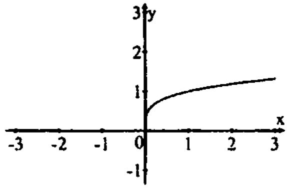
38.3. а) $y=\sqrt[3]{x}$ $\qquad y=x^{\frac{1}{3}}$





б) $y=\sqrt[4]{x}$ $\qquad y=x^{1/4}$





38.4. $f(x)=x^{\frac{5}{2}}$;

а) $f(4)=32$; б) $f\left(\frac{1}{9}\right)=\frac{1}{243}$; в) $f(0)=0$; г) $f(0{,}01)=0{,}00001$.

38.5. $f(x)=x^{-\frac{2}{3}}$;

а) $f(1)=1$; б) $f(8)=\frac{1}{4}$; в) $f\left(\frac{1}{8}\right)=4$; г) $f(0)$ – не имеет смысла.

38.6. а) $y=x^{10}$; $y(-x)=(-x)^{10}=x^{10}=y(x)\Rightarrow$ четная;

б) $y=x^{-(1/3)}$; функция определена только для положительных чисел, поэтому не является ни четной, ни нечетной;

в) $y=x^{-15}$; $y(-x)=(-x)^{-15}=-x^{-15}=-y(x)\Rightarrow$ нечетная;

г) $y=x^{\frac{4}{3}}$ — функция определена только для неотрицательных чисел, поэтому не является ни четной, ни нечетной.

38.7. а) $y=x^{8}$; $y\in[0;+\infty)$. $\qquad$ б) $y=x^{-\frac{3}{4}}$; $y\in(0;+\infty)$.

в) $y=x^{-5}$; $y\in R\quad y\neq 0$. $\qquad$ г) $y=x^{\frac{2}{5}}$; $y\in[0;+\infty)$.

38.8. а) $y=x^{12}$; убывает: $(-\infty;0]$; возрастает: $[0;+\infty)$.

б) $y=x^{-\frac{1}{6}}$; *убывает*: $(0;+\infty)$.

в) $y = x^{-11}$; убывает на $(-\infty; 0)$ и на $(0; +\infty)$.

г) $y = x^{\frac{1}{7}}$; возрастает на $[0; +\infty)$.

38.9. $y = x^{\frac{1}{4}}$. а) $x \in [0;1]$; $\max y: \begin{cases} x = 1 \\ y = 1 \end{cases}$; $\min y: \begin{cases} x = 0 \\ y = 0 \end{cases}$.

б) $x \in [1;+\infty)$, $\min y: \begin{cases} x = 0 \\ y = 0 \end{cases}$; $\max y$ не существует.

в) $x \in (2;3)$; $\min y$ *и* $\max y$ не существуют.

г) $x \in (5;16]$; $\max y: \begin{cases} x = 16 \\ y = 2 \end{cases}$; $\min y$ не существует.

38.10. $y = x^{\frac{5}{2}}$. а) $x \in [0;+\infty)$; $\min y: \begin{cases} x = 0 \\ y = 0 \end{cases}$; $\max y$ не существует;

б) $x \in [1;3)$; $\min y: \begin{cases} x = 1 \\ y = 1 \end{cases}$; $\max y$ не существует;

в) $x \in [1; 2)$; $\min y: \begin{cases} x = 1 \\ y = 1 \end{cases}$ max не существует.

г) $x \in (6;8]$; $\max y: \begin{cases} x = 8 \\ y = 128\sqrt{2} \end{cases}$; $\min y$ не существует.

38.11. $y = x^{-\frac{2}{3}}$. а) $x \in [1;8]$, $\min y = \frac{1}{4}$, $\max y = 1$;

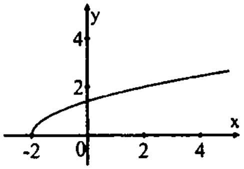
б) $x \in (3;5)$, $\min y$ *и* $\max y$ не существуют;

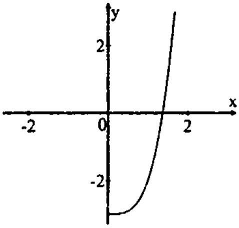
в) $x \in [1;+\infty)$, $\max y = 1$, $\min y$ не существует;

г) $x \in (0;1]$, $\max y$ не существует, $\min y = 1$.

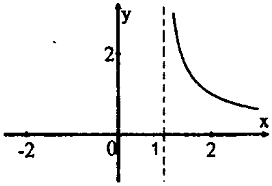
38.12.

а) $y = (x+2)^{\frac{1}{2}}$ б) $y = x^{7/2} - 3$

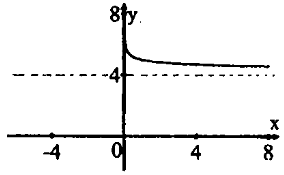




в) $y = (x-1)^{-2/3}$

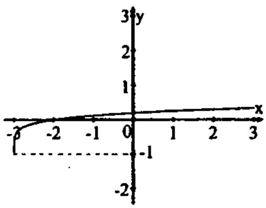


г) $y = x^{-1/3} + 4$

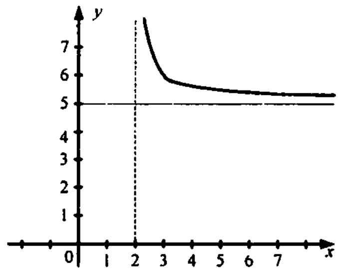


38.13.

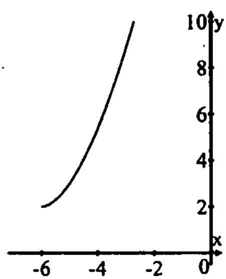
а) $y = (x+3)^{1/6} - 1$



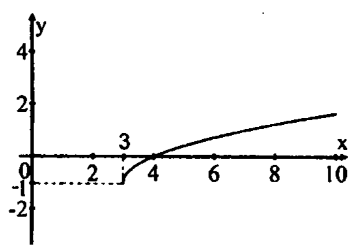
б) $y = (x-2)^{-(1/9)} + 5$



в) $y = (x+6)^{7/4} + 2$

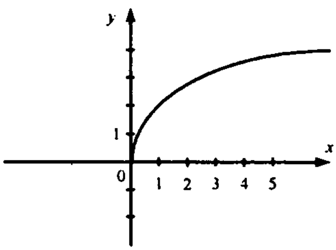


г) $y = (x-3)^{1/2} - 1$

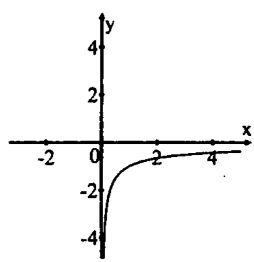


38.14.

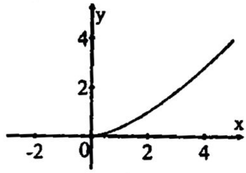
а) $y = 2x^{1/3}$



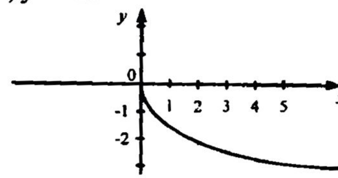
б) $y = -x^{-(3/5)}$



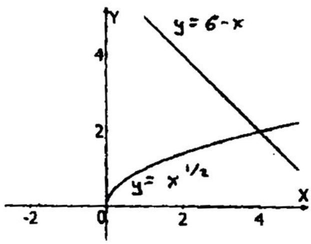
в) $y = \frac{1}{2}x^{3/2}$



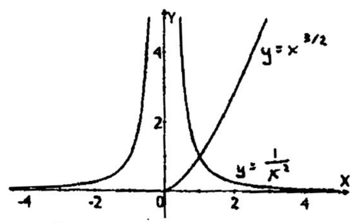
г) $y = -2x^{1/4}$



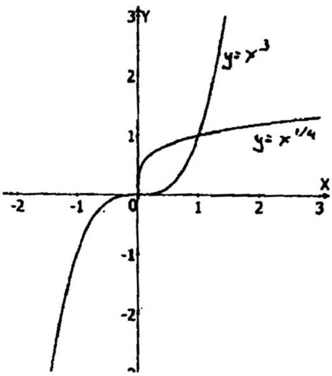
38.15. а) $x^{\frac{1}{2}} = 6 - x$, $x = 4$;



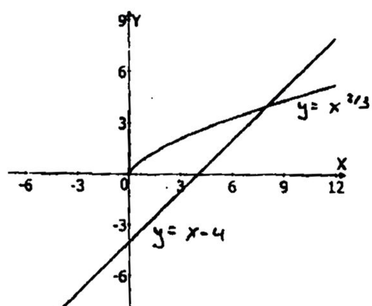
б) $x^{\frac{3}{2}} = \dfrac{1}{x^2}$, $x = 1$;



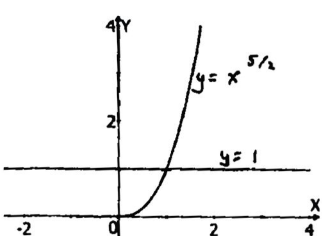
в) $x^{\frac{1}{4}} = x^3$, $x = 0$, $x = 1$;



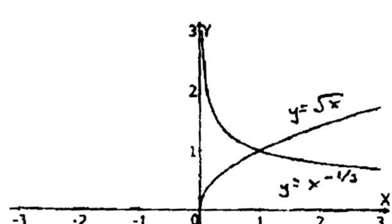
г) $x^{\frac{2}{3}} = x - 4$, $x = 8$;



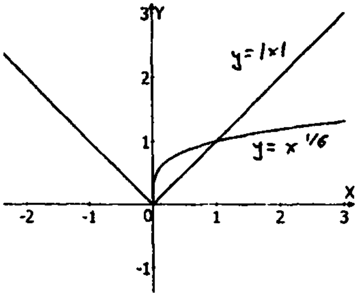
38.16. а) $\begin{cases} y = x^{\frac{5}{2}} \\ y = 1 \end{cases}$; $\begin{cases} y = 1 \\ x = 1 \end{cases}$;

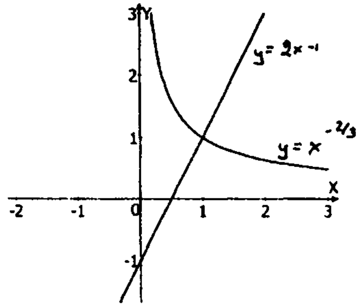


б) $\begin{cases} y = x^{-\frac{1}{3}} \\ y = \sqrt{x} \end{cases}$, $\begin{cases} x = 1 \\ y = 1 \end{cases}$;

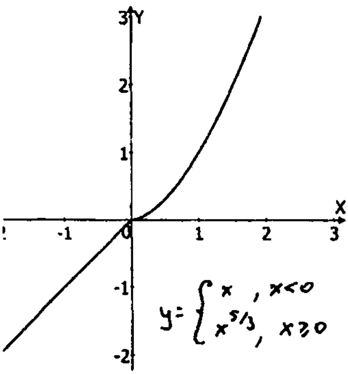


в) $\begin{cases} y = x^{\frac{1}{6}} \\ y = |x| \end{cases}$, $\begin{cases} x = 0;1 \\ y = 0;1 \end{cases}$, (0;0), (1;1); г) $\begin{cases} y = x^{-\frac{2}{3}} \\ y = 2x - 1 \end{cases}$, $\begin{cases} x = 1 \\ y = 1 \end{cases}$;

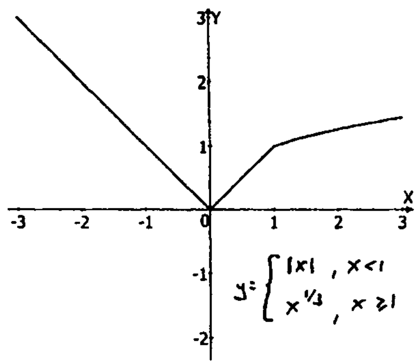




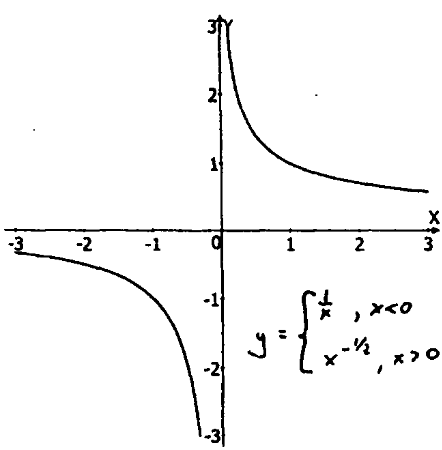
38.17.



38.18.



38.19.



38.20. $f(x)=x^{\frac{1}{4}}$; а) $f(16x)=(16x)^{\frac{1}{4}}=2x^{\frac{1}{4}}$; б) $f(81x^4)=3x$;

в) $f\left(\frac{1}{81}x\right)=\left(\frac{x}{81}\right)^{\frac{1}{4}}=\frac{x^{\frac{1}{4}}}{3}$; г) $f(x^{-8})=(x^{-8})^{\frac{1}{4}}=x^{-2}$.

38.21. $f(x)=x^{-(2/3)}$;

а) $f(8x^3)=(8x^3)^{-\frac{2}{3}}=\frac{1}{4}x^{-2}$; б) $f(x^{-6})=x^4$;

в) $f\left(\frac{x}{27}\right)=\frac{9}{x^{2/3}}$; г) $f(x^{12})=x^{-8}$.

38.22. а) $y=x^8$, $y'=8x^7$; б) $y=x^{-4}$, $y'=-4x^{-5}$;

в) $y=x^{40}$, $y'=40x^{39}$; г) $y=\frac{1}{x^6}$, $y'=-6x^{-7}$.

38.23. а) $y=x^{\frac{3}{5}}$, $y'=\frac{3}{5}x^{-\frac{2}{5}}$; б) $y=\sqrt[4]{x^5}$, $y'=\frac{5}{4}x^{\frac{1}{4}}$;

в) $y=x^{\frac{7}{2}}$, $y'=\frac{7}{2}x^{\frac{5}{2}}$; г) $y=\sqrt[5]{x}$, $y'=\frac{1}{5};\frac{1}{\sqrt[5]{x^4}}$.

38.24. а) $y=\frac{1}{\sqrt{x}}$, $y'=-\frac{1}{2\sqrt{x^3}}$; б) $y=\frac{1}{x^{\frac{3}{5}}}$, $y'=-\frac{3}{5}x^{-\frac{8}{5}}$;

в) $y=\frac{1}{\sqrt[3]{x}}$, $y'=-\frac{1}{3}\cdot\frac{1}{\sqrt[3]{x^4}}$; г) $y=\frac{1}{x^{\frac{5}{3}}}$, $y'=-\frac{5}{3}x^{-\frac{8}{3}}$;

38.25. а) $y=x\sqrt{x}$, $y'=\frac{3}{2}\sqrt{x}$; б) $y=\frac{x^2}{\sqrt{x}}$, $y'=\frac{3}{2}\sqrt{x}$;

в) $y=\frac{\sqrt[3]{x}}{x}$, $y'=-\frac{2}{3\sqrt[3]{x^5}}$; г) $y=x^2\cdot\sqrt[3]{x}$, $y'=\frac{7}{3}\sqrt[3]{x^4}$;

38.26. а) $y=2x^4+x\sqrt{x}$; $y'=8x^3+\frac{3}{2}\sqrt{x}$;

б) $y=\frac{2}{\sqrt[3]{x}}+3x^6-1$; $y'=-\frac{2}{3\sqrt[3]{x^4}}+18x^5$;

в) $y=x^5-\frac{1}{\sqrt{x}}$; $y'=5x^4+\frac{1}{2\sqrt{x^3}}$;

г) $y = x^3 - 7x\sqrt[5]{x}$; $y' = 3x^2 - \frac{42}{5} \cdot \sqrt[5]{x}$;

38.27. а) $g(x) = x^3 - 3\sqrt{x}$; $x_0 = 1$; $g'(x) = 3x^2 - \frac{3}{2\sqrt{x}}$; $g'(1) = 3 - \frac{3}{2} = \frac{3}{2}$;

б) $g(x) = \sqrt[3]{3x-1}$; $x_0 = \frac{2}{3}$; $g'(x) = \frac{1}{\sqrt[3]{(3x-1)^2}}$; $g'\left(\frac{2}{3}\right) = 1$;

в) $g(x) = x^{-1} + x^{-2}$; $x_0 = 1$; $g'(x) = -x^{-2} - 2x^{-3}$; $g'(1) = -3$;

г) $g(x) = \frac{1}{3}(5-2x)^{-3}$; $x_0 = 2$; $g'(x) = 2(5-2x)^{-4}$; $g'(2) = 2$.

38.28. а) $f(x) = 4 - x^{-\frac{3}{4}}$; $x_0 = 1$; $f'(x) = \frac{3}{4}x^{-\frac{7}{4}}$; $f'(1) = \frac{3}{4}$;

б) $f(x) = 12x^{-\frac{1}{2}} - x$; $x_0 = 9$; $f'(x) = -6x^{-\frac{3}{2}} - 1$; $f'(9) = -\frac{6}{27} - 1 = -1\frac{2}{9}$;

в) $f(x) = 2x^{2/3} - 1$; $x_0 = 8$; $f'(x) = \frac{4}{3}x^{-(1/3)}$; $f'(8) = \frac{2}{3}$;

г) $f(x) = x^{-3} + 6\sqrt{x}$; $x_0 = 1$; $f'(x) = -3x^{-4} + \frac{3}{\sqrt{x}}$; $f'(1) = -3 + 3 = 0$.

38.29. а) $g(x) = \frac{2}{3}\sqrt{4-3x}$; $x_0 = \frac{1}{3}$;

$g'(x) = -\frac{1}{\sqrt{4-3x}}$; $g'\left(\frac{1}{3}\right) = -\frac{1}{\sqrt{3}}$; $\alpha = \frac{5\pi}{6}$.

б) $g(x) = -3\left(\sqrt{2}+x\right)^{\frac{1}{3}}$; $x_0 = 1-\sqrt{2}$; $g'(x) = \left(\sqrt{2}+x\right)^{-\frac{4}{3}}$; $g'\left(1-\sqrt{2}\right) = 1$; $\alpha = \frac{\pi}{4}$.

38.30. а) $y = x^4 - 3x^3$, $a = 2$; $y = 16 - 24 + \left(4 \cdot 2^3 - 9 \cdot 2^2\right)(x-2) = -4x$.

б) $y = \sqrt[3]{3x-1}$; $a = 3$; $y' = \frac{1}{\sqrt[3]{(3x-1)^2}}$; $y = 2 + \frac{1}{4}(x-3) = \frac{1}{4}x + \frac{5}{4}$.

в) $y = 3x^3 - 5x^2 - 4$; $a = 2$; $y' = 9x^2 - 10x$;
$y = 24 - 20 - 4 + 16(x-2) = 16x - 32$.

г) $y = (2x+5)^{-\frac{1}{2}}$; $a = 2$; $y' = -(2x+5)^{-\frac{3}{2}}$; $y = \frac{1}{3} - \frac{1}{27}(x-2) = -\frac{1}{27}x + \frac{11}{27}$.

38.31. а) $y = \frac{2}{3}x\sqrt{x} - 2x$, $y' = \sqrt{x} - 2$

Убывает на $[0, 4]$; возрастает на $[4, +\infty)$. Точка минимума $\left(4; -\frac{8}{3}\right)$.

б) $y=\frac{3}{2}x^{2/3}-x$; $y'=x^{-\frac{1}{3}}-1$; возрастает на $x\in[0;1]$;

$x\geq 1$ – убывает; $x=1$ – max; $y_{\max}=\frac{1}{2}$.

38.32. а) $y=\frac{2}{3}x\sqrt{x}-2x,\ x\in[1,9]$ $y'=\sqrt{x}-2$; $y_{\text{наим}}=-\frac{8}{3}$, $y_{\text{наиб}}=0$.

б) $y=\frac{3}{2}x^{2/3}-x$; $(0;8)$; $y'=x^{-(1/3)}-1$; $y_{\max}=\frac{1}{2}$; min y не существует.

в) $y=\frac{2}{3}x\sqrt{x}-2x$; $(1;9)$; $y'=\sqrt{x}-2$; $x=4$; $y(4)=\frac{16}{3}-8=-\frac{8}{3}$ – min;

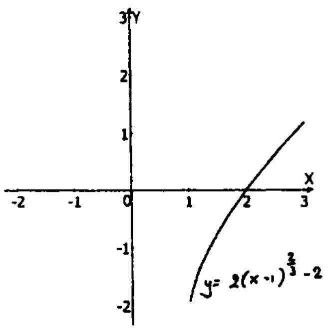
$y_{\max}$ не существует.

г) $y=\frac{3}{2}x^{\frac{2}{3}}-x$; $[0;8]$; $y'=x^{-\frac{1}{3}}-1$; $y(0)=0$; $y(8)=-2$; $y(1)=\frac{1}{2}$;

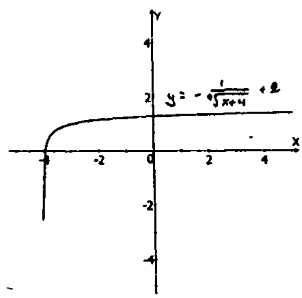
$y_{\max}=\frac{1}{2}$; $y_{\min}=-2$.

38.33.

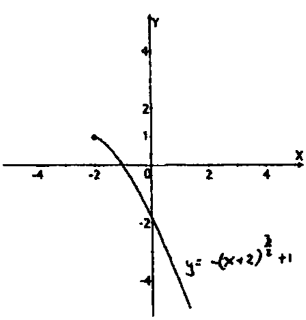
а)



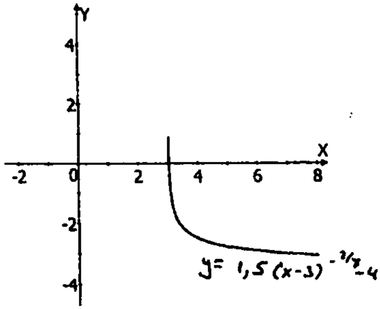
б)



в)

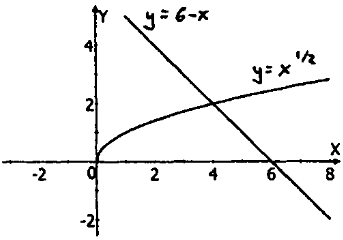


г)

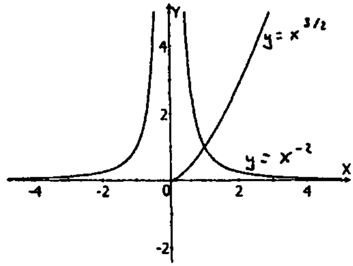


38.34.

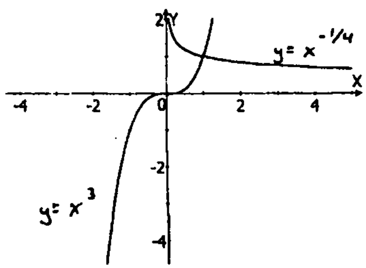
а) $x^{\frac{1}{2}} < 6 - x$; $x \in [0;4)$.



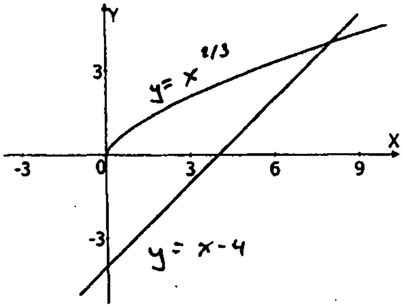
б) $x^{\frac{3}{2}} \geq -2$; $x \geq 1$.



в) $x^{-\frac{1}{4}} \leq x^3$; $x \geq 1$.



г) $x^{\frac{2}{3}} > x - 4$; $x \in [0;8)$.



38.35. а) $g(x) = 2\sqrt{x} - x$; $g'(x) = \dfrac{1}{\sqrt{x}} - 1 = 0 \quad x = 1$.

б) $g(x) = \dfrac{2}{3}x^{\frac{3}{2}} - \dfrac{12}{5}x^{\frac{5}{4}} + 2x$; $g'(x) = \sqrt{x} - 3\sqrt[4]{x} + 2 = 0$;

$\sqrt[4]{x} = 2$; $\sqrt[4]{x} = 1$; $x = 16$; $x = 1$.

в) $g(x) = \dfrac{3}{4}x^{\frac{4}{3}} - 2x$; $g'(x) = x^{\frac{1}{3}} - 2 = 0$; $x = 8$.

г) $g(x) = \dfrac{3}{4}x^{\frac{4}{3}} - \dfrac{6}{7}x^{\frac{7}{6}} - 2x$; $g'(x) = x^{\frac{1}{3}} - x^{\frac{1}{6}} - 2$; $x^{\frac{1}{6}} = 2$, $x^{\frac{1}{6}} = -1$;

$x = 64$, решений нет.

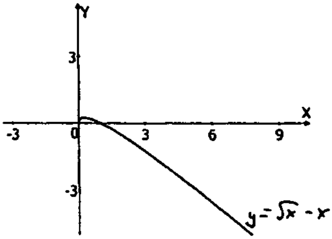
38.36. а) $f(x) = 4\sqrt[4]{x}$; $y = x - 2$; $f'(x) = x^{-\frac{3}{4}}$; $y = 4\sqrt[4]{x_0} + x_0^{-\frac{3}{4}}(x - x_0)$;

$x_0^{-\frac{3}{4}} = 1$; $x_0 = 1$; $y = 4 + x - 1 = x + 3$.

б) $f(x)=\frac{1}{x^3}$; $y=5-3x$; $f'(x)=-3\frac{1}{x^4}$; $y=\frac{1}{x_0^3}-\frac{3}{x_0^4}(x-x_0)$;

$-\frac{3}{x_0^4}=-3$; $x_0=\pm1$; $y=1-3(x-1)=-3x+4$; $y=-1-3(x+1)=-3x-4$.

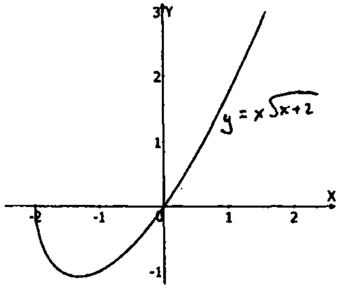
38.37. а)



$y=\sqrt{x}-x$; $y'=\frac{1}{2\sqrt{x}}-1=0$; $2\sqrt{x}=1$, $x=\frac{1}{4}$;

возрастает $x\in\left[0;\frac{1}{4}\right]$; убывает $x\ge\frac{1}{4}$; $x=\frac{1}{4}$ – max *ww*.

б)



$y=x\sqrt{x+2}$; $y'=\sqrt{x+2}+\frac{x}{2\sqrt{x+2}}=\frac{3x+4}{2\sqrt{x+2}}>0$;

$x\ge-\frac{4}{3}$ – возрастает; $x\in\left[-2;-\frac{4}{3}\right]$ – убывает; $x=-\frac{4}{3}$ – min.

38.38. а) $\underbrace{2x^5+x^3+5x-80}_{y_1}=\underbrace{\sqrt[3]{14-3x}}_{y_2}$;

$y_1'=10x^4+3x^2+5$ – возрастает, при всех x;

$y_2'=-\frac{1}{\sqrt[3]{(14-3x)^2}}$ – убывает, при всех $x\Rightarrow$ одно решение: $x=2$.

б) $\sqrt[4]{10+3x}=74-x^5-3x^3-8x$; $y_1=\sqrt[4]{10+3x}$;

$y_2 = 74 - x^5 - 3x^3 - 8x$; $y_1' = \dfrac{3}{4\sqrt[4]{(10+3x)^3}}$ – возрастает, при всех x;

$y_2' = -5x^4 - 9x^2 - 8$ – убывает, при всех $x \Rightarrow$ одно решение: $x = 2$.

38.39. а) $y = \sqrt{x}$ M(0;1); $y' = \dfrac{1}{2\sqrt{x}}$; $y = \sqrt{x_0} + \dfrac{1}{2\sqrt{x_0}}(x - x_0)$;

$1 = \sqrt{x_0} + \dfrac{1}{2\sqrt{x_0}}(-x_0)$; $1 = \sqrt{x_0} - \dfrac{1}{2}\sqrt{x_0}$; $\dfrac{1}{2}\sqrt{x_0} = 1$; $x_0 = 4$;

$y = 2 + \dfrac{1}{4}(x-4)$; $y = \dfrac{1}{4}x + 1$.

б) $y = x^{\frac{3}{2}} + 4$; M(0;0); $y' = \dfrac{3}{2}\sqrt{x}$; $y = x_0^{\frac{3}{2}} + 4 + \dfrac{3}{2}\sqrt{x_0}(x - x_0)$;

$0 = x_0^{\frac{3}{2}} + 4 + \dfrac{3}{2}\sqrt{x_0}(-x_0)$; $x_0^{\frac{3}{2}} = 8$; $x_0 = 4$; $y = 8 + 4 + \dfrac{3}{2}\cdot 2(x-4)$;

$y = 3x - 12 + 12$; $y = 3x$.