# Решение контрольных работ по алгебре за 10–11 классы

к учебному изданию «Алгебра и начала анализа. 10–11 кл.: Контрольные работы для общеобразоват. учреждений. Учеб. пособие / А.Г. Мордкович, Е.Е. Тульчинская. — М.: Мнемозина, 2000–2005»

# Контрольная работа № 1

## Вариант 1.

1. а) $\sin\frac{7\pi}{3}=\frac{\sqrt{3}}{2}$; б) $\cos\left(-\frac{5\pi}{4}\right)=-\frac{\sqrt{2}}{2}$;

в) $\operatorname{tg}\left(-\frac{13\pi}{6}\right)=-\frac{\sqrt{3}}{3}$; г) $\operatorname{ctg} 3{,}5\pi=0$.

2. а) $\sin\alpha=\frac{1}{2}$; $\alpha=(-1)^k\frac{\pi}{6}+\pi k$;

б) $\cos\alpha=-\frac{\sqrt{3}}{2}$; $\alpha=\pm\frac{5\pi}{6}+2\pi n$.

3. $\operatorname{ctg}\alpha\cdot\sin(-\alpha)+\cos(2\pi-\alpha)=-\cos\alpha+\cos\alpha=0$.

4. $$\frac{\operatorname{ctg}\alpha}{\operatorname{tg}\alpha+\operatorname{ctg}\alpha}=\frac{\frac{\cos\alpha}{\sin\alpha}}{\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}}=\frac{\frac{\cos\alpha}{\sin\alpha}}{\frac{\sin^2\alpha+\cos^2\alpha}{\cos\alpha\cdot\sin\alpha}}=\cos^2\alpha.$$

5. $2\sin 870^\circ+\sqrt{12}\cos 70^\circ-\operatorname{tg}^2 60^\circ=2\cdot\frac{1}{2}+\sqrt{12}\frac{\sqrt{3}}{2}-3=1$.

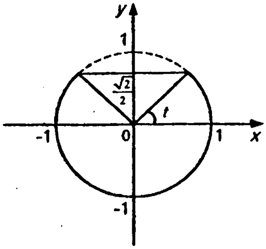
6. $\sin\alpha=\frac{4}{5}$ $\quad\frac{\pi}{2}<\alpha<\pi$;

$$\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\sqrt{1-\frac{16}{25}}=-\frac{3}{5};$$

$$\operatorname{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}=-\frac{\frac{4}{5}}{\frac{3}{5}}=-\frac{4}{3};$$

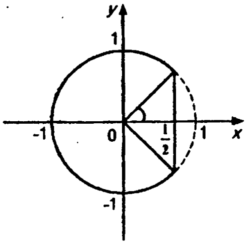
$$\operatorname{ctg}\alpha=\frac{\cos\alpha}{\sin\alpha}=\frac{-\frac{3}{5}}{\frac{4}{5}}=-\frac{3}{4}.$$

**7.** а) $\sin t > \frac{\sqrt{2}}{2}$, отсюда $\frac{\pi}{4} + 2\pi k < t < \frac{3\pi}{4} + 2\pi k,\ k \in \mathbb{Z}$.



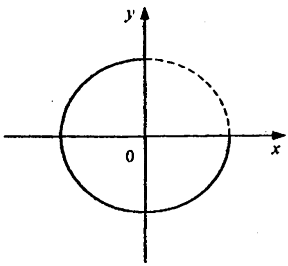

б) $\cos t > \frac{1}{2}$

$$-\frac{\pi}{3} + 2\pi k < t < \frac{\pi}{3} + 2\pi k,\ k \in \mathbb{Z}.$$




**8.** 1; cos 7; sin 7; ctg 7.

$2\pi < 7 < \frac{5\pi}{2}$, sin 7>0 и cos 7>0




Сравни $\frac{9\pi}{4} \vee 7$, $9\pi \vee 28$, $9\pi > 28 \Rightarrow 7 < \frac{9\pi}{4}$, а в этой области $\cos\alpha > \sin\alpha \Rightarrow \cos 7 > \sin 7 \Rightarrow \operatorname{ctg} 7 > 1 \Rightarrow \sin 7$, $\cos 7$, $1$, $\operatorname{ctg} 7$.

# Вариант 2.

**1** а) $\cos\frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$; б) $\sin\left(-\frac{7\pi}{4}\right) = \frac{\sqrt{2}}{2}$;

в) $\operatorname{tg}\frac{11\pi}{3} = -\sqrt{3}$; г) $\operatorname{ctg}(-3{,}5) = 0$.

**2.** а) $\sin\alpha = -\frac{1}{2}$; $\alpha = (-1)^{k+1}\frac{\pi}{6} + \pi k$;

б) $\cos\alpha = \frac{\sqrt{3}}{2}$; $\alpha = \pm\frac{\pi}{6} + 2\pi n$.

**3.** $\operatorname{tg}(-\alpha)\cos\alpha - \sin(4\pi - \alpha) = -\sin\alpha + \sin\alpha = 0$.

**4.** $\operatorname{ctg}\alpha \cdot \sin^2\alpha = \frac{\cos\alpha}{\sin\alpha} \cdot \sin^2\alpha = \sin\alpha \cdot \cos\alpha =$

$$= \left(\frac{1}{\sin\alpha \cdot \cos\alpha}\right)^{-1} = \left(\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha \cdot \cos\alpha}\right)^{-1} = (\operatorname{tg}\alpha + \operatorname{ctg}\alpha)^{-1}.$$

**5.** $4\cos 840° - \sqrt{48}\sin 600° + \operatorname{ctg}30° =$

$$= -4 \cdot \frac{1}{2} + \frac{\sqrt{3}}{2} \cdot \sqrt{48} + (\sqrt{3})^2 = 7.$$

**6.** $\cos\alpha = -\frac{4}{5}$; $\pi < \alpha \frac{3\pi}{2}$;

$$\sin\alpha = -\sqrt{1 - \cos^2\alpha} = -\sqrt{1 - \frac{16}{25}} = -\frac{3}{5};$$

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{3}{5}}{-\frac{4}{5}} = \frac{3}{4};$$

$$\operatorname{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3}.$$

**7.** а) $\sin t < \frac{\sqrt{3}}{2}$

$\pi - \frac{\pi}{3} < t < 2\pi + \frac{\pi}{3}$;

$\frac{2\pi}{3} + 2\pi k < t < \frac{7\pi}{3} + 2\pi k,\ k \in \mathbb{Z}$

б) $\cos t < -\frac{\sqrt{2}}{2}$

$\frac{3\pi}{4} + 2\pi k < t < \frac{5\pi}{4} + 2\pi k,\ k \in \mathbb{Z}$.

**8.** tg 10, sin 10, cos 10, ctg 10.

$3\pi < 10 < \frac{7\pi}{2}$, cos 10<0 и sin 10<0

$\frac{13\pi}{4} \vee 10$

$13\pi \vee 40$

$13\pi > 40 \Rightarrow \sin 10 > \cos 10 \Rightarrow$ tg 10 < 1, а ctg > 1 $\Rightarrow$

$\Rightarrow \frac{\sin 10}{\cos 10} \vee \cos 10$.

$\frac{\sin 10 - \cos^2 10}{\cos 10} \vee 0$, $\frac{\sin 10 - 1 + \sin^2 10}{\cos 10} \vee 0$.

$\sin^2 10<1$ и sin 10<0 $\Rightarrow$

$\sin^2 10 - 1 + \sin 10<0$ и cos 10<0 $\Rightarrow$ это отношение > 0, поэтому tg 10 > cos 10.

Сравни $\operatorname{tg} 10 \vee \sin 10$.

$\sin 10\left(\frac{1}{\cos 10} - 1\right) \vee 0$

$\sin 10\left(\frac{1-\cos 10}{\cos 10}\right) \vee 0$, это выражение больше нуля, т.е. tg 10> sin 10.

Поэтому: cos 10< sin 10< tg 10 < ctg 10.

## Вариант 3.

**1.** а) $\sin\frac{9\pi}{4}=\frac{\sqrt{2}}{2}$; б) $\cos\left(-\frac{4\pi}{3}\right)=-\frac{1}{2}$;

в) $\mathrm{tg}\left(-\frac{7\pi}{6}\right)=-\frac{\sqrt{3}}{3}$; г) $\mathrm{ctg}\,\frac{5\pi}{4}=1.$

**2.** а) $\sin\alpha=\frac{\sqrt{2}}{2}$; $\alpha=(-1)^k\frac{\pi}{4}+\pi k$;

б) $\cos\alpha=-\frac{1}{2}$; $\alpha=\pm\frac{2\pi}{3}+2\pi n.$

**3.** $\mathrm{tg}\alpha\cos(-\alpha)+\sin(\pi+\alpha)=\sin\alpha-\sin\alpha=0.$

**4.** $$\frac{\mathrm{tg}\,\alpha}{\mathrm{tg}\,\alpha+\mathrm{ctg}\,\alpha}=\frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{\sin\alpha}{\cos\alpha}+\frac{\cos\alpha}{\sin\alpha}}=\frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{\sin^2\alpha+\cos^2\alpha}{\cos\alpha\cdot\sin\alpha}}=\sin^2\alpha.$$

**5.** $4\sin^2 120^\circ-2\cos 600^\circ+\sqrt{27}\,\mathrm{tg}660^\circ=$

$$4\frac{3}{4}+\frac{1}{2}\cdot 2-\frac{\sqrt{3}}{3}\cdot\sqrt{27}=3+1-3=1.$$

**6.** $\sin\alpha=\frac{3}{5}$; $\frac{\pi}{2}<\alpha<\pi$;

$$\cos\alpha=-\sqrt{1-\sin^2\alpha}=-\sqrt{1-\frac{9}{25}}=-\frac{4}{5};$$

$$\mathrm{tg}\alpha=\frac{\sin\alpha}{\cos\alpha}=\frac{\frac{3}{5}}{-\frac{4}{5}}=-\frac{3}{4};$$

$$\mathrm{ctg}\alpha=\frac{\cos\alpha}{\sin\alpha}=\frac{-\frac{4}{5}}{\frac{3}{5}}=-\frac{4}{3}.$$

**7.** а) $\sin t > -\frac{1}{2}$

$$2\pi k - \frac{\pi}{6} < t < \frac{7\pi}{6} + 2\pi k,\ k \in \mathbb{Z}.$$

б) $\cos t < \frac{\sqrt{3}}{2}$,

$$\frac{\pi}{6} + 2\pi k < t < \frac{11}{6}\pi + 2\pi k,\ k \in \mathbb{Z}.$$

**8.** 1; cos 7,5; sin 7,5; tg 7,5.

$$2\pi < 7{,}5 < \frac{5\pi}{2} \Rightarrow \sin 7{,}5 > 0,$$

$\cos 7{,}5 > 0$ но $7{,}5 > \frac{9\pi}{4} \Rightarrow \sin 7{,}5 > \cos 7{,}5 \Rightarrow$

$\Rightarrow \operatorname{tg} 7{,}5 > 1 \Rightarrow \cos 7{,}5 < \sin 7{,}5 < 1 < \operatorname{tg} 7{,}5.$

## Вариант 4.

**1.** а) $\cos\frac{2\pi}{3} = -\frac{1}{2}$; б) $\sin\left(-\frac{11\pi}{4}\right) = -\frac{\sqrt{2}}{2}$;

в) $\operatorname{tg}\frac{17\pi}{6} = -\frac{\sqrt{3}}{3}$; г) $\operatorname{ctg}\left(-\frac{\pi}{4}\right) = -1$.

**2.** а) $\sin\alpha = -\frac{\sqrt{3}}{2}$; $\alpha = (-1)^{k+1}\frac{\pi}{3} + \pi k$;

б) $\cos\alpha = \frac{1}{2}$; $\alpha = \pm\frac{\pi}{3} + 2\pi n$.

**3.** $\operatorname{ctg}(-\alpha)\sin\alpha + \cos(\pi + \alpha) = -\cos\alpha - \cos\alpha = -2\cos\alpha$.

**4.** $\operatorname{tg}\alpha \cos^2\alpha = \frac{\sin\alpha}{\cos\alpha} \cdot \cos^2\alpha = \sin\alpha \cdot \cos\alpha =$

$$= \left(\frac{1}{\sin\alpha \cdot \cos\alpha}\right)^{-1} = \left(\frac{\sin^2\alpha + \cos^2\alpha}{\sin\alpha \cdot \cos\alpha}\right)^{-1} = (\operatorname{tg}\alpha + \operatorname{ctg}\alpha)^{-1}.$$

**5.** $4\sin 690° - 8\cos^2 210° + \sqrt{27}\,\mathrm{ctg}\,660° =$

$$= 2 - 2\cdot 3 - \frac{1}{\sqrt{3}}\cdot\sqrt{27} = -7.$$

**6.** $\cos\alpha = -\frac{3}{5}$;  $\frac{\pi}{2} < \alpha < \pi$;

$$\sin\alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-\frac{9}{25}} = \frac{4}{5};$$

$$\mathrm{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3};$$

$$\mathrm{ctg}\alpha = \frac{\cos\alpha}{\sin\alpha} = \frac{-\frac{3}{5}}{\frac{4}{5}} = -\frac{3}{4}.$$

**7.** а) $\sin t > -\frac{\sqrt{2}}{2}$  $-\frac{\pi}{4} + 2\pi k < t < \frac{5}{4}\pi + 2\pi k,\ k \in \mathbb{Z}$.

б) $\cos t < \frac{1}{2}$  $\frac{\pi}{3} + 2\pi k < t < \frac{5}{3}\pi + 2\pi k,\ k \in \mathbb{Z}$.

**8.** tg 10,5; cos 10,5; sin 10,5; ctg 10,5

$3\pi < 10{,}5 < \frac{7\pi}{2}$, но $\frac{13\pi}{4} < 10{,}5 \Rightarrow$ sin 10,5 < cos 10,5 и tg 10,5 > 1, ctg 10,5 < 1.

Сравни $\mathrm{ctg}10{,}5 \vee \cos 10{,}5$.

$$\cos 10{,}5\left(\frac{1}{\sin 10{,}5} - 1\right) \vee 0$$

$\cos 10{,}5\left(\frac{1-\sin 10{,}5}{\sin 10{,}5}\right) \vee 0$, выражение больше нуля, отсюда sin 10,5<cos 10,5<ctg 10,5<tg 10,5.

# Контрольная работа № 2

## Вариант 1

**1.** б) $y = \cos x$ на $\left[-\frac{2\pi}{3}; \frac{\pi}{2}\right]$

$$y_{max} = y(0) = 1$$

$$y_{min} = y\left(-\frac{2\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

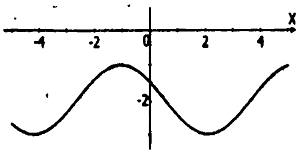
**2.** а) $\cos^2(\pi + \alpha) + \cos^2(\pi - \alpha) = 2\cos^2\alpha = 1 + \cos 2\alpha$;

б) $\dfrac{\sin\left(\frac{\pi}{2} - \alpha\right)\ \mathrm{tg}(-\alpha)}{\cos\left(\frac{\pi}{2} + \alpha\right)} = \dfrac{-\cos\alpha\ \ \mathrm{tg}\,\alpha}{-\sin\alpha} = 1.$

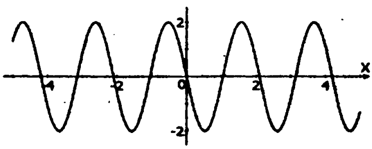
**3.** $y = \dfrac{\cos x}{x^4 - x^2 + 1}$

$y(-x) = \dfrac{\cos(-x)}{(-x)^4 - (-x)^2 + 1} = \dfrac{\cos x}{x^4 - x^2 + 1} = y(x)$ поэтому функция четная.

**4.**

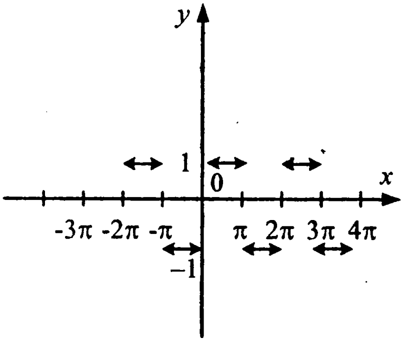



**5.**



**6.** $y = \dfrac{\sin x}{|\sin x|}$,

$$y = \begin{cases} 1, & \text{при } 2\pi k < x < \pi + 2\pi k,\ k \in \mathbb{Z} \\ -1, & \text{при } \pi + 2\pi k < x < 2\pi + 2\pi k,\ k \in \mathbb{Z} \end{cases}$$




## Вариант 2

**1.** б) $y = \sin x$ на $\left[-\dfrac{3\pi}{3}; -\dfrac{\pi}{6}\right]$

$$y_{max} = y(-\pi) = 0$$

$$y_{min} = y\left(-\frac{\pi}{2}\right) = -1$$

**2.** а) $\sin^2(\dfrac{\pi}{2} + \alpha) + \sin^2(\pi - \alpha) = \cos^2\alpha + \sin^2\alpha = 1;$

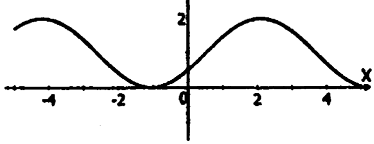
б) $\dfrac{\cos\left(\frac{\pi}{2} - \alpha\right)\ c\,tg(-\alpha)}{\sin\left(\frac{\pi}{2} - \alpha\right)} = -\dfrac{\sin\alpha}{\cos\alpha}\text{ctg}\alpha = -1.$

**3.** $y = \dfrac{\sin^3 x}{x^2 + 1}$,

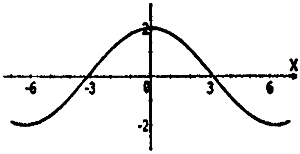
$y(-x) = \dfrac{\sin^3(-x)}{(-x)^2 + 1} = -\dfrac{\sin^3 x}{x^2 + 1} = -y(x)$ поэтому фун

нечетная.

**4.**

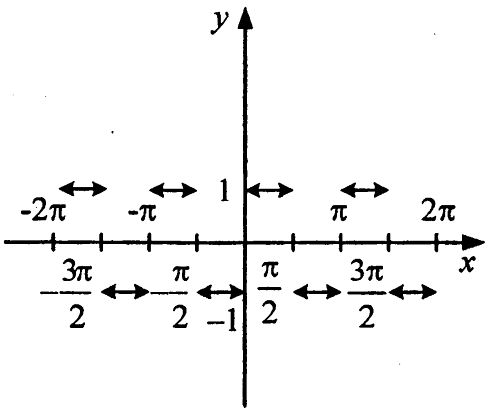



**5.**




**6.** $y = \dfrac{|\operatorname{tg} x|}{\operatorname{tg} x}$,

$$y = \begin{cases} 1, & 2\pi k < x < \dfrac{\pi}{2} + 2\pi k \text{ или} \\ & \pi + 2\pi k < x < \dfrac{3\pi}{2} + 2\pi k,\ k \in \mathbb{Z} \\ -1, & \dfrac{\pi}{2} + 2\pi k < x < \pi + 2\pi k,\ k \in \mathbb{Z} \\ & \dfrac{3\pi}{2} + 2\pi k < x < 2\pi + 2\pi k,\ k \in \mathbb{Z} \end{cases}$$



## Вариант 3

**1.** б) $y = \cos x$ на $\left[-\dfrac{\pi}{3}; \pi\right]$

$y_{\max} = y(0) = 1$

$y_{\min} = y(\pi) = -1$

**2.** а) $\sin^2(\pi + \alpha) - \sin^2(\pi - \alpha) = \sin^2\alpha - \sin^2\alpha = 0;$

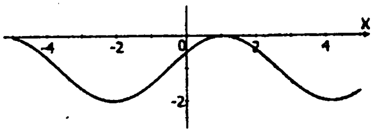
б) $\dfrac{\cos\left(\frac{\pi}{2} + \alpha\right)}{\sin\left(\pi - \alpha\right)\ \operatorname{tg}(-\alpha)} = \dfrac{-\sin\alpha\ \cos\alpha}{-\sin\alpha\ \sin\alpha} = \operatorname{ctg}\alpha.$

**3.** $y = \dfrac{\operatorname{tg} 5x}{3x^{16} - x^2 + 1}$

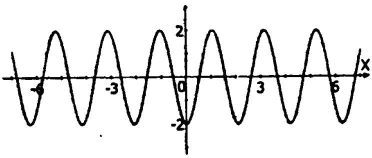
$$y(-x) = \frac{\operatorname{tg}(5(-x))}{3(-x)^{16} - (-x)^2 + 1} = -\frac{\operatorname{tg}(5x)}{3x^{16} - x^2 + 1} = -y(x)$$

функция нечетная.

**4.**

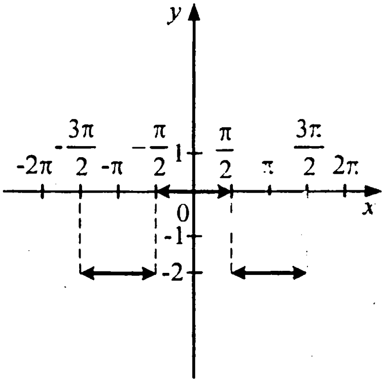



**5.**




**6.** $y = \dfrac{\cos x}{|\cos x|} - 1$

$$y = \begin{cases} 0, & -\dfrac{\pi}{2} + 2\pi k < x < \dfrac{\pi}{2} + 2\pi k,\ k \in \mathbb{Z} \\ -2, & \dfrac{\pi}{2} + 2\pi k < x < \dfrac{3\pi}{2} + 2\pi k,\ k \in \mathbb{Z} \end{cases}$$




## Вариант 4.

**1.** б) $y = \cos x$ на $\left[-\frac{2\pi}{3}; 0\right]$.

$$y_{max} = y(0) = 1$$
$$y_{min} = y\left(-\frac{\pi}{2}\right) = -1$$

**2.** а) $\cos^2(2\pi - \alpha) + \cos^2(\frac{3\pi}{2} + \alpha) = \cos^2\alpha + \sin^2\alpha = 1;$

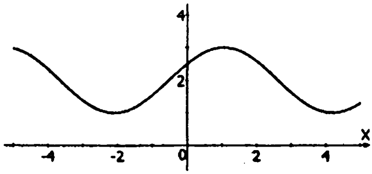
б) $$\frac{\cos\left(\frac{\pi}{2}+\alpha\right)\ \mathrm{ctg}(-\alpha)}{\sin\left(\frac{\pi}{2}-\alpha\right)} = \frac{\sin\alpha\ \mathrm{ctg}\,\alpha}{\cos\alpha} = 1.$$

**3.** $$y = \frac{\mathrm{ctg}^2\, x}{x^4 + 2x^2 + 2}$$

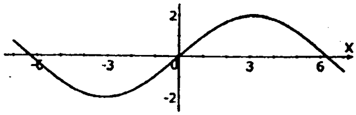
$$y(-x) = \frac{\mathrm{ctg}^2(-x)}{(-x)^4 + 2(-x)^2 + 2} = \frac{\mathrm{ctg}^2(x)}{x^4 + 2(x)^2 + 2} = y(x)$$

функция четная.

**4.**

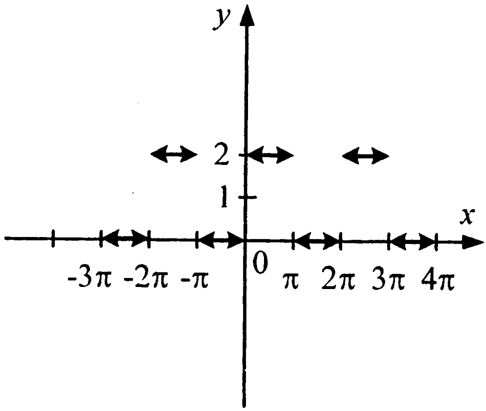


**5.**




**6.** $y = \dfrac{|\sin x|}{\sin x} + 1$

$$y = \begin{cases} 2, & 2\pi k < x < \pi + 2\pi k,\ k \in \mathbb{Z} \\ 0, & \pi + 2\pi k < x < 2\pi + 2\pi k,\ k \in \mathbb{Z} \end{cases}$$



# Контрольная работа № 3

## Вариант 1

**1.** $2\sin x + \sqrt{2} = 0;$

$\sin x = -\dfrac{\sqrt{2}}{2};$ $\quad x = (-1)^{k+1}\dfrac{\pi}{4} + 2\pi k.$

**2.** $\cos\left(\dfrac{x}{2} + \dfrac{\pi}{4}\right) + 1 = 0;$ $\quad \cos\left(\dfrac{x}{2} + \dfrac{\pi}{4}\right) = -1;$

$x = 2\pi - \dfrac{\pi}{2} + 4\pi n;$ $\quad x = \dfrac{3\pi}{2} + 4\pi n.$

**3.** $\cos(2\pi - x) - \sin\left(\dfrac{3\pi}{2} + x\right) = 1$

$\cos x + \cos x = 1$

$\cos x = -\dfrac{1}{2}$

$x = \pm\dfrac{2\pi}{4} + 2\pi k,\ k \in \mathbb{Z}$

**4.** $\sin x \cos x + 2\sin^2 x = \cos^2 x,$

$\sin x \neq 0;$ $\quad \operatorname{ctg}^2 x - \operatorname{ctg} x - 2 = 0;$

$\operatorname{ctg} x = 2;$ $\quad x = \operatorname{arcctg} 2 + \pi n$ или

$\operatorname{ctg} x = -1;$ $\quad x = \dfrac{3\pi}{4} + \pi n.$

**5.** $3\sin^2 x - 4\sin x \cos x + 5\cos^2 x = 2;$

$\cos x \neq 0;$ $\quad \operatorname{tg}^2 x - 4\operatorname{tg} x + 3 = 0;$

$\operatorname{tg} x = 3;$ $\quad x = \operatorname{arctg} 3 + \pi n$ или $\operatorname{tg} x = 1;$

$x = \dfrac{\pi}{4} + \pi k.$

**6.** $\sin 3x = \cos 3x;$ $[0,\ 4],$ $\cos 3x \neq 0;$

$\mathrm{tg} 3x = 1;$ $3x = \frac{\pi}{4} + \pi n;$ $x = \frac{\pi}{12} + \frac{\pi n}{3},$

значит, $x = \frac{\pi}{12};\ \frac{5\pi}{12};\ \frac{3\pi}{4};\ \frac{13\pi}{12}.$

## Вариант 2

**1** $2\cos x + \sqrt{3} = 0;$ $\cos x = -\frac{\sqrt{3}}{2};$

$x = \pm\frac{5\pi}{6} + 2\pi n.$

**2** $\sin(2x - \frac{\pi}{3}) + 1 = 0;$ $\cos\left(2x + \frac{\pi}{6}\right) = -1;$

$\sin(2x - \frac{\pi}{3}) = -1;$ $2x - \frac{\pi}{3} = -\frac{\pi}{2} + 2\pi n;$

$x = -\frac{\pi}{12} + \pi n.$

**3** $\sin(2\pi - x) - \cos\left(\frac{3\pi}{2} + x\right) + 1 = 0$

$-\sin x - \sin x + 1 = 0$

$2\sin x = 1$ $\sin x = \frac{1}{2}$ $x = (-1)^k \frac{\pi}{6} + \pi k,\ k \in \mathbb{Z}.$

**4.** $3\sin^2 x = 2\sin x \cos x + \cos^2 x,$

$\sin x \neq 0;$ $\mathrm{ctg}^2 x + 2\mathrm{ctg} x - 3 = 0;$

$\mathrm{ctg}\ x = -3;$ $x = -\mathrm{arcctg}\ 3 + \pi k$ или $\mathrm{ctg}\ x = 1;$

$x = \frac{\pi}{4} + \pi n.$

5. $5\sin^2 x - 25\sin x\cos x + \cos^2 x = 4,$

$\cos x \neq 0;$ $\quad tg^2 x - 2tg\, x - 3 = 0;$

$tg\, x = 3;$ $\quad x = arctg\, 3 + \pi n$ или $tg\, x = -1;$

$x = -\dfrac{\pi}{4} + \pi k.$

6. $\sin 2x = \sqrt{3}\cos 2x;$

$[-1; 6]$, $\cos 3x \neq 0$; $tg 2x = \sqrt{3}$;

$x = \dfrac{\pi}{6} + \dfrac{\pi n}{2} = \dfrac{\pi + 3\pi n}{6}$, следовательно,

$x = \dfrac{\pi}{6};$ $\quad x = \dfrac{2\pi}{3};$ $\quad x = \dfrac{7\pi}{6};$ $\quad x = \dfrac{5\pi}{3}.$

## Вариант 3

1. $2\sin x - 1 = 0;$ $\sin x = -\dfrac{1}{2};$ $\quad x = (-1)^k \dfrac{\pi}{6} + \pi k.$

2. $\cos(2x + \dfrac{\pi}{6}) + 1 = 0;$ $\quad \cos\left(2x + \dfrac{\pi}{6}\right) = -1;$

$2x + \dfrac{\pi}{6} = \pi + 2\pi n;$ $\quad x = \dfrac{5\pi}{12} + \pi n.$

3. $\sin\left(\dfrac{\pi}{2} + x\right) - \cos(\pi + x) + 1 = 0$

$\cos x + \cos x + 1 = 0$ $\quad 2\cos x = -1$

$\cos x = -\dfrac{1}{2}$ $\quad x = \pm\dfrac{2\pi}{3} + 2\pi k,\ k \in \mathbb{Z}.$

4. $3\sin^2 x - 4\cos x\sin x + \cos^2 x = 0,$

$\sin x \neq 0;$ $\quad ctg^2 x - 4ctg x + 3 = 0;$

$ctg\, x = 3;$ $\quad x = arcctg\, 3 + \pi n$ или

$ctg\, x = 1;$ $\quad x = \dfrac{\pi}{4} + \pi k.$

**5.** $\sin^2 x - 9\sin x\cos x + 3\cos^2 x = -1,$

$\cos x \neq 0;$ $2\operatorname{tg}^2 x - 9\operatorname{tg} x + 4 = 0;$

$\operatorname{tg} x = \frac{1}{2};$ $x = \operatorname{arctg}\frac{1}{2} + \pi n$ или

$\operatorname{tg} x = 4;$ $x = \operatorname{arctg} 4 + \pi n.$

**6.** $\sqrt{3}\sin 2x = \cos 2x;$ $[-1, 4],$

$\sin 2x \neq 0;$ $\operatorname{ctg} 2x = \sqrt{3};$

$x = \frac{\pi}{12} + \frac{\pi n}{2};$ $x = \frac{\pi}{12};$ $x = \frac{7\pi}{12};$ $x = \frac{13\pi}{12}.$

## Вариант 4

**1.** $2\cos x - \sqrt{2} = 0;$ $\cos x = \frac{\sqrt{2}}{2};$ $x = \pm\frac{\pi}{4} + 2\pi n.$

**2.** $\sin\left(\frac{x}{2} - \frac{\pi}{6}\right) = 1;$ $\frac{x}{2} - \frac{\pi}{6} = \frac{\pi}{2}$ $x = \frac{4\pi}{3} + 4\pi n$

**3.** $\cos\left(\frac{\pi}{2} + x\right) - \sin(\pi - x) = 1$

$-\sin x - \sin x = 1$

$\sin x = \frac{1}{2}$

$x = (-1)^k\frac{\pi}{6} + \pi k,\ k \in \mathbb{Z}.$

**4.** $6\sin^2 x = 5\sin x\cos x - \cos^2 x,$

$\sin x \neq 0;$ $\operatorname{ctg}^2 x - 5\operatorname{ctg} x + 6 = 0;$

$\operatorname{ctg} x = 2;$ $x = \operatorname{arcctg} 2 + \pi n$ или

$\operatorname{ctg} x = 3;$ $x = \operatorname{arcctg} 3 + \pi n.$

**5.** $5\sin^2 x + 2\sin x\cos x - \cos^2 x = 1,$

$\cos x \neq 0;$ $2\operatorname{ctg}^2 x + \operatorname{ctg} x - 1 = 0;$

$\mathrm{ctg}\, x = -1;$ $\quad x = -\dfrac{\pi}{4} + \pi n$ или

$\mathrm{ctg}\, x = \dfrac{1}{2};$ $\quad x = \mathrm{arcctg}\, \dfrac{1}{2} + \pi n.$

**6.** $\sin 3x + \cos 3x = 0;$ $\quad [0; 6],\ \cos 3x \neq 0;$

$\mathrm{tg} 3x = -1;$

$3x = -\dfrac{\pi}{4} + \pi n;$ $\quad x = -\dfrac{\pi}{12} + \dfrac{\pi n}{3} = \dfrac{\pi(4n-1)}{12};$

$x = \dfrac{\pi}{4};$ $\quad x = \dfrac{7\pi}{12};$ $\quad x = \dfrac{11\pi}{12};$

$x = \dfrac{5\pi}{4};$ $\quad x = \dfrac{19\pi}{12}.$

# Контрольная работа № 4

## Вариант 1

**1.** а) $\sin 58° \cos 13° - \cos 58° \sin 13° = \sin(58° - 13°) =$

$= \sin 45° = \frac{\sqrt{2}}{2}$;

б) $\cos\frac{\pi}{12}\cos\frac{7\pi}{12} - \sin\frac{\pi}{12}\sin\frac{7\pi}{12} = \cos\frac{8\pi}{12} = -\frac{1}{2}$.

**2.** а) $\cos(t - s) - \sin t \sin s = \cos t \cos s +$

$+ \sin t\ \mathrm{cins} - \sin t \sin s = \cos t \cos s$;

б) $\frac{1}{2}\cos\alpha - \sin(\frac{\pi}{6} + \alpha) =$

$= \frac{1}{2}\cos\alpha - \sin\frac{\pi}{6}\cos\alpha - \cos\frac{\pi}{6}\sin\alpha = -\frac{\sqrt{3}}{2}\sin\alpha$.

**3.** $\sin(\alpha + \beta) + \sin(\alpha - \beta) = \sin\alpha\cos\beta + \sin\alpha\cos\beta +$

$+ \sin\beta\cos\alpha - \sin\beta\cos\alpha = 2\sin\alpha\cos\beta$.

**4.** $\sin 3x \cos x + \cos 3x \sin x = 0$; $\sin 4x = 0$; $x = \frac{\pi n}{4}$.

**5.** $\sin\alpha = -\frac{12}{13}$; $\pi < \alpha < \frac{3\pi}{2}$;

$\cos\alpha = -\sqrt{1 - \sin^2\alpha} = -\sqrt{1 - \frac{144}{169}} = -\frac{5}{13}$;

$\mathrm{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{12}{13}}{-\frac{5}{13}} = \frac{12}{5}$;

$$\operatorname{tg}(\alpha + \frac{\pi}{4}) = \frac{1+\frac{12}{5}}{1-\frac{12}{5}} = -\frac{17}{5}\cdot\frac{5}{7} = -\frac{17}{7}.$$

**6.** $\cos(\frac{\pi}{4} + t) + \cos(\frac{\pi}{4} - t) = p;$

$$\cos^2(\frac{\pi}{4} + t) + 2\cos(\frac{\pi}{4} + t)\cos(\frac{\pi}{4} \cdot t) + \sin^2(\frac{\pi}{4} + t) = p^2;$$

$$\cos(\frac{\pi}{4} + \alpha)\cos(\frac{\pi}{4} - \alpha) = \frac{p^2-1}{2}.$$

## Вариант 2

**1.** а) $\sin\frac{\pi}{5}\cos\frac{3\pi}{10} + \cos\frac{\pi}{5}\sin\frac{3\pi}{10} = \sin\frac{\pi}{2} = 1;$

б) $\cos 78°\cos 108° + \sin 78°\sin 108° = \cos 30° = \frac{\sqrt{3}}{2}.$

**2.** а) $\sin(\alpha - \beta) + \cos\alpha\sin\beta = \sin\alpha\cos\beta - \sin\beta \text{cjs}\alpha +$
$+ \cos\alpha\sin\beta = \sin\alpha\cos\beta;$

б) $\frac{1}{2}\sin\alpha + \cos(\frac{\pi}{6} + \alpha) =$

$$= \frac{1}{2}\sin\alpha + \cos\frac{\pi}{6}\cos\alpha - \sin\frac{\pi}{6}\sin\alpha = \frac{\sqrt{3}}{2}\cos\alpha.$$

**3.** $\cos(\alpha + \beta) + \cos(\alpha - \beta) = \cos\alpha\cos\beta + \cos\alpha\cos\beta -$
$- \sin\alpha\sin\beta + \sin\alpha\sin\beta = 2\cos\alpha\cos\beta.$

**4.** $\cos 2x\cos x - \sin 2x\sin x = 0; \quad \cos 3x = 0; \ x = \frac{\pi}{6} + \frac{\pi n}{3}.$

**5.** $\cos\alpha = \frac{12}{13}; \qquad 0 < \alpha < \frac{\pi}{2};$

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{144}{169}} = \frac{5}{13};$$

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{5}{13}}{\frac{12}{13}} = \frac{5}{12};$$

$$\operatorname{tg}(\frac{\pi}{4} + \alpha) = \frac{1+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha} = \frac{17}{12}\cdot\frac{12}{7} = \frac{17}{7}.$$

**6.** $\sin(\frac{\pi}{3} + t) + \sin(\frac{\pi}{3} - t) = p;$

$$1 + 2\sin(\frac{\pi}{3} + t)\sin(\frac{\pi}{3} - t) = p^2;$$

$$\sin(\frac{\pi}{3} + t)\sin(\frac{\pi}{3} - t) = \frac{p^2-1}{2}.$$

## Вариант 3

**1.** а) $\sin 81^\circ\cos 21^\circ - \cos 81^\circ\sin 21^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2};$

б) $\cos\frac{5\pi}{8}\cos\frac{\pi}{8} - \sin\frac{5\pi}{8}\sin\frac{\pi}{8} = \cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}.$

**2.** а) $\cos x\cos y - \cos(x - y) = \cos x\cos y - \text{os}x\cos y -$

$- \sin x\sin y - \sin x\sin y;$

б) $\sin(\frac{\pi}{3} + \alpha) - \frac{\sqrt{3}}{2}\cos\alpha = \sin\frac{\pi}{3}\cos\alpha +$

$+ \sin\alpha\cos\frac{\pi}{3} - \frac{\sqrt{3}}{2}\cos\alpha = \text{in}\alpha\cos\frac{\pi}{3} = \frac{1}{2}\sin\alpha.$

**3.** $\sin(\alpha + \beta) - \sin(\alpha - \beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha -$

$- \sin\alpha\cos\beta + \sin\beta\cos\alpha = 2\sin\beta\cos\alpha.$

**4.** $\sin 5x\cos x - \cos 5x\sin x = 0;\quad \sin 4x = 0;\qquad x = \frac{\pi n}{4}.$

**5.** $\cos\alpha = -\frac{3}{5};\qquad \pi < \alpha < \frac{3\pi}{2};$

$\sin\alpha = -\sqrt{1-\cos^2\alpha} = -\sqrt{1-\frac{9}{25}} = -\frac{4}{5}$;

$\text{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{-\frac{4}{5}}{-\frac{3}{5}} = \frac{4}{3}$;

$\text{tg}(\frac{\pi}{4} - \alpha) = \frac{1-\text{tg}\,\alpha}{1+\text{tg}\,\alpha} = -\frac{1}{4}\cdot\frac{4}{7} = -\frac{1}{7}$.

**6.** $\cos(\frac{\pi}{6} + t) + \cos(\frac{\pi}{6} - t) = p$;

$1 + 2\cos(\frac{\pi}{6} + t)\cos(\frac{\pi}{6} - t) = p^2$;

$\cos(\frac{\pi}{6} + t)\cos(\frac{\pi}{6} - t) = \frac{p^2-1}{2}$.

## Вариант 4

**1.** а) $\sin\frac{5\pi}{14}\cos\frac{\pi}{7} + \cos\frac{5\pi}{14}\sin\frac{\pi}{7} = \sin\frac{\pi}{2} = 1$;

б) $\cos 78°\cos 18° + \sin 78°\sin 18° = \cos 60° = \frac{1}{2}$.

**2.** а) $\sin\alpha\cos\beta - \sin(\alpha - \beta) = \sin\alpha\cos\beta - \sin\alpha\cos\beta +$

$+ \sin\beta\cos\alpha = \sin\beta\cos\alpha$;

б) $\cos(\frac{\pi}{3} + x) + \frac{\sqrt{3}}{2}\sin x =$

$= \cos\frac{\pi}{3}\cos x - \sin\frac{\pi}{3}\sin x = \frac{1}{2}\cos x$.

**3.** $\cos(\alpha + \beta) - \cos(\alpha - \beta) = \cos\alpha\cos\beta - \cos\alpha\cos\beta -$

$- \sin\alpha\sin\beta - \sin\alpha\sin\beta = -2\sin\alpha\sin\beta$.

**4.** $\cos 4x\cos x + \sin 4x\sin x = 0$; $\cos 3x = 0$;

$x = \frac{\pi}{6} + \frac{\pi n}{3}$.

**5.** $\sin\alpha = \frac{4}{5}$; $\qquad \frac{\pi}{2} < \alpha < \pi$;

$$\cos\alpha = -\sqrt{1-\sin^2\alpha} = -\sqrt{1-\frac{16}{25}} = -\frac{3}{5};$$

$$\operatorname{tg}\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{4}{5}}{-\frac{3}{5}} = -\frac{4}{3};$$

$$\operatorname{tg}\left(\frac{\pi}{4} + \alpha\right) = \frac{1+\operatorname{tg}\alpha}{1-\operatorname{tg}\alpha} = -\frac{1}{3}\cdot\frac{3}{7} = -\frac{1}{7}.$$

**6.** $\sin\left(\frac{\pi}{6} + t\right) + \sin\left(\frac{\pi}{6} - t\right) = p$;

$$1 + 2\sin\left(\frac{\pi}{6} - t\right)\sin\left(\frac{\pi}{6} + t\right) = p^2;$$

$$\sin\left(\frac{\pi}{6} - t\right)\sin\left(\frac{\pi}{6} + t\right) = \frac{p^2-1}{2}.$$

# Контрольная работа № 5

## Вариант 1

**1.** $1 - \dfrac{\sin 2t \cos t}{2\sin t} = 1 - \dfrac{2\sin t \cdot \cos t \cdot \cos t}{2\sin t} = 1 - \cos^2 t = \sin^2 t.$

**2.** $\sin 5x = \sin 3x;$ $\sin x \cos 4x = 0;$

$\sin x = 0$ или $\cos 4x = 0;$

$x = \pi n$ или $x = \dfrac{\pi}{8} + \dfrac{\pi n}{4}.$

**3.** $2\cos^2(45° + 4\alpha) + \sin 8\alpha = 1 + \cos(\dfrac{\pi}{2} + 8\alpha) + \sin 8\alpha =$

$= 1 - \sin 8\alpha + \sin 8\alpha = 1.$

**4.** $\cos 70° + \sin 140° - \cos 10° =$

$= -2\sin 40° \sin 30° + \sin 40° = 0.$

**5.** $\sqrt{3}\sin x + \cos x = 1;$

$\dfrac{\sqrt{3}}{2}\sin x + \dfrac{1}{2}\cos x = \dfrac{1}{2};$ $\cos\dfrac{\pi}{6}\sin x + \sin\dfrac{\pi}{6}\cos x = \dfrac{1}{2};$

$\sin(x + \dfrac{\pi}{6}) = \dfrac{1}{2};$ $x = (-1)^k \dfrac{\pi}{6} - \dfrac{\pi}{6} + \pi k.$

**6.** $\sin 5x + \sin x + 2\sin^2 x = 1;$

$2\sin 3x \cos 2x - \cos 2x = 0;$

$\cos 2x(2\sin 3x - 1) = 0;$

$\cos 2x = 0;$ или $\sin 3x = \dfrac{1}{2};$

$x = \dfrac{\pi}{4} + \dfrac{\pi n}{2}$ или $x = (-1)^k \dfrac{\pi}{18} + \dfrac{\pi k}{3}.$

## Вариант 2

**1.** $\dfrac{\cos 2t}{\cos t + \sin t} - \cos t = \dfrac{\cos^2 t - \sin^2 t}{\cos t + \sin t} -$

$- \cos t \dfrac{(\cos t - \sin t)(\cos t + \sin t)}{\cos t + \sin t} = \cos t =$

$= \cos t - \sin t - \cos t = - \sin t.$

**2.** $\cos 8x = \cos 6x$; $\sin x \sin 7x = 0$;

$\sin x = 0$ или $\sin 7x = 0$; $\alpha = \pi n$, или $\alpha = \dfrac{\pi n}{7}$.

**3.** $2\sin^2(45° - 2t) + \sin 4t = 1 - \cos(\dfrac{\pi}{2} - 4t) + \sin 4t =$

$= 1 - \cos\dfrac{\pi}{2}\cos 4 - \sin\dfrac{\pi}{2}\sin 4t + + \sin 4t = 1.$

**4.** $\sin 72° + \cos 222° - \sin 12° =$

$= 2\sin 30°\cos 42° - \cos 42° = 0.$

**5.** $\sqrt{3}\sin x - \cos x = 1$; $\dfrac{\sqrt{3}}{2}\sin x - \dfrac{1}{2}\cos x = \dfrac{1}{2}$; ;

$\sin x \cos\dfrac{\pi}{6} - \sin\dfrac{\pi}{6}\cos x = \dfrac{1}{2}$

$\sin(x - \dfrac{\pi}{6}) = \dfrac{1}{2}$; $\qquad x = (-1)^k\dfrac{\pi}{6} + \dfrac{\pi}{6} + \pi k.$

**6.** $2\cos^2 3x + \cos 3x + \cos 9x = 1$;

$\cos 6x + 2\cos 3x \cos 6x = 0$;

$\cos 6x(1 + 2\cos 3x) = 0$; $\cos 6x = 0$ или

$\cos 3x = -\dfrac{1}{2}$;

$x = \dfrac{\pi}{12} + \dfrac{\pi n}{6}$ или $x = \pm\dfrac{2\pi}{9} + \dfrac{2\pi n}{3}$.

## Вариант 3

1. $\dfrac{\sin 2t \sin t}{2\cos t} = 1 - \dfrac{2\sin t\cos t \sin t}{2\cos t} = 1 - \sin^2 t = \cos^2 t.$

2. $\sin 7x = \sin 5x;$ $\quad\quad$ $\sin x \cos 6x = 0;$

$\sin x = 0;$ $\quad\quad$ $\cos 6x = 0;$

$x = \pi n$ $\quad$ или $\quad$ $x = \dfrac{\pi}{12} + \dfrac{\pi n}{6}.$

3. $2\cos^2(45° + 3\alpha) + \sin 6\alpha = 1;$

$1 - \cos(\dfrac{\pi}{2} + 6\alpha) + \sin 6\alpha =$

$= 1 - \cos\dfrac{\pi}{2}\cos 6\alpha - \sin\dfrac{\pi}{2}\sin 6\alpha + \sin 6\alpha = 1.$

4. $\cos 50° + \sin 160° - \cos 10° =$

$= -2\sin 20°\sin 30° + \sin 20° = 0.$

5. $\sin x + \sqrt{3}\cos x = 1;$

$\dfrac{1}{2}\sin x + \dfrac{\sqrt{3}}{2}\cos x = \dfrac{1}{2};\ \cos\dfrac{\pi}{3}\sin x + \sin\dfrac{\pi}{3}\cos x = \dfrac{1}{2}$

$\sin(x + \dfrac{\pi}{3}) = \dfrac{1}{2};$

$x = (-1)^k\dfrac{\pi}{6} - \dfrac{\pi}{3} + \pi k.$

6. $\sin 6x + \sin 2x + 2\sin^2 x = 1;$
$2\sin 4x \cos 2x - \cos 2x = 0;$
$\cos 2x(2\sin 4x - 1) = 0;$

$\cos 2x = 0$ $\quad$ или $\quad$ $\sin 4x = \dfrac{1}{2};$

$x = \dfrac{\pi}{4} + \dfrac{\pi n}{2}$ $\quad$ или $\quad$ $x = (-1)^k\dfrac{\pi}{24} + \dfrac{\pi k}{4}.$

## Вариант 4

**1.** $\dfrac{\cos 2t}{\cos t - \sin t} - \sin t =$

$= \dfrac{\cos^2 t - \sin^2 t}{\cos t - \sin t} - \sin t \dfrac{(\cos t - \sin t)(\cos t + \sin t)}{\cos t - \sin t} - \sin t =$

$= \cos t + \sin t - \sin t = \cos t.$

**2.** $\cos 6x = \cos 4x$; $\sin x \sin 5x = 0$;

$\sin x = 0$ или $\sin 5x = 0$; $x = \dfrac{\pi n}{5}$.

**3.** $2\sin^2(45° - 3t) + \sin 6t = 1 - \cos(\dfrac{\pi}{2} - 6t) + \sin 6t =$

$= 1 - \cos\dfrac{\pi}{2}\cos 6t - \sin\dfrac{\pi}{2}\sin 6t + \sin 6t = 1.$

**4.** $\sin 84° + \cos 234° - \sin 24° =$

$= 2\sin 30°\cos 54° - \cos 54° = 0.$

**5.** $\sin x - \sqrt{3}\cos x = 1$;

$\dfrac{1}{2}\sin x - \dfrac{\sqrt{3}}{2}\cos x = \dfrac{1}{2}$; $\cos\dfrac{\pi}{3}\sin x - \sin\dfrac{\pi}{3}\cos x = \dfrac{1}{2}$;

$\sin(x - \dfrac{\pi}{3}) = \dfrac{1}{2}$; $x = (-1)^k\dfrac{\pi}{6} + \dfrac{\pi}{3} + \pi k.$

**6.** $2\cos^2 2x + \cos 2x + \cos 6x = 1$;

$2\cos 2x \cos 4x + \cos 4x = 0$;

$\cos 4x(2\cos 2x + 1) = 0$;

$\cos 4x = 0$ или $\cos 2x = -\dfrac{1}{2}$;

$x = \dfrac{\pi}{8} + \dfrac{\pi n}{4}$ или $x = \pm\dfrac{\pi}{3} + \pi n.$

# Контрольная работа № 6

## Вариант 1

**1.** а) $y = x^5$; $\quad y' = 5x^4$;

б) $y = 3$; $\quad y' = 0$;

г) $y = 3 - 2x$; $\quad y' = -2$;

в) $y = \dfrac{4}{x}$; $\quad y' = -\dfrac{4}{x^2}$;

д) $y = 2\sqrt{x} + 3\sin x$; $\quad y' = \dfrac{1}{\sqrt{x}} + 3\cos x$.

**2.** $y = \dfrac{x^{10}}{10} - \dfrac{x^7}{7} + x\sqrt{3} - 2$, $x_0 = 1$

$$y' = x^9 - x^6 + \sqrt{3}$$

$$y'\big|_{x_0=1} = 1^9 - 1^6 + \sqrt{3} = \sqrt{3}$$

$$y' = \operatorname{tg}\alpha \Rightarrow \operatorname{tg}\alpha = \sqrt{3} \Rightarrow \alpha_1 = \frac{\pi}{3}.$$

**3.** $f(x) = 2\sin x + 3x^2 - 2\pi x + 3$;

$f'(x) = 2\cos x + 6x - 2\pi$; $f'(\frac{\pi}{3}) = 1 + 2\pi - 2\pi = 1$.

**4.** $s = t^5 - t^3$; $\quad t = 2$;

$s' = 5t^4 - 3t^2$; $\quad s'(2) = 80 - 12 = 68$.

**5.** $f(x) = 12x - x^3$; $\quad f'(x) = 12 - x^2 \le 0$;

$x \in (-\infty; -2\sqrt{3}] \cup [2\sqrt{3}; +\infty)$.

**6.** $f(x) = \cos 2x + x\sqrt{3}$; $\quad x \in [0; 4\pi]$;

$f'(x) = -2\sin 2x + \sqrt{3} = 0$;

$\sin 2x = \frac{\sqrt{3}}{2}$; $\quad x = (-1)^k \frac{\pi}{6} + \frac{\pi k}{2}$;

$x = \frac{\pi}{6}$; $\frac{\pi}{3}$; $\frac{7\pi}{6}$; $\frac{4\pi}{3}$;

$x = \frac{13\pi}{6}$; $\frac{7\pi}{3}$; $\frac{19\pi}{6}$; $\frac{10\pi}{3}$.

## Вариант 2

**1.** а) $y = x^4$; $\quad y' = 4x^3$;

б) $y = 4$; $\quad y' = 0$;

г) $y = 3x + 2$; $\quad y' = 3$;

в) $y = -\frac{3}{x}$; $\quad y' = \frac{3}{x^2}$;

д) $y = 2\cos x - 4\sqrt{x}$; $\quad y' = -2\sin x - \frac{2}{\sqrt{x}}$.

**2.** $y = \frac{x^{12}}{12} + \frac{x^3}{3} + x + 2$, $x_0 = -1$.

$y = x^{11} + x^2 + 1$

$y'\Big|_{x_0 = 1} = -1 + 1 + 1 = 1$

$\operatorname{tg}\alpha = 1 \Rightarrow \alpha = \frac{\pi}{4}$.

**3.** $f(x) = 1{,}5x^2 - \frac{\pi x}{2} + 5 - 4\cos x$;

$f'(x) = 3x - \frac{\pi}{2} + 4\sin x$;

$f'(\frac{\pi}{6}) = \frac{\pi}{2} - \frac{\pi}{2} + 2 = 2$.

**4.** $s = t^4 - 2t^2$; $\quad t = 3$;

$s = 4t^3 - 4t$; $\quad s'(3) = 96$;

**5.** $f(x) = 6x^2 - x^3$; $f'(x) = 12x - 3x^2 > 0$;

$x(4 - x) > 0$; $x \in (0; 4)$.

**6.** $f(x) = \sin 2x - x\sqrt{3}$; $x \in [0; 4\pi]$;

$f'(x) = 2\cos 2x - \sqrt{3} = 0$;

$\cos 2x = \dfrac{\sqrt{3}}{2}$;

$x = \pm\dfrac{\pi}{12} + \pi n$;

$x = \dfrac{\pi}{12}; \dfrac{11\pi}{12}; \dfrac{13\pi}{12}; \dfrac{23\pi}{12}$;

$x = \dfrac{25\pi}{12}; \dfrac{35\pi}{12}; \dfrac{37\pi}{12}; \dfrac{47\pi}{2}$.

## Вариант 3

**1.** а) $y = x^6$; $y' = 6x^5$;

б) $y = 2$; $y' = 0$;

г) $y = 3 - 5x$; $y' = -5$;

в) $y = \dfrac{5}{x}$; $y' = -\dfrac{5}{x^2}$;

д) $y = 8\sqrt{x} + 0{,}5\cos x$; $y' = \dfrac{4}{\sqrt{x}} - \dfrac{1}{2}\sin x$.

**2.** $y = \dfrac{x^8}{8} - \dfrac{x^5}{5} - x\sqrt{3} - 3$, $x_0 = 1$

$y' = x^7 - x^4 - \sqrt{3}$

$y'|_{x_0=1} = 1 - 1 - \sqrt{3} = -\sqrt{3}$

$\operatorname{tg}\alpha = -\sqrt{3} \Rightarrow \alpha = \dfrac{2\pi}{3}$.

**3.** $f(x) = 2\cos x - x^2 - \dfrac{\pi x}{3} + 5$;

$f'(x) = -2\sin x + 2x - \frac{\pi}{3}$;

$f'(\frac{\pi}{6}) = -1 + \frac{\pi}{3} - \frac{\pi}{3} = -1.$

**4.** $s = t^4 - t^2$ $\quad t = 3;$ $\quad s' = 4t^3 - 2t;$ $\quad s'(3) = 102;$

**5.** $f(x) = 81x - 3x^3;$ $\quad f'(x) = 81 - 9x^2 < 0;$

$x^2 > 9;$ $\quad x \in (-\infty; -3) \cup (3; +\infty).$

**6.** $f(x) = \cos 2x - x\sqrt{3}$; $\quad x \in [0; 4\pi];$

$f'(x) = -2\sin 2x - \sqrt{3} = 0;$

$\sin 2x = -\frac{\sqrt{3}}{2}$;

$x = (-1)^{k+1}\frac{\pi}{6} + \frac{\pi n}{2}$;

$x = \frac{2\pi}{3}; \frac{5\pi}{6}; \frac{5\pi}{3}; \frac{11\pi}{6}$; $x = \frac{8\pi}{3}; \frac{17\pi}{6}; \frac{11\pi}{3}; \frac{23\pi}{6}.$

## Вариант 4

**1.** а) $y = x^7;$ $\quad y' = 7x^6;$

б) $y = 5;$ $\quad y' = 0;$

г) $y = 4x + 5;$ $\quad y' = 4;$

в) $y = -\frac{6}{x}$; $\quad y' = \frac{6}{x^2}$;

д) $y = \sin x + \frac{\sqrt{x}}{2}$; $\quad y' = \cos x + \frac{1}{4\sqrt{x}}.$

**2.** $y = \frac{x^9}{9} + \frac{x^6}{6} - x + 3$, $x_0 = -1$

$y' = x^8 + x^5 - 1$

$y'\big|_{x_0=-1} = 1 - 1 - 1 = -1$

$\operatorname{tg} \alpha = -1 \Rightarrow \alpha = \frac{3\pi}{4}.$

**3.** $f(x) = 1{,}5x^2 + 6\sin x - \pi x + 4;$

$f'(x) = 3x + 6\cos x - \pi;$

$f'(\frac{\pi}{3}) = 3;$

**4.** $s = t^6 - 4t^4 \qquad t = 2;$

$s' = 6t^5 - 16t^3 = 2t^3(3t^2 - 8);$
$s'(2) = 64.$

**5.** $f(x) = 7{,}5x^2 - x^3 \qquad f'(x) = 15x - 3x^2 \geq 0;$

$x(3x - 15) \leq 0; \qquad x \in [0; 5].$

**6.** $f(x) = \sin 2x + x; \qquad x \in [0; 4\pi];$

$f'(x) = 2\cos 2x + 1 = 0; \quad \cos 2x = -\frac{1}{2};$

$x = \pm\frac{\pi}{3} + \pi n; \qquad x = \frac{\pi}{3}; \ \frac{2\pi}{3}; \ \frac{4\pi}{3}; \ \frac{5\pi}{3};$

$x = \frac{7\pi}{3}; \ \frac{8\pi}{3}; \ \frac{10\pi}{3}; \ \frac{11\pi}{3}.$

# Контрольная работа № 7

## Вариант 1

**1.** $y = x^3 - 3x^2 + 4;$

$y' = 3x^2 - 6x = 3x(x - 2);$

а) возрастает: $x \in (-\infty; 0] \cup [2; +\infty);$

убывает: $x \in [0; 2];$

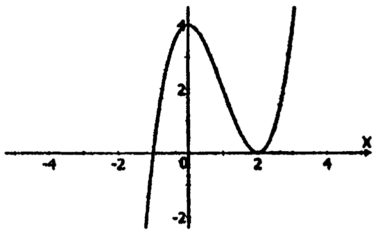
б) max: в точке $x = 0$, min в точке $x = 2;$

в) $y(-1) = -1 - 3 + 4 = 0;$

$y(4) = 64 - 48 + 4 = 20;$

$y_{max} = y(4) = 20;$ $\qquad$ $y_{min} = y(2) = y(-1) = 0.$

**2.**




**3.** $y = 4\sqrt{x};$ $\qquad$ $x = 4;$ $\qquad$ $y(4) = 8;$

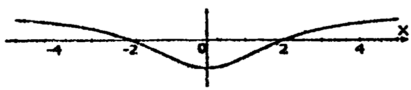
$y'(x) = \frac{2}{\sqrt{x}};$ $\qquad$ $y'(4) = 7;$

$y_{кас} = 8 + x - 4 = x + 4.$

**4.** Пусть $a$ и $b$ – стороны участка;

$\begin{cases} ab = 144 \\ a + b = y \end{cases}$; $\begin{cases} a + \frac{144}{a} = y \\ b = \frac{144}{a} \end{cases}$; $y' = 1 - \frac{144}{a^2} = 0$; $\begin{cases} a = 12 \\ b = 12 \end{cases}$.

**5.**



## Вариант 2

1. $y = 0{,}5x^4 - 4x^2$;

2. $y' = 2x^3 - 8x = 2x(x^2 - 4) = 2x(x - 4)(x + 4)$;

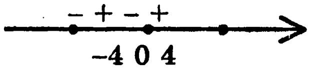
   а) возрастает: $x \in [-4;\ 0] \cup [4;\ +\infty)$;

   убывает: $x \in (-\infty;\ -4] \cup [0;\ 4]$;

   б) max: в точке $x = 0$, min в точках $x = -4$, $x = 4$;

   в) $y(0) = 0$;

   $y(-4) = y(4) = 64$;

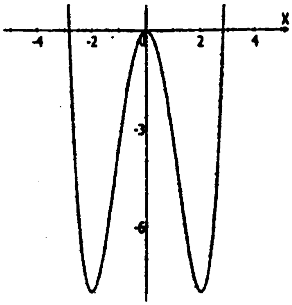
   
   

   $y(-1) = 0{,}5 - 4 = -3{,}5$;

   $y_{max} = y(4) = = y(-4) = 64$;

   $y_{min} = y(-1) = -3{,}5$.

3.




4. $y = \dfrac{6}{x}$; $x = 3$; $y(3) = 2$; $y' = -\dfrac{6}{x^2}$;

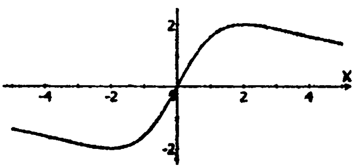
   $y'(3) = -\dfrac{2}{3}$; $y_{кас} = 3 - \dfrac{2}{3}(x - 3)\ -\dfrac{2}{3}x + 5.$

5. Пусть $a$ и $b$ – катеты треугольника;

   $\begin{cases} ab = 12 \\ a^2 + b^2 = y \end{cases}$; $\begin{cases} a = \frac{12}{b} \\ y = \frac{144}{b^2} + b^2 \end{cases}$; $y' = -\frac{28b}{b^3} + 2b = 0$, $b^4 = 144$; $\begin{cases} b = 2\sqrt{3} \\ a = 2\sqrt{3} \end{cases}$;

   $S = a^2 + b^2 = 12 + 12 = 24см^2$.

6.



## Вариант 3

**1.** $y = x^3 + 3x^2 - 4;$

$y' = 3x^2 + 6x = 3x(x + 2);$

а) возрастает: $x \in (-\infty; -2] \cup [0; +\infty);$

убывает: $x \in [-2; 0];$

б) max: в точке $x = -2$, min в точке $x = 0;$

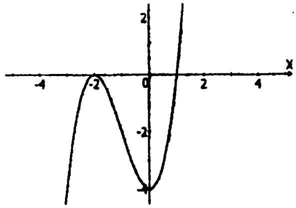
в) $y(-2) = 0;$ $y(0) = -4;$

$y(-4) = -20;$ $y(1) = 0;$

$y_{max} = y(-2) = y(1) = 0;$

$y_{min} = y(-4) = -20.$

**2.**



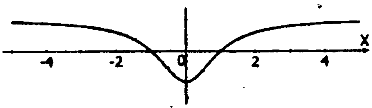

**3.** $y = 2\sqrt{x}$ ; $x = 1;$ $y(1) = 2;$ $y'(x) = \dfrac{1}{\sqrt{x}};$

$y'(1) = 1;$ $y_{max} = 2 + x - 1 = x + 1.$

**4.** Пусть $a$ и $b$ – катеты треугольника;

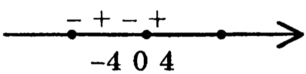
$$\begin{cases} ab = 16 \\ a^2 + b^2 = y \end{cases}; \quad \begin{cases} a = \dfrac{16}{b} \\ \dfrac{256}{b^2} + b^2 = y \end{cases}; \quad \begin{matrix} y' = 2b - \dfrac{512}{b^3} = 0 \\ b^4 = 256 \end{matrix}; \quad \begin{cases} b=4 \\ a=4 \end{cases}.$$

**5.**




## Вариант 4

**1.** $y = 0{,}25x^4 - 2x^2;$




$y' = x^3 - 4x = x(x^2 - 4) = x(x - 4)(x + 4);$

а) возрастает: $x \in [-4; 0] \cup [4; +\infty)$;

убывает: $x \in (-\infty; -4] \cup [0; 4]$;

б) min: в точке $x = 0$, max в точках $x = -4$, $x = 4$;

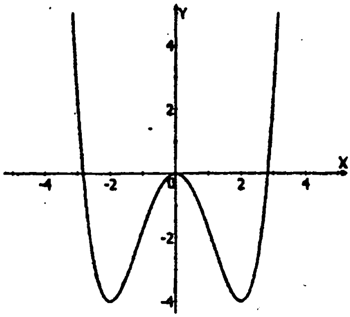
в) $y(-4) = y(4) = 32$; $y(0) = 0$;

$y(1) = -\frac{7}{4}$; $y(-3) = 2\frac{1}{4}$;

$y_{max} = y(-4) = y(4) = 32$;

$y_{min} = y(1) = -\frac{7}{4}$.

2.




3. $y = \frac{9}{x}$; $x = 3$;

$y(3) = 3$; $y' = -\frac{9}{x^2}$;

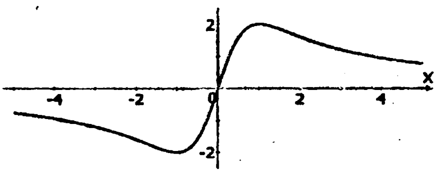
$y'(3) = -1$; $y_{кас} = 3 - x + 3 = 6 - x$.

4. Пусть стороны основания равны $a$, а высота равна $b$.

$\begin{cases} 2a + b = 36 \\ a^2 b = y \end{cases}$; $\begin{cases} b = 36 - 2a \\ 36a^2 - 2a^3 = y \end{cases}$; $\begin{matrix} y' = 72a - 6a^2 = 0 \\ 6a(12 - a) = 0 \end{matrix}$; $\begin{cases} a = 12 \\ b = 12 \end{cases}$.

$V = a^2 в = 12 \cdot 12 \cdot 12 = 1728$ см$^3$.

5.



# Контрольная работа № 8

## Вариант 1

**1.** $F(x) = x^4 - 3\sin x;$

$F'(x) = 4x^3 - 3\cos x = f(x).$

**2.** $\int(\frac{4}{x^2} + 3\sin x)dx = -\frac{4}{x} - 3\cos x.$

**3.** а) $\int_1^4 \frac{dx}{\sqrt{x}} = 2\sqrt{x}\Big|_1^4 = 4 - 2 = 2;$

б) $\int_0^{\pi/4} \cos 2x\, dx = \frac{1}{2}\sin 2x\Big|_0^{\pi/4} = \frac{1}{2}.$

**4.** $y = 1 - x^3;$ $y = 0;$ $x = -1;$

$\begin{cases} y = 1 - x^3 \\ y = 0 \end{cases};$ $\begin{cases} x=1 \\ y = 0 \end{cases};$

$S = \int_{-1}^{1}(1 - x^3)dx = \left(x - \frac{x^4}{4}\right)\Big|_{-1}^{1} = \frac{3}{4} + \frac{5}{4} = 2.$

**5.** $y = 0{,}5x^2 + 2;$ $x = 0;$ $x_0 = -2;$ $y(-2) = 4;$

$y' = x;$ $y'(-2) = -2;$

$y = 4 - 2(x + 2) = -2x;$

$\begin{cases} y = -2x \\ y = 0{,}5x^2 + 2 \end{cases};$ $\begin{matrix} x^2 + 4x + 4 = 0 \\ x = -2 \end{matrix};$

$S = \int_{-2}^{0}(0{,}5x^2 + 2)dx - 2 \cdot 4 \cdot \frac{1}{2} = \left(\frac{x^3}{6} + 2x\right)\Big|_{-2}^{0} - 4 =$

$= -4 + \frac{8}{6} + 4 = \frac{4}{3}.$

6. $y = \dfrac{\sqrt{3}}{\cos^2 x} + \sin 3x + \dfrac{1}{\pi}$;

$Y = \sqrt{3}\,\mathrm{tg}\,x - \dfrac{1}{3}\cos 3x + \dfrac{x}{\pi} + c$;

$-1 = -\dfrac{1}{3} + c$; $\qquad c = -\dfrac{2}{3}$;

$Y = \sqrt{3}\,\mathrm{tg}\,x - \dfrac{\cos 3x}{3} + \dfrac{x}{\pi} - \dfrac{2}{3}$;

$Y(\dfrac{\pi}{6}) = 1 - \dfrac{1}{6} - \dfrac{2}{3} = \dfrac{1}{6}$.

## Вариант 2

1. $F(x) = x^5 + \cos x$; $\qquad F'(x) = 5x^4 - \sin x = f(x)$.

2. $\int (\dfrac{1}{x^2} - 2\cos x)dx = -\dfrac{1}{x} - 2\sin x$.

3. а) $\int_0^1 x^7 dx = \dfrac{x^7}{8}\Big|_0^1 = \dfrac{1}{8}$;

б) $\int_{\pi/2}^{3\pi/2} \sin\dfrac{x}{2}\,dx = -2\cos\dfrac{x}{2}\Big|_{\pi/2}^{3\pi/2} = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$.

4. $y = 2 - x^2$;

$y = 0$; $x = -1$; $x = 0$;

$S = \int_{-1}^{0} (2 - x^2)dx = \left(2x - \dfrac{x^3}{3}\right)\Big|_{-1}^{0} = 2 - \dfrac{1}{3} = \dfrac{5}{3}$.

5. $y = x^3 + 2$; $\qquad x_0 = 1$; $\qquad x = 0$;

$y(1) = 3$; $\qquad y' = 3x^2$;

$y'(1) = 3$; $\qquad y = 3 + 3(x - 1) = 3x$;

$S = \int_0^1 (x^3 + 2)dx - \dfrac{3}{2} = -\dfrac{3}{2} + \left(\dfrac{x^4}{4} + 2x\right)\Big|_0^1 = -\dfrac{3}{2} + \dfrac{9}{4} = \dfrac{3}{4}$.

6. $y = \frac{3}{\sin^2 x} + \cos 2x - \frac{2}{\pi}$;

$Y = -3\text{ctg}\, x + \frac{1}{2}\sin 2x + -\frac{2x}{\pi} + c$;

$0 = -1 + c$; $c = 1$;

$Y = -3\text{ctg}\, x + \frac{1}{2}\sin 2x + -\frac{2x}{\pi} + 1$;

$Y(\frac{\pi}{4}) = -3 + \frac{1}{2} - \frac{1}{2} + 1 = -2$.

## Вариант 3

1. $F(x) = x^3 - 2\sin x$;

$F'(x) = 3x^2 - 2\cos x = f(x)$.

2. $\int (\frac{3}{x^2} + 5\cos x)dx = -\frac{3}{x} + 5\sin x$.

3. а) $\int_{0,25}^{2,25} \frac{dx}{\sqrt{x}} = 2\sqrt{x}\Big|_{0,25}^{2,25} = 3 - 1 = 2$;

б) $\int_0^{\pi/2} \sin 2x\, dx = -\frac{1}{2}\cos 2x\Big|_0^{\pi/2} = \frac{1}{2} - \frac{1}{2} = 0$.

4. $y = 2 - x^3$; $y = 0$; $x = 1$;

$x = 0$; $S = \int_0^1 (2 - x^3)dx = \left(2x - \frac{x^4}{4}\right)\Big|_0^1 = \frac{7}{8}$.

5. $y = 1{,}5x^2 + 3$; $x_0 = 2$; $x = 0$; $y(x_0) = 9$; $y' = 3x$;

$y'(2) = 6$; $y = 9 + 6(x - 2) = 6x - 3$;

$S = \int_0^2 (1{,}5x^2 + 3)dx - \int_0^2 (6x - 3)dx =$

$= \left(\frac{x^3}{2} + 3x - 3x^2 + 3x\right)\Big|_0^2 = \left(\frac{x^3}{3} - 3x^2 + 6x\right)\Big|_0^2 =$

$= \frac{8}{3} - 12 + 12 = \frac{8}{3}$.

**6.** $y = 12\cos 4x + \frac{8}{\pi} - \frac{1}{\sin^2 x}$;

$Y = 3\sin 4x + \frac{8x}{\pi} + \text{ctg } x + c$;

$0 = 2 + 1 + c$; $c = -3$;

$Y = 3\sin 4x + \frac{8x}{\pi} + \text{ctg } x - 3$;

$Y(\frac{\pi}{2}) = 4 - 3 = 1.$

## Вариант 4

**1.** $F(x) = x^6 - 2\cos x$;

$F'(x) = 6x^5 + 2\sin x = f(x).$

**2.** $\int(\frac{5}{x^2} - 4\sin x)dx = -\frac{5}{x} + 4\cos x.$

**3.** а) $\int_0^1 x^{10}dx = \frac{x^{11}}{11}\Big|_0^1 = \frac{1}{11}$;

б) $\int_{\pi/2}^{3\pi/2} \cos\frac{x}{2}\,dx = 2\sin\frac{x}{2}\Big|_{\pi/2}^{3\pi/2} = \sqrt{2} - \sqrt{2} = 0.$

**4.** $y = 1 - x^2$;

$y = 0$;

$S = 2\int_0^1(1 - x^2)dx = \left(2x - \frac{2x^3}{3}\right)\Big|_0^1 = 1\frac{1}{3}.$

**5.** $y = x^3 - 3$; $x_0 = 1$; $x = 0$; $y(1) = -2$; $y' = 3x^2$;

$y'(x_0) = 3$; $y = -2 + 3(x - 1) = 3x - 5$;

$S = \left|\int_0^1(3x - 5)dx - \int_0^1(x^3 - 3)dx\right| =$

$$= \left|\left(\frac{3x^2}{2} - 5x - \frac{x^4}{4} + 3x\right)\Big|_0^1\right| =$$

$$= \left|\left(\frac{3x^2}{2} - 2x - \frac{x^4}{4}\right)\Big|_0^1\right| = \left|\frac{3}{2} - 2 - \frac{1}{4}\right| = \frac{3}{4}.$$

**6.** $y = 3\sin 3x + \dfrac{6}{\pi} - \dfrac{\sqrt{3}}{\cos^2 x}$;

$Y = -\cos 3x + \dfrac{6x}{\pi} - \sqrt{3}\,\mathrm{tg}\,x + c$; $5 = -1 + c$; $c = 6$;

$Y = -\cos 3x + \dfrac{6x}{\pi} - \sqrt{3}\,\mathrm{tg}\,x + 6$;

$Y(\dfrac{\pi}{6}) = 1 - 1 + 6 = 6.$

# Контрольная работа № 9

## Вариант 1

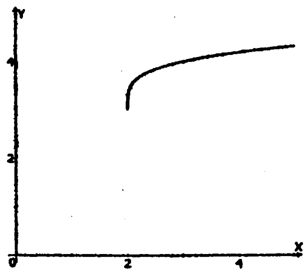
**1.** а) $\sqrt{\frac{1}{9}}+\sqrt[3]{-2\frac{10}{27}}+\sqrt[4]{256}=\frac{1}{3}-\frac{4}{3}+4=3;$

б) $\sqrt[6]{3^7 4^5}\cdot\sqrt[6]{3^5 4}=3^2\cdot 4=36.$

**2.** $\left(\sqrt[4]{x}-2\sqrt[4]{y}\right)\left(\sqrt[4]{x}+2\sqrt[4]{y}\right)+\dfrac{2\sqrt[8]{y^7}}{\sqrt[8]{y^3}}=$

$=\sqrt{x}-4\sqrt{y}+2\sqrt{y}=\sqrt{x}-2\sqrt{y}.$

**3.**




возрастает при $x\geq 2$, $x<2$ не определена. $\min y=y(2)=3$.

**4.** $\sqrt[3]{x}=10-x$ указывает, что $x=8$, т.к. $\sqrt[3]{x}$ – возрастает, а $y=10-x$ убывает, то существует только одно пересечение, т.е. 1 корень, значит, других корней нет.

**5.** $\sqrt[3]{243m^5}+\sqrt[4]{16m^4}-\sqrt{36m^2}$, $m=-\frac{1}{7}$;

$3m+2|m|-6|m|=-\frac{3}{7}+\frac{2}{7}-\frac{6}{7}=-1.$

**6.** $\sqrt[3]{32x^2}+\sqrt[3]{16x}=4$; $\sqrt[3]{2x}=y$; $y^2+y-2=0$;

$y=-2$; $x=-4$ или $y=1$; $x=\frac{1}{2}$.

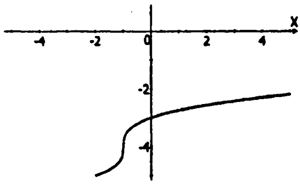
## Вариант 2

1. а) $\sqrt{0,64}+\sqrt[3]{-15\frac{5}{8}+\sqrt[4]{81}}=0,8-\frac{5}{2}+3=1,3;$

б) $\sqrt[5]{2^3\cdot 7^2}\cdot\sqrt[5]{2^{12}\cdot 7^3}=2^3\cdot 7=56.$

2. $\left(2\sqrt[6]{a}-\sqrt[6]{b}\right)^2+4\sqrt[12]{a^7b^8}:\sqrt[12]{a^5b^6}=$

$4\sqrt[3]{a}+\sqrt[3]{b}-4\sqrt[6]{ab}+4\sqrt[6]{ab}=4\sqrt[3]{a}+\sqrt[3]{b}.$

3. 


возрастает на R.

4. $\sqrt[4]{x}=3-2x$; $x=1.$

Аналогично с вариантом 1 № 4 один корень.

5. $\sqrt[4]{625c^{-4}}-\sqrt[5]{32c^{-5}}+\sqrt{36c^{-2}}$, $c=-\frac{1}{13}$;

$5|c|-2c+6|c|=\frac{11}{13}+\frac{2}{13}=1.$

6. $12-\sqrt[3]{16y}=\sqrt[3]{32y^2}$; $\sqrt[3]{4y^2}+\sqrt[3]{2y}-6=0;$

$\sqrt[3]{2y}=x$; $x^2+x-6=0$; $x=-3$; $2y=-27;$

$y=-\frac{27}{2}$ или $x=2;$

$2y=8;$ $y=4.$

## Вариант 3

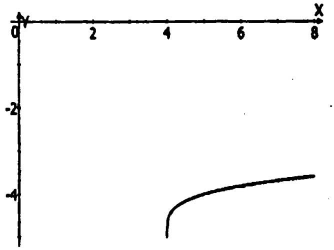
1. а) $\sqrt{\frac{1}{16}}+\sqrt[3]{-1\frac{61}{64}+\sqrt[4]{625}}=\frac{1}{4}+5-\frac{5}{4}=4;$

б) $\sqrt[8]{5^9\cdot 9^7}\cdot\sqrt[8]{5^7\cdot 9}=5^2\cdot 9=225.$

**2.** $\left(3\sqrt[6]{a}+\sqrt[6]{b}\right)\left(3\sqrt[6]{a}-\sqrt[6]{b}\right)+\sqrt[9]{5b^8}:\sqrt[9]{5b^5}=$

$=9\sqrt[3]{a}-\sqrt[3]{b}+\sqrt[3]{b}=9\sqrt[3]{a}$.

**3.**




возрастает, $x \geq 4$, $x < 4$ не определена.

**4.** $\sqrt[3]{x}=-x-2$; $x=-1$

Аналогично с вариантом 1 № 4 один корень.

**5.** $\sqrt[5]{1024x^5}+\sqrt[4]{81x^4}-\sqrt{81x^2}, x=-0{,}1;$

$5x+3|x|-3|x|=-0{,}5.$

**6.** $\sqrt[5]{128x^4}+\sqrt[5]{64x^2}=4;$ $\sqrt[5]{2x^2}=y;$

$2y^2+y-4=0;$ $y^2+y-2=0;$

$y=-2;$ $2x^2=-32$ – решений нет;

$y=1;$ $2x^2=1;$

$x=\pm\dfrac{\sqrt{2}}{2}.$

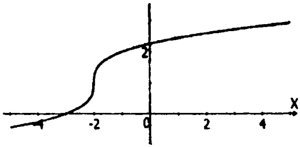
## Вариант 4

**1.** а) $\sqrt{0{,}81}+\sqrt[3]{-4\dfrac{12}{125}}+\sqrt[4]{16}=0{,}9-\dfrac{8}{5}+2=1{,}3;$

б) $\sqrt[4]{3^5\cdot7^3}\cdot\sqrt[4]{3^3\cdot7}=3^2\cdot7=63;$

**2.** $\left(\sqrt[4]{x}+3\sqrt[4]{y}\right)^2+6\sqrt[5]{x^5y^7}:\sqrt[5]{x^3y^5}=$

$\sqrt{x}+9\sqrt{y}+6\sqrt[4]{xy}-6\sqrt[4]{xy}=\sqrt{x}+9\sqrt{y}.$

**3.** 


возрастает на R.

**4.** $\sqrt[4]{x} = 4 - 3x$ ; $x = 1$

Аналогично с вариантом 1 № 4 один корень.

**5.** $\sqrt[4]{81y^4} - \sqrt[5]{32y^5} + \sqrt{16y^2}$, $\quad y=-\dfrac{2}{9}$;

$$3|y| - 2y + 4|y| = \frac{14}{9} + \frac{4}{9} = 2.$$

**6.** $4 + \sqrt[5]{64y^2} = \sqrt[5]{128y^4}$ ;

$\sqrt[5]{2y^2} = x \geq 0$; $\qquad x^2 - x - 2 = 0$;

$x = 2$; $\qquad y = \pm 4$; $\qquad x = -1$ – решений нет.

# Контрольная работа № 10

## Вариант 1

**1.** а) $2^{-3} = \frac{1}{8}$;  в) $32^{\frac{1}{5}} - 81^{\frac{1}{4}} = 2 - 3 = 1$;

б) $\left(\frac{2}{5}\right)^{-1} = \frac{5}{2}$;  г) $\left(2^{\frac{5}{3}} - 1\right)\left(2^{\frac{10}{3}} + 2^{\frac{5}{3}} + 1\right) = 2^5 - 1 = 31.$

**2.** в) $\left(x^{-\frac{7}{2}}y^{\frac{1}{6}}\right) : \left(x^{-\frac{11}{4}}y^{\frac{2}{3}}\right) = \frac{x^{-\frac{7}{2}}y^{\frac{1}{6}}}{x^{-\frac{11}{4}}y^{\frac{2}{3}}} = x^{\left(-\frac{7}{2}+\frac{11}{4}\right)} \cdot y^{\left(\frac{1}{6}-\frac{2}{3}\right)} =$

$= x^{-\frac{3}{4}} \cdot y^{-\frac{1}{2}}.$

**3.** $\frac{ab^{\frac{1}{4}} + a^{\frac{1}{3}}b}{a^{\frac{2}{3}} + b^{\frac{3}{4}}} = \frac{a^{\frac{1}{3}}b^{\frac{1}{4}}\left(a^{\frac{2}{3}} + b^{\frac{3}{4}}\right)}{\left(a^{\frac{2}{3}} + b^{\frac{3}{4}}\right)} = a^{\frac{1}{3}}b^{\frac{1}{4}}$. При $a = 125$,

$b = 81$ выражение равно: $(125)^{\frac{1}{3}}(81)^{\frac{1}{4}} = 5 \cdot 3 = 15$.

**4.** $y = x^{-\frac{1}{2}}$;  $x = 1$;  $x = 4$;

$y = 0$;  $S = \int_1^4 x^{-\frac{1}{2}}dx = 2\sqrt{x}\Big|_1^4 = 4 - 2 = 2.$

**5.** $\left(\frac{b^{\frac{1}{2}} + 3}{b^{\frac{3}{2}} - 3b} - \frac{b^{\frac{1}{2}} - 3}{b^{\frac{3}{2}} + 3b}\right)\frac{b - 9}{b^{\frac{1}{2}}} =$

$= \frac{\left(b^{\frac{1}{2}} + 3\right)^2 - \left(b^{\frac{1}{2}} - 3\right)^2}{b(b - 9)} \cdot \frac{b - 9}{b^{\frac{1}{2}}} = \frac{12b^{\frac{1}{2}}}{b \cdot b^{\frac{1}{2}}} = \frac{12}{b}.$

## Вариант 2

**1.** а) $4^{-3}=\frac{1}{64}$; в) $16^{\frac{1}{2}}-125^{\frac{1}{3}}=2-5=-3$;

б) $\left(\frac{3}{7}\right)^{-1}=\frac{7}{3}$; г) $\left(2+3^{\frac{2}{3}}\right)\left(4-2\cdot 3^{\frac{2}{3}}+4^{\frac{2}{3}}\right)=8+3^2=17.$

**2.** в) $\left(a^{\frac{3}{4}}b^{-\frac{11}{3}}\right):\left(a^{\frac{7}{8}}b^{\frac{5}{6}}\right)=a^{\left(\frac{3}{4}-\frac{7}{8}\right)}b^{\left(-\frac{11}{3}-\frac{5}{6}\right)}=a^{-\frac{1}{8}}b^{-\frac{27}{6}}$.

**3.** $\dfrac{a^{\frac{5}{3}}-a^{\frac{2}{3}}b^{\frac{1}{2}}}{ab^{\frac{1}{4}}-b^{\frac{3}{4}}}=\dfrac{a^{\frac{2}{3}}\left(a-b^{\frac{1}{2}}\right)}{b^{\frac{1}{4}}\left(a-b^{\frac{1}{2}}\right)}=\dfrac{a^{\frac{2}{3}}}{b^{\frac{1}{4}}}$; при $a=27$ и $b=81$:

$\dfrac{(27)^{\frac{2}{3}}}{(81)^{\frac{1}{4}}}=\dfrac{9}{3}=3$.

**4.** $y=\dfrac{1}{x^6}$; $x=1$; $x=2$; $y=0$;

$S=\int_1^2\dfrac{1}{x^6}dx=-\dfrac{1}{5x^5}\Big|_1^2=\dfrac{1}{160}+\dfrac{1}{5}=\dfrac{31}{160}$.

**5.** $\left(\dfrac{3}{a-3a^{\frac{1}{2}}}-\dfrac{a^{\frac{3}{2}}}{a^2-9a}\right):\dfrac{a^{\frac{1}{2}}}{a^{\frac{1}{2}}+3}=$

$=\dfrac{3a+9a^{\frac{1}{2}}-a^{\frac{3}{2}}}{a^2-9a}\cdot\dfrac{a^{\frac{1}{2}}+3}{a^{\frac{1}{2}}}=\dfrac{(a^{\frac{1}{2}}+3)(3a^{\frac{1}{2}}-a+9)}{a^2-9a}=$

$=\dfrac{(a^{\frac{1}{2}}+3)(3a^{\frac{1}{2}}-a+9)}{(a-3a^{\frac{1}{2}})a^{\frac{1}{2}}(a^{\frac{1}{2}}+3)}=\dfrac{3a^{\frac{1}{2}}-a+9}{a^{\frac{1}{2}}-3a}$

## Вариант 3

**1.** а) $3^{-2}=\dfrac{1}{9}$; б) $\left(\dfrac{1}{4}\right)^{-1}=4$;

в) $64^{\frac{1}{3}}-49^{\frac{1}{2}}=4-7=3$;

г) $\left(3^{\frac{1}{3}}+2^{\frac{2}{3}}\right)\left(3^{\frac{2}{3}}-3^{\frac{1}{3}}\cdot 2^{\frac{2}{3}}+2^{\frac{4}{3}}\right)=4+3=7.$

**2.** в) $\left(x^{\frac{2}{3}}y^{-\frac{2}{5}}\right):\left(x^{\frac{11}{12}}y^{-\frac{14}{15}}\right)=x^{\left(\frac{2}{3}-\frac{11}{12}\right)}\cdot y^{\left(-\frac{2}{5}+\frac{14}{15}\right)}=x^{-\frac{1}{4}}y^{\frac{8}{15}}.$

**3.** $\dfrac{ab^{\frac{1}{3}}+a^{\frac{3}{4}}b^{\frac{2}{3}}}{a^{\frac{1}{4}}+b^{\frac{1}{3}}}=\dfrac{a^{\frac{3}{4}}b^{\frac{1}{3}}\left(a^{\frac{1}{4}}+b^{\frac{1}{3}}\right)}{\left(a^{\frac{1}{4}}+b^{\frac{1}{3}}\right)}=a^{\frac{3}{4}}b^{\frac{1}{3}}$, при $a=16$,

$b=125$: $16^{\frac{3}{4}}\cdot 125^{\frac{1}{3}}=8\cdot 5=40$.

**4.** $y=x^{-\frac{1}{3}}$; $x=1$; $x=8$; $y=0$;

$S=\int_1^8\left(x^{-\frac{1}{3}}\right)dx=\dfrac{3}{2}x^{\frac{2}{3}}\Big|_1^8=6-\dfrac{3}{2}=4{,}5.$

**5.** $\left(\dfrac{a^{\frac{1}{2}}+4}{a^{\frac{3}{2}}-4a}-\dfrac{a^{\frac{1}{2}}-4}{a^{\frac{3}{2}}+4a}\right)\dfrac{a-16}{a^{\frac{1}{2}}}=$

$=\dfrac{\left(a^{\frac{1}{2}}+4\right)^2-\left(a^{\frac{1}{2}}-4\right)^2}{a(a-16)}\cdot\dfrac{a-16}{a^{\frac{1}{2}}}=\dfrac{16a^{\frac{1}{2}}}{a\cdot a^{\frac{1}{2}}}=\dfrac{16}{a}.$

## Вариант 4

**1.** а) $4^{-2}=\dfrac{1}{16}$; б) $\left(\dfrac{1}{5}\right)^{-1}=5$;

в) $27^{\frac{1}{3}} - 25^{\frac{1}{2}} = 3 - 5 = -2;$

г) $\left(1 - 2^{\frac{4}{3}}\right)\left(1 + 2^{\frac{4}{3}} + 2^{\frac{8}{3}}\right) = 1 - 2^4 = -15.$

**2.** в) $\left(a^{-\frac{7}{8}} b^{\frac{11}{6}}\right) : \left(a^{-\frac{3}{4}} b^{3}\right) = a^{\left(-\frac{7}{8}+\frac{3}{4}\right)} b^{\left(\frac{11}{6}-3\right)} = a^{-\frac{1}{8}} b^{-\frac{7}{6}}.$

**3.** $$\frac{a^{\frac{1}{3}}b - a^{\frac{5}{3}}}{a^{\frac{4}{3}}b^{\frac{3}{4}} - b^{\frac{7}{4}}} = \frac{a^{\frac{1}{3}}\left(b - a^{\frac{4}{3}}\right)}{b^{\frac{3}{4}}\left(a^{\frac{4}{3}} - b\right)} = -\frac{a^{\frac{1}{3}}}{b^{\frac{3}{4}}}$$

при $a = 64$, $b = 16$: $-\frac{64^{\frac{1}{3}}}{16^{\frac{3}{4}}} = -\frac{4}{8} = -\frac{1}{2}.$

**4.** $y = \frac{1}{x^4}$; $\quad x = 1$; $\quad x = 2$; $\quad y = 0$;

$S = \int_1^2 \frac{1}{x^4} dx = -\frac{1}{3x^3}\Big|_1^2 = -\frac{1}{24} + \frac{1}{3} = \frac{7}{24}.$

**5.** $$\left(\frac{4}{b - 4b^{\frac{1}{2}}} - \frac{b^{\frac{3}{2}}}{b^2 - 16b}\right) : \frac{b^{\frac{1}{2}}}{b^{\frac{1}{2}} + 4} = \frac{4b + 16b^{\frac{1}{2}} - b^{\frac{3}{2}}}{b^2 - 16b} : \frac{b^{\frac{1}{2}}}{b^{\frac{1}{2}} + 4} =$$

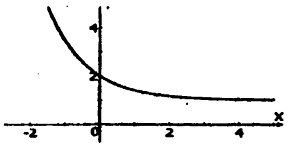
$$= \frac{4b^{\frac{1}{2}} + 16 - b}{b(b^{\frac{1}{2}} - 4)}.$$

В учебнике, вероятно, опечатка. Надо не умножать, а делить.

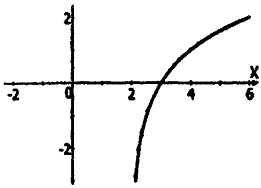
# Контрольная работа № 11

## Вариант 1

**1.** а)




б)




**2.** б) $\dfrac{100\cdot 4^{x^2}}{5^{5x}}=\dfrac{32^x}{25^{x^2}}$;

$\dfrac{4^{x^2+1}}{5^{5x-2}}=\dfrac{2^{5x}}{5^{2x^2}}$;

$\dfrac{2^{2x^2+2}}{5^{5x-2}}=\dfrac{2^{5x}}{5^{2x^2}}$; $\quad 10^{2x^2}\cdot 4=\dfrac{10^{5x}}{25}$;

$100=10^{5x-2x^2} \Rightarrow 5x-2x^2=2$

$2x^2-5x+2=0 \qquad D=25-16=9$

$x_{1,2}=\dfrac{5\pm 3}{4}$; $x_1=2$; $x_2=\dfrac{1}{2}$.

**3.** $\left(\dfrac{1}{2}\right)^{x^2-5}>\left(\dfrac{1}{16}\right)^{x}$; $\left(\dfrac{1}{2}\right)^{x^2-5}>\left(\dfrac{1}{2}\right)^{4x}$;

$x^2-4x-5<0$; $\qquad x\in(-1;\ 5)$.

**4.** $\log_7 49\sqrt[3]{7}=\log_7(7^2\cdot 7^{\frac{1}{3}})=\log_7 7^{\frac{7}{3}}=\dfrac{7}{3}$.

**5.** $\dfrac{2^x+10}{4}=\dfrac{9}{2^{x-2}}$; $\qquad 2^{2x}+10\cdot 2^x-144=0$;

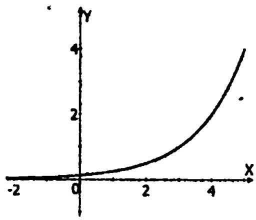
$2^x=-18$ – решений нет; $\quad 2^x=8\ x=3$

**6.** $36^x - 2\cdot18^x \geq 8\cdot9^x$; $\left(\dfrac{6}{3}\right)^{2x} - 2\cdot\left(\dfrac{6}{3}\right)^{x} - 8 \geq 0$;

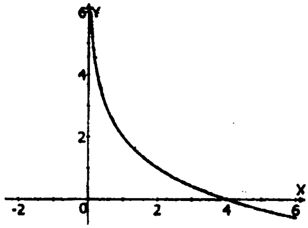
$\left(\dfrac{6}{3}\right)^{x} \in(-\infty;-2]\cup[4;+\infty)$; $x\geq2$.

## Вариант 2

**1.** а)




б)




**2.** б) $\dfrac{36\cdot27^{x^2}}{4^{5x}} = \dfrac{3^{10x}}{6\cdot8^{x^2}}$;

$(8\cdot27)^{x^2}\cdot6\cdot36 = 36^{5x}$;

$6^{3x^2}\cdot6^3 = 6^{10x}$; $\qquad 6^{3x^2+3-10x} = 1$;

$3x^2 - 10x + 3 = 0$;

$D = 100 - 36 = 64$;

$x_{1,2} = \dfrac{10\pm8}{6}$; $x_1 = 3$, $x_2 = \dfrac{1}{3}$.

**3.** $\left(\dfrac{3}{7}\right)^{2x^2} < \left(\dfrac{9}{49}\right)^{4}$; $x^2>4$; $x\in(-\infty;-2)\cup(2;+\infty)$.

**4.** $\log_2 16\sqrt[4]{2} = \log_2 2^4 2^{\frac{1}{4}} = \log_2 2^{\frac{17}{4}} = \dfrac{17}{4}$.

**5.** $3\cdot5^{2x-1} - 50\cdot5^{x-3} = 0{,}2$;

$75\cdot5^{2x} - 50\cdot5^x - 25 = 0$; $\qquad 3\cdot5^{2x} - 2\cdot5^x - 1 = 0$;

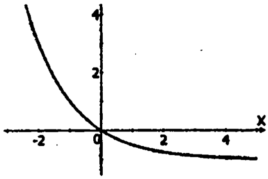
$5^x = \dfrac{1-2}{3}$ – решений нет; $\qquad 5^x = 1$; $\qquad x = 0$.

**6.** $9 \cdot 4^x + 8 \cdot 12^x \geq 36^x$; $\quad \left(\frac{6}{2}\right)^{2x} - 8 \cdot \left(\frac{6}{2}\right)^x - 9 \leq 0$;

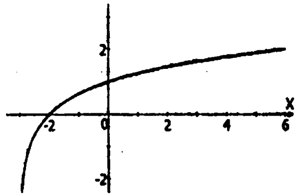
$3^x \in [-1; 9]$; $\quad x \in (-\infty; 2]$.

## Вариант 3

**1.** а)




б)




**2.** б) $\frac{8 \cdot 7^{3x^2}}{14^x} = \frac{14^4}{8^{x^2-1}}$; $\quad 7^{3x^2} \cdot 8^{x^2} = 14^{x+4} = 7^{x+4} \cdot 2^{x+4}$;

$7^{3x^2-x-4} \cdot 2^{3x^2-x-4} = 1 \quad 3x^2 - x - 4 = 0$;

$D = 1 + 48 = 49$;

$x_{1,2} = \frac{1 \pm 7}{6}$; $x_1 = -1$, $\quad x_2 = \frac{4}{3}$.

**3.** $\left(\frac{1}{4}\right)^{x^2-6} < \left(\frac{1}{2}\right)^{10x}$;

$x^2 - 5x - 6 > 0$;

$x \in (-\infty; -1) \cup (6; +\infty)$.

**4.** $\log_5 125\sqrt{5} = \log_5 5^3 5^{\frac{1}{2}} = \log_5 5^{\frac{7}{2}} = \frac{7}{2}$.

**5.** $\frac{3^x + 3}{4} = \frac{3}{3^{x-2}}$; $3^{2x} + 3 \cdot 3^x - 108 = 0$; $3^x = -12$ – решений нет; $3^x = 9$; $x = 2$.

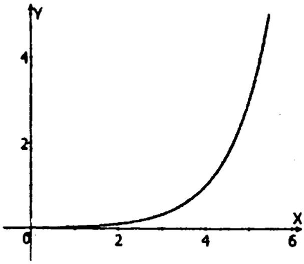
**6.** $20^x + 4 \cdot 10^x \geq 5 \cdot 5^x$;

$2^{2x} + 4 \cdot 2^x - 5 \geq 0$;

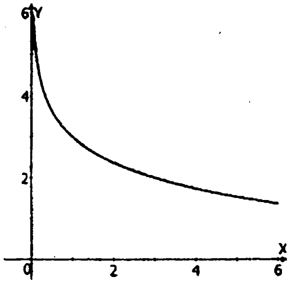
$2^x \in (-\infty; -5] \cup [1; +\infty)$; $\quad x \geq 0$.

# Вариант 4

**1.** а)




б)




**2.** б) $\dfrac{27\cdot 3^{3x-1}}{25^{x^2}}=\dfrac{9^{x^2}}{5\cdot 5^{3x+1}}$;

$3^{3x+2}\cdot 5^{3x+2}=(9\cdot 25)^{x^2}$

$(15)^{3x+2}=(15)^{2x^2}$

$2x^2-3x-2=0$

$D=9+16=25$

$x_{1,2}=\dfrac{3\pm 5}{4}$; $x_1=2$, $x_2=-\dfrac{1}{2}$.

**3.** $\left(\dfrac{3}{5}\right)^{3x^2-1}\geq\left(\dfrac{9}{25}\right)^{13}$;

$3x^2\leq 27$; $x\in[-3;\ 3]$.

**4.** $\log_3 81\sqrt[4]{3}=\log_3 3^4 3^{\frac{1}{4}}=\log_3 3^{\frac{17}{4}}=\dfrac{17}{4}$.

**5.** $8\cdot 2^{2x-1}-28\cdot 2^{x-3}=\dfrac{1}{2}$;

$8\cdot 2^{2x}-7\cdot 2^x-1=0$;

$2^x=\dfrac{7-9}{16}$ – решений нет; $2^x=1$; $x=0$.

**6.** $9\cdot 6^x+8\cdot 18^x>54^x$; $3^{2x}-8\cdot 3^x-9<0$;

$3^x\in(-1;\ 9)$; $x\in(-\infty;\ 2)$.

# Контрольная работа № 12

## Вариант 1

1. $\log_3 45 - \log_3 5 + 9^{\log_3 5} = \log_3 \frac{45}{5} + 3^{\log_3 25} =$

$= \log_3 9 + 25 = 2 + 25 = 27$.

2. $\log_{\frac{1}{4}}(2x - 5) > -1$; ОДЗ, $x > \frac{5}{2}$; $2x - 5 < 4$; $x \in \left(\frac{5}{2}; \frac{9}{2}\right)$.

3. $y = xe^x$; $y' = e^x(x + 1) = 0$; $x = -1$ – экстремум.

4. $\begin{cases} \log_{\sqrt{2}}(x - y) = 2 \\ 2^x \cdot 5^{x-2y} = 40 \end{cases}$; $\begin{cases} x = 2 + y \\ 2^{2+y} \cdot 5^{2+y-2y} = 40 \end{cases}$;

$\left(\frac{2}{5}\right)^y = \left(\frac{2}{5}\right)$; $\begin{cases} y = 1 \\ x = 3 \end{cases}$.

5. $y = \ln 2x$; $y' = \frac{1}{x}$; $y_{кас} = \ln 2x_0 + \frac{1}{x_0}(x - x_0)$;

$\ln 2x_0 - 1 = 0$; $x_0 = \frac{e}{2}$; $y^{кас} = \frac{2x}{e}$.

## Вариант 2

1. $\log_2 56 - \log_2 7 + 16^{\log_2 3} = \log_2 8 + 2^{\log_2 3^4} =$
$= 3 + 3^4 = 81 + 3 = 84$.

2. $\log_{\frac{1}{3}}(2 - 3x) > -2$; ОДЗ, $x < \frac{2}{3}$;

$2 - 3x > 9$; $x < -\frac{7}{3}$.

3. $y = (2x - 1)e^x$; $y' = e^x(2x + 1) = 0$;

$x = -\dfrac{1}{2}$ – точка экстремума.

4. $\begin{cases} \log_2(x+y) + 2\log_4(x-y) = 3 \\ 3^{2+\log_3(2x-y)} = 45 \end{cases}$; ;

$\begin{cases} y = 2x - 5 \\ \log_2(3x-5) + \log_2(5-x) = 3 \end{cases}$

$-3x^2 + 20x - 25 = 8$; $3x^2 - 20x + 33 = 0$;

$\begin{cases} x = 3 \\ y = 1 \end{cases}$ или $\begin{cases} x = 11/3 \\ y = 2\,1/3 \end{cases}$.

5. $y = \ln 3x$; $y_{кас} = \ln 3x_0 + \dfrac{1}{x_0}(x - x_0)$;

$\ln 3x_0 - 1 = 0$; $x_0 = \dfrac{e}{3}$; $y_{кас} = \dfrac{3x}{e}$.

## Вариант 3

1. $\log_5 75 - \log_5 3 + 25^{\log_5 2} = \log_5 25 + 5^{\log_5 2^2} = 2 + 4 = 6$.

2. $\log_{\frac{1}{2}}(2x+1) > -2$; ОДЗ, $x > -\dfrac{1}{2}$; $2x + 1 < 4$;

$x < \dfrac{3}{2}$; $x \in \left(-\dfrac{1}{2}; \dfrac{3}{2}\right)$.

3. $y = x^2e^x$; $y' = e^x(x^2 + 2x) = 0$;

$x = 0$ и $x = -2$ – точки экстремума.

4. $\begin{cases} \log_{\sqrt{2}}(x+y) = 2 \\ 3^x \cdot 7^y = 21 \end{cases}$; $\begin{cases} x = 2 - y \\ 3^{2-y} \cdot 7^y = 21 \end{cases}$;

$$\left(\tfrac{7}{3}\right)^{y} = \left(\tfrac{7}{3}\right); \begin{cases} y = 1 \\ x = 1 \end{cases}.$$

**5.** $y = \ln\dfrac{x}{2}$; $\qquad y_{\text{кас}} = \ln\dfrac{x_0}{2} + \dfrac{1}{x_0}(x - x_0)$;

$\ln\dfrac{x_0}{2} = 1$; $\qquad x_0 = 2e$; $\qquad y_{\text{кас}} = \dfrac{x}{2e}$.

## Вариант 4

**1.** $\lg 300 - \lg 3 + 100^{\lg 6} = \lg 100 + 10^{\lg 6^2} = 2 + 36 = 38$.

**2.** $\log_{\frac{2}{3}}(2 - 5x) < -2$; $\qquad$ ОДЗ, $x < \dfrac{2}{5}$;

$2 - 5x > \dfrac{9}{4}$; $\qquad x < -\dfrac{1}{20}$.

**3.** $y = x \cdot e^{-x}$; $\qquad y' = e^{-x}(1 - x) = 0$;

$x = 1$ – точка экстремума.

**4.** $\begin{cases} \log_2(x + y) + 2\log_4(x - y) = 5 \\ 3^{1+2\log_3(x-y)} = 48 \end{cases}$; $\qquad \begin{cases} x^2 - y^2 = 32 \\ (x - y)^2 = 16 \end{cases}$;

т.к. $x - y > 0$, то $\begin{cases} x = 4 + y \\ 8y = 16 \end{cases}$; $\qquad \begin{cases} y = 2 \\ x = 6 \end{cases}$.

**5.** $y = \ln ex$; $\qquad y_{\text{кас}} = \ln ex_0 + \dfrac{1}{x_0}(x - x_0)$;

$\ln ex_0 = 1$; $\qquad x_0 = 1$; $\qquad y_{\text{кас}} = x$.

# Контрольная работа № 13

## Вариант 1

**1.** а) $\sqrt{2x+3}+\sqrt{4-x}=\sqrt{3x+7}$; ОДЗ: $x\in[-\frac{3}{2};\ 4]$;

$x+7+2\sqrt{(2x+3)(4-x)}=3x+7$;

$\begin{cases}-2x^2+5x+12=x^2\\ x\geq 0\end{cases}$;

$3x^2-5x-12=0$; $x=\frac{5\text{-}13}{6}$; не подходит, значит, $x=3$;

б) $2\sin^2\frac{x}{2}+5\cos\frac{x}{2}=4$; $2\cos^2\frac{x}{2}-5\cos\frac{x}{2}+2=0$;

$\cos\frac{x}{2}=\frac{5+3}{4}$ — не подходит, значит,

$\cos\frac{x}{2}=\frac{1}{2}$; $x=\pm\frac{2\pi}{3}+4\pi n$.

**2.** $\log_2(3x-1)-\log_2(5x+1)<\log_2(x-1)-2$;

ОДЗ: $x>1$; $12x-4<5x^2-4x-1$;

$5x^2-16x+3>0$ $x\in\left|\frac{8+3\sqrt{5}}{5};+\infty\right|$.

**3.** $2x^2\geq|x^2-x|+2$;

1. $x\in(-\infty;\ 0]\cup[1;+\infty)$;

$x^2+x-2\geq 0$;

$x\in(-\infty;\ -2]\cup[1;+\infty)$;

2. $x\in[0;1]$;

$3x^2-x-2\geq 0$; $x\in\{1\}$.

Итого:

$x\in(-\infty;\ -2]\cup[1;+\infty)$.

**3.** $(x^2+8x+15)(\log_{1/2}(1+\cos^2\frac{\pi x}{4}))\geq 1$.

**4.** Т.к., $\begin{cases}\log_{1/2}(1+\cos^2\frac{\pi x}{4})\in[-1;0]\\ x^2+8x+15\geq -1\end{cases}$;

то неравенство возможно только при

$$\begin{cases} x^2+8x+15\ge -1 \\ \log_{1/2}(1+\cos^2\frac{\pi x}{4}) = -1 \end{cases}; \quad x=-4.$$

## Вариант 2

**1.** а) $\sqrt{2x+9}+\sqrt{1-2x}=\sqrt{4-3x}$; ОДЗ: $x\in\left[-\frac{9}{2};\frac{1}{2}\right]$;

$10+2\sqrt{(2x+9)(1-2x)}=4-3x$;

$$\begin{cases} 4(-4x^2-16x+9)=36+9x^2+36x \\ x\le -2 \end{cases};$$

$25x^2+100x=0$; $x=0$ не подходит, значит, $x=-4$.

б) $5\sin 2x-1=2\cos^2 2x$; $2\sin^2 2x+5\sin 2x-3=0$;

$\sin 2x=\frac{-7-5}{4}$;

не подходит, значит, $\sin 2x=\frac{1}{2}$; $x=(-1)^k\frac{\pi}{12}+\frac{\pi k}{2}$.

**2.** $\log_{1/2}(3x-4)-\log_{1/2}(3x+4)<\log_{1/2}(x-2)+2$;

ОДЗ: $x>2$;

$12x-16>3x^2-2x-8$; $3x^2-14x+8<0$; $x\in(2;4)$.

**3.** $3x^2\ge|x^2+2x|+12$; 1. $x\in(-\infty;-2]\cup[0;+\infty)$;

$x^2-x-6\ge 0$; $x\in(-\infty;-2]\cup[3;+\infty)$;

2. $x\in[-2;0]$; $2x^2+x-6\ge 0$; $x\in\{-2\}$.

Итого: $x\in(-\infty;-2]\cup[3;+\infty)$.

**4.** $(10x-x^2-24)(\log_5(4\sin^2\frac{\pi x}{2}+1))\ge 1$;

Т.к., $\begin{cases} 10x-x^2-24\le 1 \\ \log_5(4\sin^2\frac{\pi x}{2}+1)\in[0;1], \end{cases}$; то неравенство возможно

только при $\begin{cases} 10x-x^2-24=1 \\ \log_5(4\sin^2\frac{\pi x}{2}+1)=1 \end{cases}$; $x=5$.

## Вариант 3

**1.** а) $\sqrt{2x+1}=2\sqrt{x}-\sqrt{x-3}$; ОДЗ: $x\geq 3$;

$3x-2+2\sqrt{(2x+1)(x-3)}=4x$;

$\begin{cases}8x^2-20x-12=x^2+4x+4\\ x\geq -2\end{cases}$;

$7x^2-24x-16=0$; $x=\dfrac{12-16}{7}$

— не подходит, значит, $x=4$;

б) $2\sin^2 3x+5\cos 3x+1=0$;

$2\cos^2 3x-5\cos 3x-3=0$; $\cos 3x=\dfrac{5+7}{4}$ — не подходит, значит, $\cos 3x=-\dfrac{1}{2}$; $x=\pm\dfrac{2\pi}{9}+\dfrac{2\pi n}{3}$

**2.** $\log_{1/3}(2x+1)-\log_{1/3}(2x+25)>3+\log_{1/3}(x+2)$;

ОДЗ: $x>-\dfrac{1}{2}$; $54x+27<2x^2+29x+50$;

$2x^2-25x+23>0$; $x\in(-\dfrac{1}{2};1)\cup(11{,}5;+\infty)$.

**3.** $3x^2+|x^2-2x|\leq 12$; 1. $x\in(-\infty;0]\cup[2;+\infty)$;

$2x^2-x-6\leq 0$; $x\in[-\dfrac{3}{2};0]\cup\{2\}$;

2. $x\in[0;2]$; $x^2+x-6\leq 0$; $x\in[0;2]$.

Итого: $x\in[-\dfrac{3}{2};2]$.

**4.** $(12x-x^2-35)\lg(9+\cos^2\dfrac{\pi x}{3})\geq 1$.

Т.к., $\begin{cases}12x-x^2-35\leq 1\\ \lg(9+\cos^2\dfrac{\pi x}{3})\leq 1,\end{cases}$

то неравенство возможно только при $\begin{cases} 12x - x^2 - 35 = 1 \\ \lg(9+\cos^2\frac{\pi x}{3}) = 1, \end{cases}$ $x=6$.

## Вариант 4

**1.** а) $\sqrt{8x+1} - \sqrt{3+x} = \sqrt{3x-2}$; ОДЗ: $x \geq \frac{2}{3}$;

$8x + 1 = 4x + 1 + 2\sqrt{(3+x)(3x-2)}$; $\begin{cases} 3x^2 + 7x - 6 = 4x^2 \\ x \geq 0 \end{cases}$;

$x^2 - 7x + 6 = 0$; $x = 6$ или $x = 1$;

б) $4(\cos^2\frac{x}{3} + \sin\frac{x}{3}) = 1$; $4\sin^2\frac{x}{3} - 4\sin\frac{x}{3} - 3 = 0$;

$\sin\frac{x}{3} = \frac{2+4}{4}$ — не подходит, значит, $\sin\frac{x}{3} = -\frac{1}{2}$;

$x = (-1)^{k+1}\frac{\pi}{2} + 3\pi k$.

**2.** $\log_3(5 - 2x) - \log_3(25 - x) > \log_3(x + 5) - 2$;

ОДЗ: $x \in (-5; 2{,}5)$; $45 - 18x > -x^2 + 20x + 125$;
$x^2 - 38x - 80 > 0$; $x \in (-5; 2)$.

**3.** $18 - 2x^2 \geq |x^2 + 3x|$; 1. $x \in (-\infty; -3] \cup [0; +\infty)$;

$x^2 + x - 6 \leq 0$; $x \in \{-3\} \cup [0; 2]$;
2. $x \in [-3; 0]$; $x^2 - 3x - 18 \leq 0$; $x \in [-3; 0]$.
Итого: $x \in [-3; 2]$.

**4.** $(x^2 + 6x + 8)(\log_{1/4}(3+\sin^2\frac{\pi x}{6}) \geq 1$.

Т.к., $\begin{cases} x^2 + 6x + 8 \geq -1 \\ \log_{1/4}(3+\sin^2\frac{\pi x}{6}) \in [-1; \log_{1/4} 3], \end{cases}$, то неравенство

возможно только при $\begin{cases} x^2 + 6x + 8 = -1 \\ \log_{1/4}(3+\sin^2\frac{\pi x}{6}) \end{cases}$; $x = -3$.