

Решение самостоятельных и контрольных работ по алгебре за 10 класс

к учебному пособию
«Дидактические материалы
по алгебре и началам анализа для 10 класса.
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М.: Просвещение, 1999 г.»

учебно-методическое пособие

ВАРИАНТ 1.

С-1

1. $60^\circ = \frac{\pi}{3}$; $144^\circ = \frac{\pi}{180} \cdot 144 = \frac{4\pi}{5}$.
2. $\frac{3\pi}{4} = 135^\circ$; $\frac{5\pi}{18} = \frac{5 \cdot 180^\circ}{18} = 50^\circ$.
3.
 - а) $49^\circ = \frac{\pi}{180} \cdot 49 = \frac{49\pi}{180}$; $\sin 49^\circ \approx 0,7547$; $\cos 49^\circ \approx 0,6560$;
 - б) $76^\circ, 7' = \frac{\pi}{180} \cdot \left(76 + \frac{7}{60}\right) = \frac{4567\pi}{10800}$;
 $\sin 76^\circ, 7' \approx 0,9728$ $\cos 76^\circ, 7' \approx 0,2315$.
4.
 - а) $0,8600 \approx 49^\circ$; б) $1,2369 \approx 71^\circ$.

С-2

1.
$$\frac{\sin^4 \alpha - 2 \sin^2 \alpha \cos^2 \alpha + \cos^4 \alpha}{(\sin \alpha + \cos \alpha)^2} = 1 - \sin 2\alpha$$
$$\frac{(\sin^2 \alpha - \cos^2 \alpha)^2}{(\sin \alpha + \cos \alpha)^2} = (\sin \alpha - \cos \alpha)^2 = 1 - \sin 2\alpha$$
2.
 - а) $\cos 700^\circ \operatorname{tg} 380^\circ = \cos 20^\circ \operatorname{tg} 20^\circ = \sin 20^\circ > 0$;
 - б) $\cos(-1)\sin(-2) = -\cos(1)\sin(2) < 0$.
3. $\cos(\alpha) = \frac{2}{\sqrt{5}}$, $0 < \alpha < \frac{\pi}{2}$; $\sin(\alpha) = \frac{1}{\sqrt{5}}$, $\operatorname{tg} \alpha = \frac{1}{2}$

С-3

1.
 - а) $\sin\left(-\frac{23\pi}{6}\right) = -\sin\left(4\pi - \frac{\pi}{6}\right) = -\sin\left(-\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$;
 - б) $\operatorname{ctg}(-600^\circ) = -\operatorname{ctg}(-120^\circ) = -\frac{1}{\sqrt{3}}$.

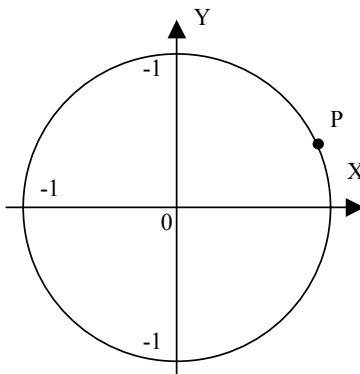
2. $1 + \operatorname{ctg}(\pi + \alpha) \operatorname{tg}\left(\frac{3\pi}{2} - \alpha\right) = 1 + \operatorname{ctg} \alpha \operatorname{ctg} \alpha = \frac{1}{\sin^2 \alpha}.$
3. $\cos(2\alpha + \pi) = \cos^2\left(\alpha - \frac{\pi}{2}\right) + \cos(\alpha + \pi) \sin\left(\alpha + \frac{\pi}{2}\right).$
 $\sin^2 \alpha - \cos^2 \alpha = -\cos 2\alpha = \cos(2\alpha + \pi); \cos^2\left(\alpha - \frac{\pi}{2}\right) = \sin^2 \alpha;$
 $\cos(\alpha + \pi) \sin\left(\alpha + \frac{\pi}{2}\right) = -\cos \alpha \cdot \cos \alpha = -\cos^2 \alpha.$

C-4

1. $4 \sin 37^0 30' \cos 37^0 30' \sin 15^0 = 2 \sin 75^0 \sin 15^0 = \sin 30^0 = \frac{1}{2}.$
2. $\cos \alpha = \frac{7}{25}, \frac{3\pi}{2} < \alpha < 2\pi;$
 $\sin \alpha = -\frac{24}{25}, \sin 2\alpha = -\frac{336}{625} = \cos 2\alpha \operatorname{tg} 2\alpha.$
3. $(\sin \alpha - \cos \alpha)^2 - 1 + 4 \sin 2\alpha = -\sin 2\alpha + 4 \sin 2\alpha = 3 \sin 2\alpha.$

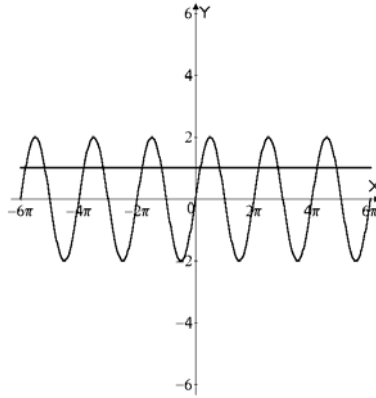
C-5

1. абсцисса : $\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$; ордината : $\sin \frac{\pi}{6} = \frac{1}{2}.$



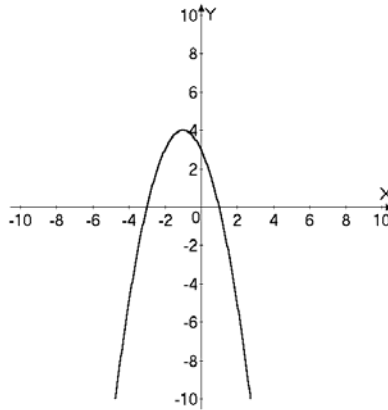
2. а) II; б) IV.

3. $2 \sin x = 1, x = (-1)^k \frac{\pi}{6} + \pi k.$



C-6

1. a) $f(x) = \frac{3}{x^2 - 4}$; ОДЗ $x^2 - 4 \neq 0, x \neq \pm 2$;
 б) $\sqrt{4x^2 - 1} = f(x)$; ОДЗ $4x^2 - 1 \geq 0, x \in \left(-\infty; -\frac{1}{2}\right] \cup \left[\frac{1}{2}; +\infty\right).$
2. $f(x) = (x-1)^4$; $f(2) = 1, f(1 + \sqrt{x}) = (\sqrt{x})^4 = x^2.$
- 3.



C-7

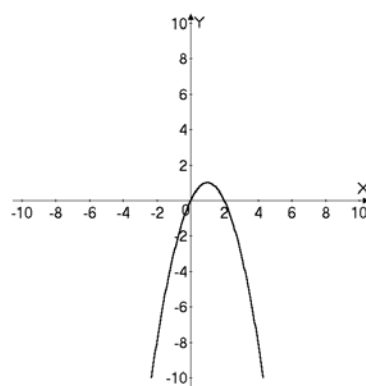
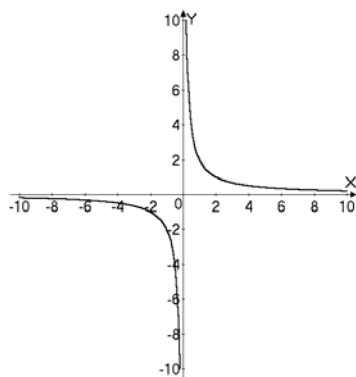
- $f(x) = x^4 - 2x^2 - \sin^2 3x$;
 $f(-x) = (-x)^4 - 2(-x)^2 - \sin^2(-3x) = x^4 - 2x^2 - \sin^2 3x = f(x)$.
- $f(x) = x^3 - 3x + \sin 2x$;
 $f(-x) = (-x)^3 - 3(-x) + \sin(-2x) = -x^3 + 3x - \sin 2x = -f(x)$.

C-8

- a)** $\cos 177^\circ = -\cos 3^\circ$; **б)** $\sin 3521^\circ = -\sin 79^\circ = -\cos 11^\circ$;
в) $\operatorname{ctg} \frac{45\pi}{7} = \operatorname{ctg} \frac{3\pi}{7} = \operatorname{tg} \frac{\pi}{14}$.
- $\sin(2x + 4\pi) - 2\sin(x + \pi)\cos(x - \pi) = \sin 2x - 2\sin x \cos x = 0$
- a)** $\sin \frac{2x}{3}$, $T = 3\pi$; **б)** $\cos 7x$, $T = \frac{2\pi}{7}$ **в)** $\operatorname{tg}\left(\frac{1}{3}x + \frac{\pi}{8}\right)$, $T = 3\pi$.

C-9

- a)** убывает на обл. опр;



A

Б

б) возрастает: $x \in \left(-\infty; \frac{1}{4}\right]$; убывает $x \in \left[\frac{1}{4}; +\infty\right)$.

1. $y = \frac{1}{2} \sin x$.

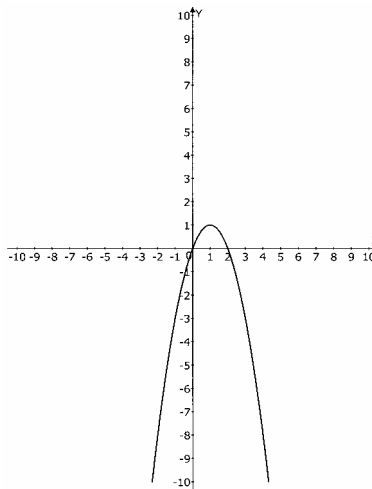
возрастает: $\left[-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n\right]$; убывает: $\left[\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right]$.

2. $\cos 1 \vee \cos 3$; $\cos 57^0 > \cos 171^0$.

C-10

1. $y = 2x - x^2$; а) (1;1).

б)



в) $2x - x^2 < -3$; $x^2 - 2x - 3 > 0$; $(x-3)(x+1) > 0$;

$x \in (-\infty; -1) \cup (3; +\infty)$.

2.

$y = \frac{1}{3} \sin x - 1$;

$y' = \frac{1}{3} \cos x = 0$ $x = \frac{\pi}{2} + \pi n$; $n \in \mathbb{Z}$;

$x = \frac{\pi}{2} + 2\pi n$ — точки максимума ;

$x = -\frac{\pi}{2} + 2\pi n$ — точки минимума ;

Экстремумы: $y\left(\frac{\pi}{2} + 2\pi n\right) = -\frac{2}{3}$; $y\left(-\frac{\pi}{2} + 2\pi n\right) = -1\frac{1}{3}$.

С-11

обл.опр: $x \in [-10; 10]$; обл. зн.: $x \in [-3; 7]$;
 функция возрастает на: $[-10; -6] \cup [-3; 6]$;
 функция убывает на: $[-6; -3] \cup [6; 10]$;
 $y > 0$ при $x \in [-10; 3) \cup (3; 10]$; $y < 0$, $x \in (-3; 3)$; $y = 0$ при $x = -3$ и $x = 3$;
 $y_{\max} = y(-6) = y(6) = 7$; $y_{\min} = y(0) = -3$.

С-12

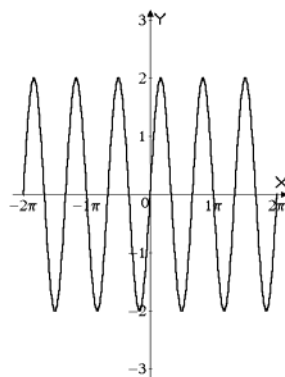
1.

$f(x) = \frac{1}{\cos 2x}$; ОДЗ: $\cos 2x \neq 0$; $x \neq \frac{\pi}{4} + \frac{\pi n}{2}$, $n \in \mathbb{Z}$, значит, функция

определена всюду на \mathbb{R} , кроме точек $x = \frac{\pi}{4} + \frac{\pi n}{2}$.

2.

$$y = 2 \sin 3x.$$



а) $x \in \mathbb{R}$; б) $y \in [-2; 2]$; в) $x = \frac{\pi n}{3}$; $n \in \mathbb{Z}$;

г) точка максимума $x = \frac{\pi}{6} + \frac{2}{3}\pi n$; $n \in \mathbb{Z}$,

значит $y_{\max} = 2 \sin\left(3\left(\frac{\pi}{6} + \frac{2}{3}\pi n\right)\right) = 2 \sin\left(\frac{\pi}{2} + 2\pi\right)$;

точки минимума $x = -\frac{\pi}{6} + \frac{2}{3}\pi n$, $n \in \mathbb{Z}$, значит, $y_{\min} = 2 \sin\left(-\frac{\pi}{2} + \pi n\right)$.

C-13

1.

a) $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$; **б)** $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$;

в) $\arctg 1 + \arccos 1 = \frac{\pi}{4} + 0 = \frac{\pi}{4}$; **г)** $\sin\left(2 \arccos \frac{\sqrt{3}}{2}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$.

2.

a) $\arcsin(-0,9) \approx -1,1198$; **б)** $\arccos 0,179 \approx 1,3908$;

в) $\arctg \frac{1}{\pi} \approx 0,3082$.

C-14

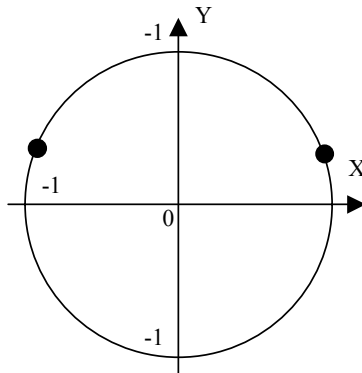
1.

a) $\cos x = -\frac{\sqrt{3}}{2}$, $x = \mp \frac{5\pi}{6} + 2\pi n$; **б)** $\sin 3x = -1$, $x = -\frac{\pi}{6} + \frac{2\pi n}{3}$;

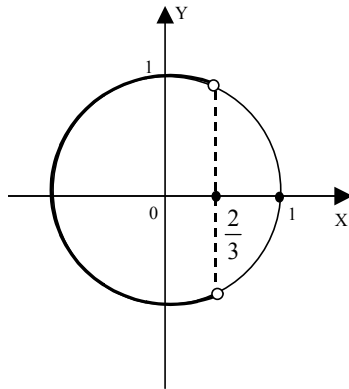
в) $\tg\left(x - \frac{\pi}{4}\right) = \sqrt{3}$, $x = \frac{7\pi}{12} + \pi n$.

C-15

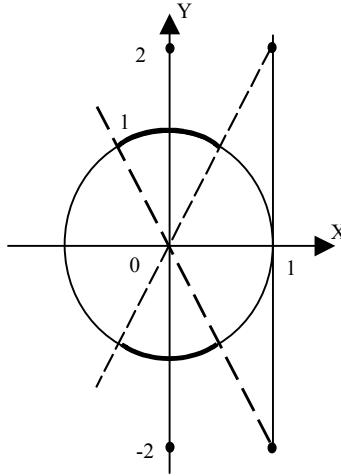
a)



б)



в)



C-16

а) $\sin x \leq \frac{\sqrt{3}}{2}$, $x \in \left[-\frac{4\pi}{3} + 2\pi n; \frac{\pi}{3} + 2\pi n \right]$;

б) $\operatorname{tg} 3x > \sqrt{3}$, $x \in \left(\frac{\pi}{9} + \frac{\pi n}{3}; \frac{\pi}{6} + \frac{\pi n}{3} \right)$.

C-17

а) $2 \cos^2 x - \cos x - 1 = 0$; $D=1+8=9$, $\cos x = \frac{1+3}{4} = 1$ или

$$\cos x = \frac{1-3}{4} = -\frac{1}{2}, \quad x = 2\pi n; \quad x = \pm \frac{2\pi}{3} + 2\pi n.$$

б) $2 \cos^2 x + 2 \sin x = 2,5$;

$$2 \sin^2 x - 2 \sin x + 0,5 = 0;$$

$$\frac{D}{4} = 1 - 1 = 0;$$

$$\sin x = \frac{1}{2}, \quad x = (-1)^n \frac{\pi}{6} + \pi n.$$

C-18

a) $\sin x = -\sqrt{3} \cos x$; $\operatorname{tg} x = -\sqrt{3}$, $x = -\frac{\pi}{3} + \pi n$.

б) $\sin^2 x - 4 \sin x \cos x + 3 \cos^2 x = 0$; $\operatorname{tg}^2 x - 4 \operatorname{tg} x + 3 = 0$;

$\operatorname{tg} x = 1$, $x = \frac{\pi}{4} + \pi n$; $\operatorname{tg} x = 3$, $x = \operatorname{arctg} 3 + \pi n$.

C-19

$$\begin{cases} x + y = \frac{\pi}{2} \\ \sin^2 x + \cos^2 y = 1 \end{cases}; \begin{cases} x = \frac{\pi}{2} - y \\ \cos y = \pm \frac{\sqrt{2}}{2} \end{cases}; \begin{cases} y = \frac{\pi}{4} + \frac{\pi n}{2} \\ x = \frac{\pi}{4} - \frac{\pi n}{2} \end{cases}.$$

C-20

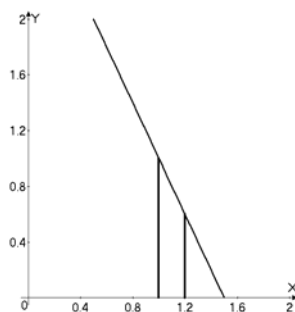
a) $1 - \cos 2x = \sin 2x$; $2 \sin^2 x - \sin 2x = 0$; $\sin x(\sin x - \cos x) = 0$;

$\sin x = 0$ или $\sin x = \cos x$; $x = \pi n$, $x = \frac{\pi}{4} + \pi n$.

б) $\sin x \cos 2x + \cos x \sin 2x = \frac{1}{2}$; $\sin 3x = \frac{1}{2}$; $x = (-1)^k \frac{\pi}{18} + \frac{\pi k}{3}$.

C-21

1.



$f(x) = 3 - 2x$, $\Delta f(x_0) = 3 - 2x_0 - 2\Delta x + 2x_0 - 3$;

$\Delta f(x_0) = -2\Delta x$, $\Delta x = 0,2$, $\Delta f(x_0) = -0,4$.

2.

$$f(x) = x^2 - x \frac{\Delta f(x_0)}{\Delta x} = \frac{\Delta x^2 + 2\Delta x x_0 - \Delta x}{\Delta x} = \Delta x + 2x_0 - 1;$$

$$x_0 = 0, \Delta x = 0,1, \frac{\Delta f(x_0)}{\Delta x} = -0,9;$$

$$\Delta x = 0,001, \frac{\Delta f(x_0)}{\Delta x} = -0,999;$$

$$\Delta x = 0,00001, \frac{\Delta f(x_0)}{\Delta x} = -0,99999;$$

$$x_0 = 0, \lim_{\Delta x \rightarrow 0} \frac{\Delta f(x_0)}{\Delta x} = 2x_0 - 1 = -1.$$

C-22

1.

$$x(t) = t^2 + 5, V = 2t, V(2) = 4 \text{ м/с.}$$

2.

$$\text{а) } f(x) = 4 - 7x, f'(x) = -7; \text{ б) } f(x) = \frac{3}{x}, f'(x) = -\frac{3}{x^2}.$$

C-23

$$\text{а) } f(-1) = 3, g(-1) \text{ - неопред.; б) да; в) для } f(x) \text{ не сущ.}$$

$$\lim_{x \rightarrow -1} g(x) = 1.$$

C-24

1.

$$\text{а) } \lim_{x \rightarrow 2} y = \lim_{x \rightarrow 2} 13f(x) - g(x) = 3 \lim_{x \rightarrow 2} f(x) - \lim_{x \rightarrow 2} g(x) = 9 + 1 = 10;$$

$$\text{б) } \lim_{x \rightarrow 2} y = \lim_{x \rightarrow 2} (3f(x)g^2(x)) = 3 \lim_{x \rightarrow 2} f(x) \cdot \lim_{x \rightarrow 2} g^2(x) = 3 \cdot 3 \cdot 1 = 9.$$

2)

$$\text{а) } \lim_{x \rightarrow 1} (3x^3 - x^2 + 3) = 3 \lim_{x \rightarrow 1} x^3 - \lim_{x \rightarrow 1} x^2 + 3 = 3 \cdot 1 - 1 + 3 = 5;$$

$$\text{б) } \lim_{x \rightarrow 2} \frac{3x+1}{x^2+1} = \frac{3 \lim_{x \rightarrow 2} x + 1}{\lim_{x \rightarrow 2} x^2 + 1} = \frac{3 \cdot 2 + 1}{4 + 1} = 1 \frac{2}{5}.$$

C-25

1.

а) $f(x) = x^5 - 2\sqrt{x}$, $f'(x) = 5x^4 - \frac{1}{\sqrt{x}}$; б) $f(x) = \frac{x^2 - 1}{x^2 + 1}$,

$$f'(x) = \frac{2x(x^2+1) - 2x \cdot (x^2 - 1)}{(x^2 + 1)^2} = \frac{2x + 2x}{(x^2 + 1)^2} = \frac{4x}{(x^2 + 1)^2}.$$

2.

$$f(x) = 3x - 4x^3, f'(x) = 3 - 12x^2; f'(1) = -9, f'(5) = -297;$$

$$f'(x) = 3 - 12x^2, f'(x+2) = 3 - 12(x+2)^2.$$

3.

$$f(x) = 6x - 3x^2; f'(x) = 6 - 6x > 0, x < 1.$$

C-26

1.

$$f(x) = 100x^{10} - 10x^{100}; f'(x) = 1000x^9 - 1000x^{99}; f'(1) = 0$$

2.

а) $f(x) = x^2 - 3x + 1$; $f'(x) = 2x - 3$, $f'(x) = 0$, при $2x - 3 = 0$;

$$x = 1\frac{1}{2};$$

$$f'(x) > 0 \text{ при } x > \frac{3}{2} = 1\frac{1}{2}; f'(x) < 0 \text{ при } x < \frac{3}{2} = 1\frac{1}{2};$$

б) $f(x) = \frac{x-3}{2x+5}$, $f'(x) = \frac{2x+5-2x+6}{(2x+5)^2} = \frac{11}{(2x+5)^2}$;

$f'(x) = 0$ не существует; $f'(x) > 0$ всегда, значит, не существует x , при которых $f'(x) < 0$.

C-27

1.

$$f(x) = \frac{3x+1}{9x^2-1}; \text{ ОДЗ: } 9x^2-1 \neq 0; x \neq \pm \frac{1}{3},$$

$$\text{значит, } x \in (-\infty; -\frac{1}{3}) \cup (-\frac{1}{3}; \frac{1}{3}) \cup (\frac{1}{3}; \infty).$$

2.

$$f(x) = \frac{x}{x-1}, \quad g(x) = \sqrt{x}; \quad f(g(x)) = \frac{\sqrt{x}}{\sqrt{x}-1}, \quad g(f(x)) = \sqrt{\frac{x}{x-1}}.$$

3.

a) $f(x) = (4-3x)^{100}, \quad f'(x) = -300(4-3x)^{99};$

б) $g(x) = \sqrt{x^2+1}, \quad g'(x) = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}.$

C-28

a) $f(x) = \sin 2x - \cos 3x, \quad f'(x) = 2 \cos 2x + 3 \sin 3x;$

б) $f(x) = \operatorname{tg} x - \operatorname{ctg}\left(x + \frac{\pi}{4}\right), \quad f'(x) = \frac{1}{\cos^2 x} + \frac{1}{\sin^2\left(x + \frac{\pi}{4}\right)};$

в) $f(x) = \sin^2 x, \quad f'(x) = 2 \sin x \cos x.$

C-29

1.

$f(x) = \frac{x^4 - 3x^2}{x(x-2)};$ функция непрерывна при $x \in (-\infty; 0) \cup (0; 2) \cup (2; +\infty).$

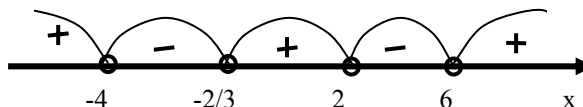
2.

a) $2x^2 - 8 > 0, \quad x^2 > 4;$

$(x-2)(x+2) > 0;$

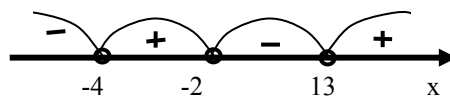
$x \in (-\infty; -2) \cup (2; +\infty);$

б) $\frac{(x-2)(x+4)(x-6)}{3x+2} \leq 0;$



$x \in \left[-4; -\frac{2}{3}\right) \cup [2; 6].$

$$\begin{aligned} \text{в) } \frac{x^2 - 11x - 26}{x + 4} &> 0; \\ \frac{(x - 13)(x + 2)}{x + 4} &> 0; \\ x &\in (-4; -2) \cup (13; +\infty). \end{aligned}$$

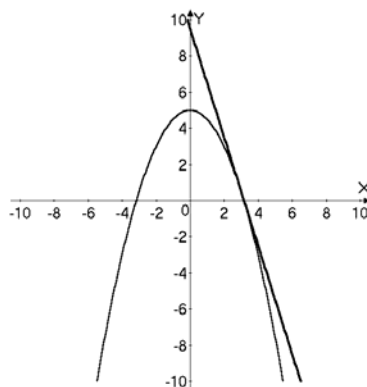


C-30

1. $f(x) = x^3 + 27 = 0$, $x = -3$; $f'(x) = 3x^2$, $f'(-3) = 27$ – тангенс угла наклона касательной.

2.

$$\begin{aligned} f(x) &= 5 - \frac{1}{2}x^2, \quad f(3) = 5 - \frac{9}{2} = \frac{1}{2}; \\ f'(x) &= -x, \quad f'(3) = -3; \\ y &= \frac{1}{2} - 3(x - 3) = -3x + 9,5. \end{aligned}$$



C-31

1. $\sqrt{1 + 0,0008} \approx 1 + 0,0004 = 1,0004$.
2. $1,00007^{500} \approx 1,035$.

C-32

1. $S(t) = 16t - 2t^3$, $V(t) = 16 - 6t^2$; $a(t) = -12t$, $V(2) = -8$, $a(2) = -24$.
2. $L(t) = V_0 t - \frac{gt^2}{2}$, $L'(t) = V_0 - gt$; $L'(t) = 60 - 10t = 0$, $t = 6$;
 $L(t) = 6 \cdot 60 - 5 \cdot 36 = 180 \text{ м.}$

C-33

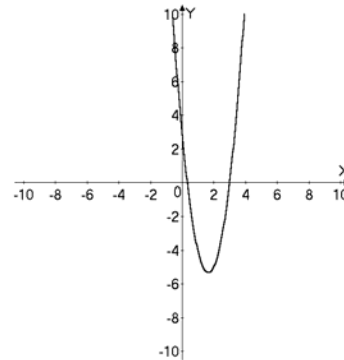
1. $f(x) = x + \frac{9}{x}$, $f'(x) = 1 - \frac{9}{x^2} > 0$; $x \in (-3; 3)$, значит функция $f(x)$ возрастает при $x \in (-3; 3)$; убывает при $x \in (-\infty; -3) \cup (3; +\infty)$.
2. $y = x^3 - 6x^2 - 15x - 3$; $y' = 3x^2 - 12x - 15$, $y' = 0$ при $x^2 - 4x - 5 = 0$; $x = 5$, $x = -1$;
 $y(-1) = 1 - 6 + 15 - 3 = 5$ - max;
 $y(5) = 125 - 150 - 75 - 3 = -103$ - min.

C-34

1. $f(x) = \frac{1}{3}x - x^3$, $f'(x) = \frac{1}{3} - 3x^2$; $f'(x)$ при $x = \pm \frac{1}{3}$ - экстремумы;
 функция возрастает: $x \in \left[-\frac{1}{3}; \frac{1}{3}\right]$; убывает: $x \in \left(-\infty; -\frac{1}{3}\right] \cup \left[\frac{1}{3}; +\infty\right)$.

C-35

1.
 $y = 3x^2 - 10x + 3$;
 вершина параболы
 $x_{\text{в}} = \frac{10}{6} = \frac{5}{3}$ - минимум;
 $y_{\text{в}}\left(\frac{5}{3}\right) = -5\frac{1}{3}$;
 функция убывает при $\left(-\infty; \frac{5}{3}\right]$;
 функция возрастает при $\left[\frac{5}{3}; +\infty\right)$.



2.
 а) $x^2 - 17x - 18 \leq 0$; $x \in [-1; 18]$;
 б) $9x^2 - 12x + 4 > 0$; $\frac{D}{4} = 36 - 36 = 0$, значит, $9x^2 - 12x + 4$ всегда больше нуля.

С-36

$$f(x) = \frac{2x-3}{2+x} - 1,$$

$$f'(x) = \frac{2x+4-2x+3}{(x+2)^2} = \frac{7}{(x+2)^2};$$

возрастает при

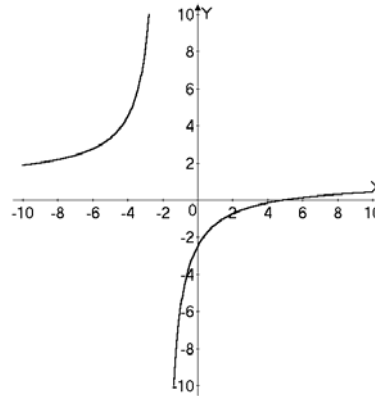
$$x \in (-\infty; -2) \cup (2; \infty);$$

$$\text{ОДЗ: } x \in (-\infty; -2) \cup (2; \infty);$$

множество значений:

$$y \in (-\infty; 1) \cup (1; \infty);$$

экстремумов нет.



С-37

1.

$$y = \frac{x^4}{4} - 8x^2; \quad y' = x^3 - 16x; \quad y' = 0 \text{ при } x = 0, \quad x = \pm 4;$$

$$y(0) = 0, \quad y(-1) = \frac{1}{4} - 8 = -7\frac{3}{4}; \quad y(2) = 4 - 32 = -28;$$

наибольшее значение $y = y(0) = 0$;

наименьшее значение $y = y(2) = -28$.

2.

Введем функцию $f(y) = x^2 + y^2$, тогда из условия $x + y = 10$

получаем, что $f(y) = (10 - y)^2 + y^2 = 2y^2 - 20y + 100$; $f'(y) = 4y - 20$;

Найдем критические точки $f(y): f'(y) = 0$ при $4y - 20 = 0; y = 5$;

$f(5) = 50 - 100 + 100 = 50$ – минимум, тогда $x = 10 - y = 5$, а искомое разбиение: $10 = 5 + 5$.

С-38

1.

$$\sin \alpha = \frac{2}{\sqrt{5}}, \quad \frac{\pi}{2} < \alpha < \pi; \quad \cos \alpha = -\frac{1}{\sqrt{5}}, \quad \operatorname{tg} \alpha = -2;$$

$$\operatorname{tg}\left(\alpha - \frac{\pi}{4}\right) = \frac{\operatorname{tg} \alpha - 1}{1 + \operatorname{tg} \alpha} = \frac{-3}{-1} = 3.$$

$$2. \quad \frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{\cos \alpha \cos \beta} = \frac{2 \sin \alpha \cos \beta}{\cos \alpha \cos \beta} = 2 \operatorname{tg} \alpha .$$

$$3. \quad \cos 75^0 + \cos 15^0 = 2 \cos 45^0 \cos 30^0 = \sqrt{2} \cdot \frac{1}{2} = \frac{\sqrt{2}}{2} .$$

C-40

1.

$$a) \quad 2 \arccos \left(-\frac{\sqrt{3}}{2} \right) = \frac{5\pi}{3} ;$$

$$b) \quad \arcsin \frac{1}{\sqrt{2}} - \operatorname{arctg}(-\sqrt{3}) = \frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12} .$$

2.

$$a) \quad \sin \left(x - \frac{3\pi}{5} \right) = -1 ; \quad x = -\frac{\pi}{2} + \frac{3\pi}{5} + 2\pi n = \frac{\pi}{10} + 2\pi n ;$$

$$b) \quad \cos(2x) = \sin x ; \quad 2 \sin^2 x + \sin x - 1 = 0, \quad D = 1 + 8 = 9;$$

$$\sin x = \frac{-1+3}{4} = \frac{1}{2} \quad \text{и} \quad \sin x = -1 ; \quad x = (-1)^k \frac{\pi}{6} + \pi k \quad \text{и} \quad x = -\frac{\pi}{2} + 2\pi n .$$

3.

$$a) \quad \cos 2x \leq -\frac{1}{2}, \quad \frac{2\pi}{3} + 2\pi n \leq 2x \leq \frac{4\pi}{3} + 2\pi n ; \quad x \in \left[\frac{\pi}{3} + \pi n; \frac{2\pi}{3} + \pi n \right] ;$$

$$b) \quad \operatorname{tg} \left(x + \frac{\pi}{3} \right) > \sqrt{3}, \quad x \in \left(\pi n; \frac{\pi}{6} + \pi n \right) .$$

C-41

$$\begin{cases} \sin x + \cos y = 1 \\ \cos^2 x + \sin^2 y = \frac{3}{2} \end{cases} \quad \begin{cases} \sin x = 1 - \cos y \\ 1 - 1 - \cos^2 y + 2 \cos y + \sin^2 y = \frac{3}{2} \end{cases} ;$$

$$\begin{cases} \sin^2 y - \cos^2 y + 2 \cos y = \frac{3}{2} \\ \sin x = 1 - \cos y \end{cases} ; \quad \begin{cases} 2 \cos^2 y - 2 \cos y + \frac{1}{2} = 0 \\ \sin x = 1 - \cos y \end{cases} ;$$

$$\begin{cases} \cos y = \frac{1}{2} \\ \sin x = \frac{1}{2} \end{cases}, \begin{cases} y = \pm \frac{\pi}{3} + 2\pi n \\ x = (-1)^k \frac{\pi}{6} + \pi k \end{cases}.$$

C-42

1.

a) $2x^2 - 3x - 5 \leq 0$, $D=9+40=49$;

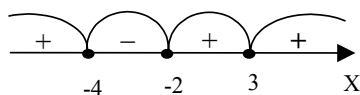
$$x = \frac{3 \pm 7}{4}; x_1 = \frac{5}{2} \quad x_2 = -1; x \in \left[-1; \frac{5}{2}\right].$$

б) $x^2 + 4x + 1 > 0$, $D/4=4-1=3$;

$$x = -2 \pm \sqrt{3}; x \in (-\infty; -2 - \sqrt{3}) \cup (-2 + \sqrt{3}; +\infty).$$

2.

a) $(x+2)^3(x-3)^2(x+4) \leq 0$; $x \in [-4; -2] \cup \{3\}$;

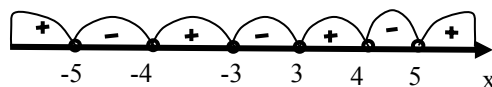


б) $\frac{16}{x^2-9} - \frac{9}{x^2-16} < 0$; $\frac{16x^2 - 256 - 9x^2 + 81}{(x^2-9)(x^2-16)} < 0$;

$$\frac{7x^2 - 175}{(x^2-9)(x^2-16)} < 0$$

$$\frac{x^2 - 25}{(x^2-9)(x^2-16)} < 0$$

$$\frac{(x-5)(x+5)}{(x-3)(x+3)(x-4)(x+4)} < 0; x \in (-5; -4) \cup (-3; 3) \cup (4; 5).$$



C-43

a) $y = 2x^6 + 20\sqrt{x}$; $y' = 12x^5 + \frac{10}{\sqrt{x}}$;

б) $y = x \operatorname{ctgx}$; $y' = \operatorname{ctgx} - \frac{x}{\sin^2 x}$;

$$\text{в)} y = \operatorname{tg} \frac{x}{7}; y' = \frac{1}{7 \cos^2 \frac{x}{7}};$$

$$\text{г)} y = \cos x^2, y' = (-\sin x^2)2x;$$

$$\text{д)} y = \frac{1}{x^9} - \frac{3}{x^3}, y' = -\frac{9}{x^{10}} + \frac{9}{x^4}.$$

C-44

1.

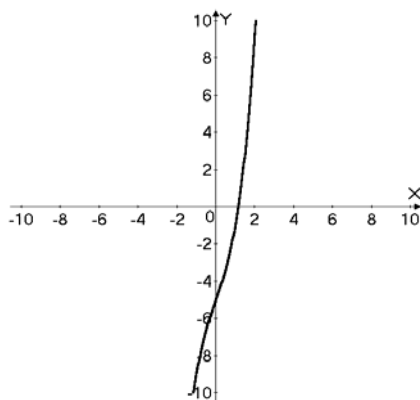
$f(x) = \cos(x+3); f'(x) = -\sin(x+3); f'(-3) = -\sin 0^0 = 0$ – тангенс угла наклона.

2.

$$\text{а)} 1,0007^{300} \approx 1,23; \text{б)} \sin \frac{\pi}{20} \approx 0,157.$$

C-45

1.



$$f(x) = x^3 + 3x - 5;$$

$$f'(x) = 3x^2 + 3;$$

Экстремумов нет, всегда возрастает.

2.

$$y = 4x + \frac{9}{x}; y' = 4 - \frac{9}{x^2} \quad y' = 0 \text{ при } x = \pm \frac{3}{2};$$

$$y\left(\frac{1}{2}\right) = 2 + 18 = 20, \quad y\left(\frac{3}{2}\right) = 6 + 6 = 12;$$

$$y(4) = 16 + \frac{9}{4} = 18,25; \text{ наибольшее значение: } y(4) = 18,25;$$

$$\text{наименьшее значение: } y\left(\frac{3}{2}\right) = 12.$$

3.

$$S(t) = 2t^3 - 2t + 3; \quad S'(t) = 6t^2 - 2; \quad S''(t) = a = 12t;$$

$$F = ma = 12 \cdot 5 \cdot 3 = 180H.$$

ВАРИАНТ 2.

С-1

$$1. \quad 75^0 = \frac{\pi}{180} \cdot 75 = \frac{5\pi}{12}; \quad 168^0 = \frac{\pi}{180} \cdot 168 = \frac{14\pi}{15}.$$

$$2. \quad \frac{5\pi}{6} = 150^0; \quad \frac{17\pi}{36} = 85^0.$$

3.

$$31^0 = \frac{31\pi}{180}; \quad \sin 31^0 \approx 0,595; \quad \cos 31^0 \approx 0,857; \quad 86^0 23' = \frac{5183\pi}{10800};$$

$$\sin 86^0 23' \approx 0,998; \quad \cos 86^0 23' \approx 0,017.$$

4.

$$\text{а) } 0,54 \approx 30^0 56'; \quad \text{б) } 1,4327 \approx 82^0 5'.$$

С-2

$$1. \quad (\sin^4 \alpha + 2 \sin^2 \alpha \cos^2 \alpha + \cos^4 \alpha) + \sin^2 \alpha + \cos^2 \alpha = 2;$$
$$(\sin^2 \alpha + \cos^2 \alpha)^2 + 1 = 2.$$

2.

$$\text{а) } \sin 300^0 \cos 400^0 < 0; \quad \text{б) } \sin(-1) \cos(-2) > 0.$$

$$3. \quad \sin \alpha = \frac{1}{5}, \quad \frac{\pi}{2} < \alpha < \pi; \quad \cos \alpha = -\frac{2\sqrt{6}}{5}.$$

C-3

1. а) $\cos \frac{17\pi}{3} = \cos \frac{\pi}{3} = \frac{1}{2}$; б) $\operatorname{tg} 600^0 = -\operatorname{tg} 120^0 = \operatorname{ctg} 30^0 = \sqrt{3}$.
2. $1 + \operatorname{tg}(\pi + \alpha) \operatorname{ctg}\left(\frac{3\pi}{2} - \alpha\right) = 1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}$.
3. $\sin(\pi - \alpha) \cos\left(\frac{3\pi}{2} + \alpha\right) - \sin^2\left(\alpha + \frac{\pi}{2}\right) =$
 $\sin^2 \alpha - \cos^2 \alpha = -\cos 2\alpha = \cos(\pi - 2\alpha)$.

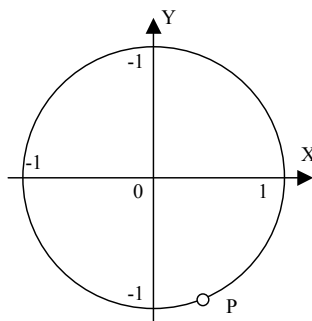
C-4

1. $4 \sin 7^0 30' \cos 7^0 30' \sin 75^0 = 2 \sin 15^0 \cos 15^0 = \sin 30^0 = \frac{1}{2}$.
2. $\sin \alpha = \frac{24}{25}$, $0 < \alpha < \frac{\pi}{2}$; $\cos \alpha = \frac{7}{25}$, $\sin 2\alpha = \frac{336}{625}$;
 $\cos 2\alpha = -\frac{527}{625}$; $\operatorname{ctg} 2\alpha = -\frac{527}{625} \cdot \frac{625}{336} = -\frac{527}{336}$.
3. $(\sin \alpha + \cos \alpha)^2 + 1 - \sin 2\alpha = 1 + \sin 2\alpha + 1 - \sin 2\alpha = 2$.

C-5

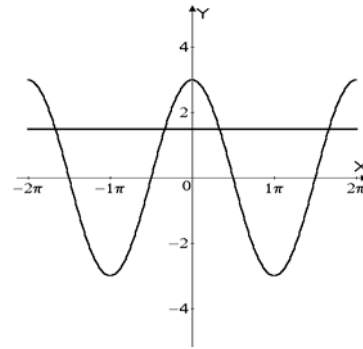
1. см.рис.

абсцисса : $\cos\left(-\frac{\pi}{3}\right) = \frac{1}{2}$; ордината : $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$.



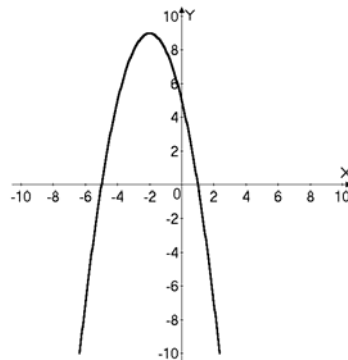
2.
а) II; б) III

3.
см.рис;
 $3 \cos x = 1,5; \cos x = \frac{1}{2};$
 $x = \pm \frac{\pi}{3} + 2\pi n.$



C-6

1.
а) $f(x) = \frac{5}{3x^2 - 2x}$; ОДЗ: $3x^2 - 2x \neq 0; x \neq 0, x \neq \frac{2}{3}$, значит,
 $x \in (-\infty; 0) \cup \left(0; \frac{2}{3}\right) \cup \left(\frac{2}{3}; \infty\right);$
б) $f(x) = \sqrt{9x^2 - 4}$; ОДЗ: $9x^2 - 4 \geq 0; x \in \left(-\infty; -\frac{2}{3}\right] \cup \left[\frac{2}{3}; +\infty\right)$
2. $f(x) = (x+1)^6; f(1) = 64; f(\sqrt{x}-1) = x^3$
3.



C-7

1.

$$f(x) = \sqrt{\frac{1-x^2}{1+x^2}}; \quad f(-x) = \sqrt{\frac{1-(-x)^2}{1+(-x)^2}} = \sqrt{\frac{1-x^2}{1+x^2}} = f(x).$$

2.

$$g(x) = 7x^3 + \sin \frac{x}{2};$$

$$g(-x) = 7(-x)^3 + \sin\left(-\frac{x}{2}\right) = -\left(7x^3 + \sin \frac{x}{2}\right) = -g(x).$$

C-8

1.

а) $\operatorname{tg} 139^\circ = -\operatorname{tg} 41^\circ$; б) $\cos 2743^\circ = -\cos 43^\circ$;

в) $\sin \frac{49\pi}{5} = -\sin \frac{\pi}{5}$.

2.

$$\cos\left(4x + \frac{\pi}{2}\right) + 2\sin(2x - \pi)\cos(2x + \pi) = -\sin 4x + \\ + 2\cos 2x \sin 2x = 0.$$

3.

а) $f(x) = \cos \frac{3x}{2}$, $T = \frac{4\pi}{3}$; б) $f(x) = \operatorname{tg} 5x$, $T = \frac{\pi}{5}$;

в) $f(x) = \sin \frac{x}{3}$, $T = 6\pi$.

C-9

1.

а) см.рис

$$f(x) = -\frac{4}{x}$$

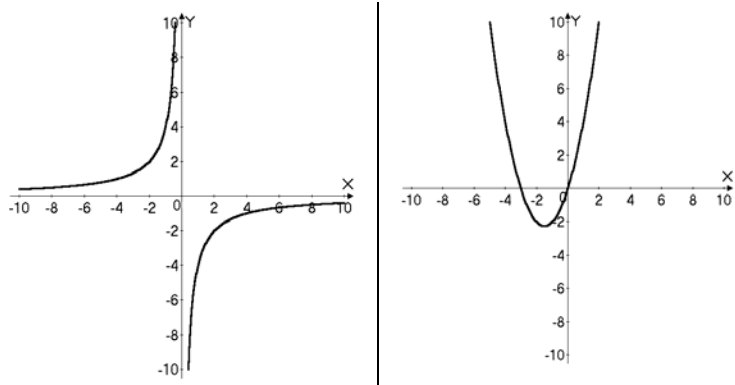
возрастает на обл. опред.

б) см.рис

$$f(x) = 3x + x^2$$

убывает при $x \in (-\infty; -1,5]$;

возрастает при $x \in [-1,5; +\infty)$.



2. $f(x) = \frac{1}{2} \cos \frac{x}{2}$;

возрастает: $[-2\pi + 4\pi n; 4\pi n]$; убывает: $[4\pi n; 2\pi + 4\pi n]$.

3. $\sin 1 < \sin 3$; $\sin 1 > \sin 3$.

C-10

1.

$y = 3x + x^2$;

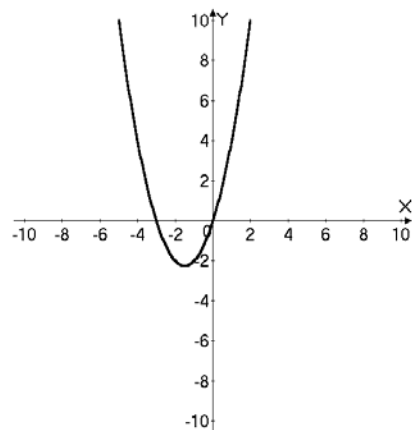
а) $x = -\frac{3}{2}$ — точка минимума;

б) см. рис;

в) $x^2 + 3x > 4$;

$x^2 + 3x - 4 > 0$;

$x \in (-\infty; -4) \cup (1; +\infty)$.



2.

$y = \frac{1}{5} \cos x + 1$;

$x = 2\pi n$ — точка максимума;

$x = \pi + 2\pi n$ — точка минимума;

$y(2\pi n) = 1\frac{1}{5}$; $y(\pi + 2\pi n) = \frac{4}{5}$.

С-11

обл.опр $[-6;10]$; обл.зн $[-3;6]$;
 возрастает при $x \in [-6;-2] \cup [5;10]$; убывает при $x \in [-2;5]$;
 наименьшего значения $y = -3$ функция достигает при $x = 5$;
 наибольшего значения $y = 6$ функция достигает при $x = 10$;
 точка максимума $x = -2$; точка минимума $x = 5$;
 экстремумы: $y_{\min} = -3$; $y_{\max} = 4$;
 функция равна 0 при $x = -6$; $x = 1$; $x = 8$.

С-12

1. $f(x) = \frac{1}{2 \sin 3x}$; ОДЗ: $2^0 < \alpha < \frac{\pi}{2}$;

$x \neq \frac{\pi n}{3}$

2. $y = \frac{1}{2} \cos 2x$

см.рис

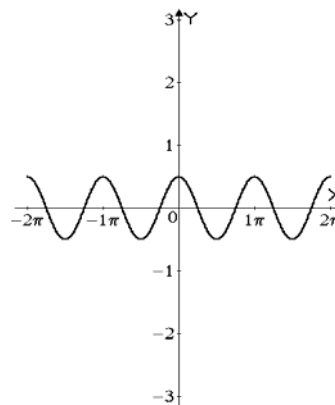
а) $x \in R$

б) $y \in \left[-\frac{1}{2}; \frac{1}{2}\right]$; в) $\frac{\pi}{4} + \frac{\pi n}{2} = x$;

г) $x = \pi n$ — точка максимума;

$x = \frac{\pi}{2} + \pi n$ — точка минимума;

экстремумы: $y(\pi n) = \frac{1}{2}$; $y\left(\frac{\pi}{2} + \pi n\right) = -\frac{1}{2}$.



С-13

1.

а) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$; б) $\arccos(-1) = \pi$;

в) $\arctg(-1) + \arcsin(-1) = -\frac{\pi}{4} - \frac{\pi}{2} = -\frac{3\pi}{4}$;

г) $\cos\left(2 \arcsin \frac{1}{2}\right) = \cos \frac{\pi}{3} = \frac{1}{2}$.

2.

а) $\arcsin 0,8 \approx 0,9273$; **б)** $\arccos(-0,273) \approx 1,8473$; **в)** $\arctg \pi \approx 1,26$.

C-14

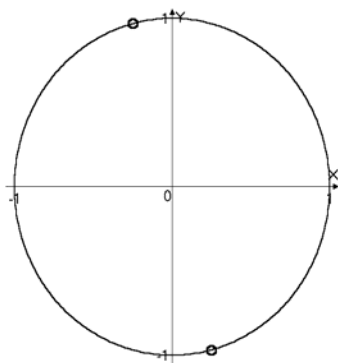
а) $\operatorname{tg} x = -\sqrt{3}$; $x = -\frac{\pi}{3} + \pi n$;

б) $\cos^2 2x = 1$; $\cos 2x = \pm 1$; $x = \frac{\pi n}{2}$;

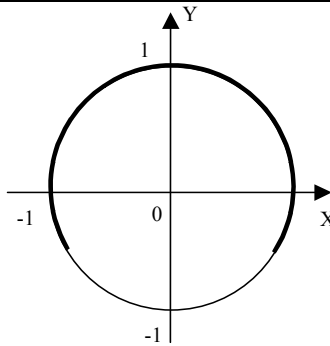
в) $\sin\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$; $x = -\frac{\pi}{4} + (-1)^k \frac{\pi}{4} + \pi k$.

C-15

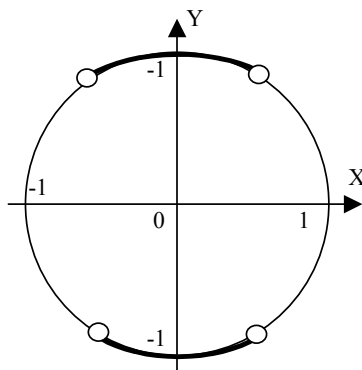
а) см.рис.



б) см.рис.



в) см.рис.



C-16

$$\text{a) } \cos x > -\frac{\sqrt{3}}{2}; \quad x \in \left(-\frac{5\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n\right);$$

$$\text{б) } \operatorname{tg} \frac{x}{2} \leq -1; \quad \frac{x}{2} \in \left(-\frac{\pi}{2} + \pi n; -\frac{3\pi}{4} + \pi n\right]; \quad x \in \left(\pi + 2\pi n; \frac{3\pi}{2} + 2\pi n\right).$$

C-17

$$\text{a) } 2\sin^2 x + \sin x - 1 = 0; \quad D = 1 + 8 = 9;$$

$$\sin x = -1; \quad \sin x = \frac{1}{2};$$

$$x = -\frac{\pi}{2} + 2\pi n; \quad x = (-1)^k \frac{\pi}{6} + \pi k.$$

$$\text{б) } 2\sin^2 x - 2\cos x = \frac{5}{2}; \quad 2\cos^2 x + 2\cos x + \frac{1}{2} = 0; \quad \cos x = -\frac{1}{2};$$

$$x = \pm \frac{2\pi}{3} + 2\pi n.$$

C-18

$$\text{a) } \sin 2x = -\cos 2x; \quad \operatorname{tg} 2x = -1; \quad x = -\frac{\pi}{8} + \frac{\pi n}{2};$$

$$\text{б) } \sin^2 x + 2\sin 2x + 3\cos^2 x = 0; \quad \cos x \neq 0;$$

$$\sin^2 x + 4\sin x \cos x + 3\cos^2 x = 0;$$

$$\operatorname{tg}^2 x + 4\operatorname{tg} x + 3 = 0; \quad \operatorname{tg} x = -3 \quad \operatorname{tg} x = -1$$

$$x = \operatorname{arctg}(-3) + \pi n; \quad x = -\frac{\pi}{4} + \pi k.$$

C-19

$$\begin{cases} x - y = \pi \\ \cos x - \cos y = \sqrt{3} \end{cases}; \quad \begin{cases} x = \pi + y \\ \cos(\pi + y) - \cos y = \sqrt{3} \end{cases};$$

$$\begin{cases} x = \pi + y \\ \cos y = -\frac{\sqrt{3}}{2} \end{cases}; \quad \begin{cases} y = \pm \frac{5\pi}{6} + 2\pi n \\ x = \pi \pm \frac{5\pi}{6} + 2\pi n \end{cases}.$$

C-20

a) $1 + \cos 2x = \sin 2x$;

$$1 + 2 \cos^2 x - 1 = 2 \sin x \cdot \cos x ; \quad \cos x (\cos x - \sin x) = 0$$

$$\cos x = 0 ; \quad \operatorname{tg} x = 1 ;$$

$$x = \frac{\pi}{2} + \pi n ; \quad x = \frac{\pi}{4} + \pi n ;$$

объединяя полученные результаты получим: $x = \frac{\pi}{8} + (-1)^k \frac{\pi}{8} + \frac{\pi k}{2}$.

б) $\sin 3x \sin x + \cos 3x \cos x = -1 ; \cos 2x = -1 ; x = \frac{\pi}{2} + \pi n$

C-21

1.

см.рис.

$$f(x) = 4 - 3x ; \Delta f(x_0) = -3\Delta x ;$$

$$x_0 = -1, \Delta x = 0,3 ; \Delta f(x_0) = -0,9 .$$

2.

$$f(x) = x^2 + x ;$$

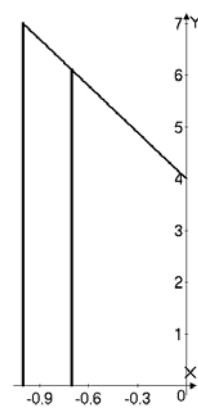
$$\frac{\Delta f(x_0)}{\Delta x} = (\Delta x^2 + 2x_0\Delta x + \Delta x) / \Delta x = \Delta x + 2x_0 + 1$$

$$x_0 = 0, \Delta x = 0,1 ; \frac{\Delta f(x_0)}{\Delta x} = 1,1 ;$$

$$\Delta x = 0,001, \frac{\Delta f(x_0)}{\Delta x} = 1,001 ;$$

$$\Delta x = 0,00001 \frac{\Delta f(x_0)}{\Delta x} = 1,00001 ;$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta f(x_0)}{\Delta x} = 2x_0 + 1 = 1, \text{ так как } x_0 = 0 .$$



C-22

1.

$$x(t) = 100 - t^2 ; V(t) = -2t ; V(4) = -8 \text{ м/с.}$$

2.

a) $f(x) = 5 - 6x ; f'(x) = -6 ;$ **б)** $f(x) = -\frac{1}{x} ; f'(x) = \frac{1}{x^2} .$

C-23

- а) $f(1)=1, g(1)=2$;
б) для f существ, для g нет;
в) $\lim_{x \rightarrow 1} g(x) = 2$, для f несущ.

C-24

1. а) $\lim_{x \rightarrow -3} y = 3 \lim_{x \rightarrow -3} f(x) - 2 \lim_{x \rightarrow -3} g(x) = -3 \cdot 2 - 2 \cdot 5 = -16$;
б) $\lim_{x \rightarrow -3} y = 2 \lim_{x \rightarrow -3} f^2(x) \cdot \lim_{x \rightarrow -3} g(x) = 2 \cdot 4 \cdot 5 = 40$.
2. а) $\lim_{x \rightarrow -1} (x^3 - 4x - 3) = -1 + 4 - 3 = 0$;
б) $\lim_{x \rightarrow 2} \frac{4x+1}{x^2-1} = \frac{9}{3} = 3$.

C-25

1. а) $f(x) = 2x^7 + 4\sqrt{x}$; $f'(x) = 14x^6 + \frac{2}{\sqrt{x}}$;
б) $f(x) = \frac{x^2+1}{x^2-3}$; $f'(x) = \frac{2x^3 - 6x - 2x^3 - 2x}{(x^2-3)^2} = \frac{-8x}{(x^2-3)^2}$.
2. $f(x) = 2x^2 + x^3$; $f'(x) = 4x + 3x^2$; $f(2) = 8 + 12 = 20$;
 $f(4) = 16 + 3 \cdot 16 = 64$; $f(x-3) = (x-3)(-5+3x)$.
3. $f(x) = 4x + 2x^2$; $f'(x) = 4 + 4x \leq 0$; $x \leq -1$.

C-26

1. $f(x) = 50x^5 + 5x^{50}$; $f'(x) = 250x^4 + 250x^{49}$; $f'(-1) = 250 - 250 = 0$.
2. а) $f(x) = x^2 + 3x - 3$; $f'(x) = 2x + 3$; $f'(x) = 0$ при $x = -1,5$;
 $f'(x) > 0$ при $x > -1,5$; $f'(x) < 0$ при $x < -1,5$.

б) $f(x) = \frac{2x-3}{x+2}$; $f'(x) = \frac{2x+4-2x+3}{(x+2)^2} = \frac{7}{(x+2)^2}$
 $f'(x) = 0$ нет решений; $f'(x) > 0$ при $x \in (-\infty; -2) \cup (-2; \infty)$;
 $f' < 0$ ни при каких x .

C-27

1.

$f(x) = \frac{4x-1}{1-16x^2}$; ОДЗ: $1-16x^2 \neq 0$; $x = \pm \frac{1}{4}$, значит,
 $x \in \left(-\infty; -\frac{1}{4}\right) \cup \left(-\frac{1}{4}; \frac{1}{4}\right) \cup \left(\frac{1}{4}; \infty\right)$.

2.

$f(g(x)) = \frac{\sqrt{x}+1}{\sqrt{x}+2}$; $g(f(x)) = \sqrt{\frac{x+1}{x+2}}$.

3.

а) $f(x) = (3-2x)^{160}$; $f'(x) = -320(3-2x)^{159}$;

б) $g(x) = \sqrt{1-x^2}$; $g'(x) = \frac{-x}{\sqrt{1-x^2}}$.

C-28

а) $f(x) = \cos 2x - \sin 3x$; $f'(x) = -2 \sin 2x - 3 \cos 3x$;

б) $f(x) = \operatorname{ctgx} + \operatorname{tg}(x - \pi/4)$; $f'(x) = -\frac{1}{\sin^2 x} + \frac{1}{\cos^2(x - \pi/4)}$;

в) $f(x) = \cos^2 x$; $f'(x) = -2 \sin x \cos x$.

C-29

1.

$f(x) = \frac{x^4+3x^3}{x(x+2)}$; ОДЗ: $x \neq 0$, $x \neq -2$, значит, промежутки

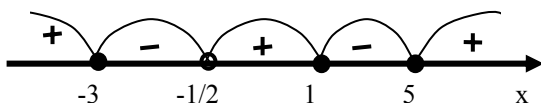
непрерывности: $x \in (-\infty; -2) \cup (-2; 0) \cup (0; \infty)$.

2.

а) $3x^2 - 27 < 0$;
 $(x-3)(x+3) < 0$; $x \in (-3; 3)$;



б) $\frac{(x-1)(x+3)(x-5)}{2x+1} \geq 0$;

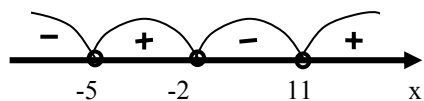


$x \in (-\infty; -3] \cup \left(-\frac{1}{2}; 1\right] \cup [5; +\infty)$;

в) $\frac{x^2 - 9x - 22}{x+5} > 0$;

$\frac{(x-11)(x+2)}{x+5} > 0$;

$x \in (-5; -2) \cup (11; +\infty)$.



C-30

1. $f(x) = x^3 - 27$; $f(x) = 0$ при $x=3$, значит, $x=3$ – точка пересечения графика с осью абсцисс; $f'(x) = 3x^2$, $f'(3) = 27$ – тангенс угла наклона касательной в этой точке.

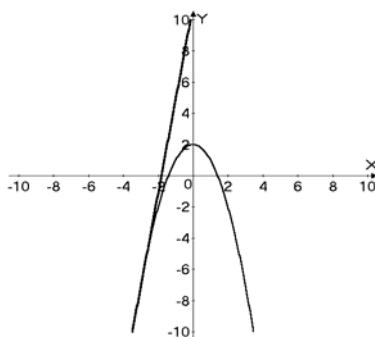
2.

$f(x) = 2 - x^2$; $f(-3) = -7$;

$f'(x) = -2x$, $f'(-3) = 6$;

уравнение касательной

$y = -7 + 6(x+3) = 6x + 11$.



C-31

1. $\sqrt{1 - 0,0016} \approx 1 - 0,008 = 0,992$.

2. $0,9996^{300} \approx 0,88$.

C-32

1. $S(t) = 12t - 3t^3$, $V(t) = 12 - 9t^2$; $a(t) = -18t$, $V(1) = 3$, $a(1) = -18$.
2. $h(t) = 40t - 5t^2$; $h'(t) = 40 - 10t = 0$; $h'(t) = 0$ при $t = 4$, $h(4) = 160 - 80 = 80$ м – наибольшая высота, которой достигнет тело.

C-33

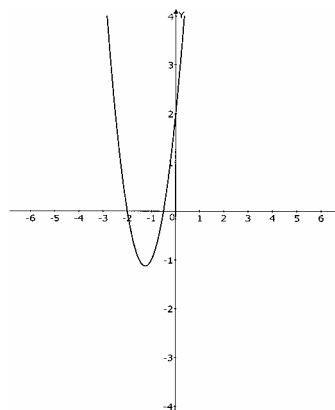
1. $f(x) = x + \frac{4}{x}$; $f'(x) = 1 - \frac{4}{x^2} > 0$; $f'(x) = 0$ при $x \in [-\infty; -2] \cup (2; \infty)$, значит, на этих промежутках данная функция возрастает; $f'(x) < 0$ при $x \in (-2; 2)$, значит, на этих промежутках данная функция убывает.
2. $y = x^3 - 6x^2 - 15x + 7$; $y' = 3x^2 - 12x - 15 = 0$; $x^2 - 4x - 5 = 0$; $x_{\min} = 5$ $x_{\max} = -1$.

C-34

$f(x) = 48x - x^3$; $f'(x) = 48 - 3x^2$; $f'(x) = 0$ при $x = \pm 4$ – экстремумы; функция возрастает при $x \in [-4; 4]$; убывает при $x \in (-\infty; -4] \cup [4; +\infty)$.

C-35

1.



$$y = 2x^2 + 5x + 2; \quad \text{см.рис}; \quad D=25-16=9;$$

$$x_1 = -2, \quad x_2 = -\frac{1}{2}; \quad \text{нули: } (-2;0), \left(-\frac{1}{2};0\right), (0;2);$$

$$\text{убывает: } \left(-\infty; -\frac{5}{4}\right]; \quad \text{возрастает: } \left[-\frac{5}{4}; +\infty\right); \quad x = -\frac{5}{4} - \min$$

2.

$$\text{а) } x^2 + 15x - 16 \geq 0; \quad (x+16)(x-1) \geq 0; \quad x \in (-\infty; -16] \cup [1; +\infty);$$

$$\text{б) } 4x^2 + 12x + 9 \leq 0; \quad (2x+3)^2 \leq 0; \quad \text{неравенству удовлетворяет только}$$

$$x = -\frac{3}{2}.$$

С-36

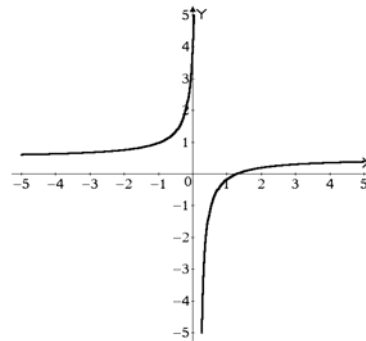
см.рис

$$y = \frac{x+3}{1-2x} + 1$$

$$\text{ОДЗ: } x \neq \frac{1}{2}$$

$$\text{возрастает: } x \neq \frac{1}{2}$$

экстремумов нет



С-37

$$1. \quad y = 2x^4 - 8x; \quad x \in [-2; 1]; \quad y' = 8x^3 - 8; \quad y' = 0 \quad \text{при } y = 1;$$

$$y(1) = -6; \quad y(-2) = 48; \quad \text{значит, } y = 6 - \text{наименьшее значение функции;}$$

$$y = 48 - \text{наибольшее значение функции.}$$

2. Введем функцию $f(x) = x^2 y$, тогда из условия $x + y = 18$, где x и y искомые неотрицательные слагаемые, получаем

$$f(x) = x^2(18 - x) = 18x^2 - x^3; \quad f'(x) = 36x - 3x^2, \quad \text{найдем критические}$$

точки функции $f(x): f'(x) = 0$ при $36x - 3x^2 = 0$; $x = 0$ – посторонний

корень, т.к. $x > 0$ по условию, значит, $x = 12$; $f(12) = 864$ – максимум,

тогда $y = 18 - x = 18 - 12 = 6$, а искомое разбиение: $12 + 6 = 18$.

C-38

1.

$$\cos \alpha = \frac{2}{\sqrt{5}}, \quad 0 < \alpha < \frac{\pi}{2}; \quad \sin \alpha = \frac{1}{\sqrt{5}},$$

$$\operatorname{tg} \alpha = \frac{1}{2}; \quad \operatorname{tg}\left(\alpha + \frac{\pi}{4}\right) = \frac{\operatorname{tg} \alpha + 1}{1 - \operatorname{tg} \alpha} = \frac{3}{2} \cdot 2 = 3.$$

$$2. \quad \frac{\sin \alpha \cos(\pi + \alpha) \cos(\pi - 2\alpha)}{\cos 4\alpha} = \frac{\cos \alpha \sin \alpha \cos 2\alpha}{\cos 4\alpha} = \frac{1}{4} \operatorname{tg} 4\alpha.$$

$$3. \quad \sin 75^0 - \sin 15^0 = 2 \sin 30^0 \cos 45^0 = 2 \cdot \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}.$$

C-39

а)

см.рис;

$$y = \sin 3x, \quad x \in R;$$

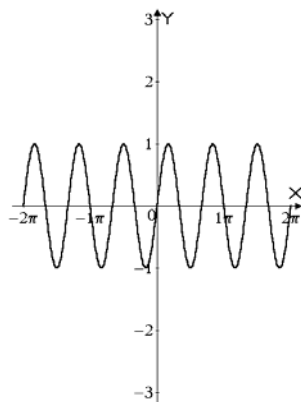
$$\text{нули: } \left(\frac{\pi n}{3}; 0\right);$$

$$\text{возрастает: } \left[-\frac{\pi}{6} + \frac{2\pi n}{3}; \frac{\pi}{6} + \frac{2\pi n}{3}\right];$$

$$\text{убывает: } \left[\frac{\pi}{6} + \frac{2\pi n}{3}; \frac{\pi}{2} + \frac{2\pi n}{3}\right];$$

$$\text{max: } x = \frac{\pi}{6} + \frac{2\pi n}{3};$$

$$\text{min: } x = -\frac{\pi}{6} + \frac{2\pi n}{3}.$$



б)

см.рис;

$$y = \cos \frac{x}{4}, \quad x \in R, \quad y \in [-1; 1];$$

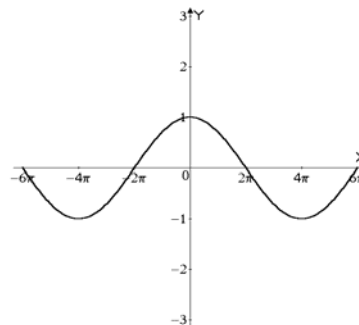
$$\text{нули: } (2\pi + 4\pi n; 0); (0; 1);$$

$$\text{возрастает: } [-4\pi + 8\pi n; 8\pi n];$$

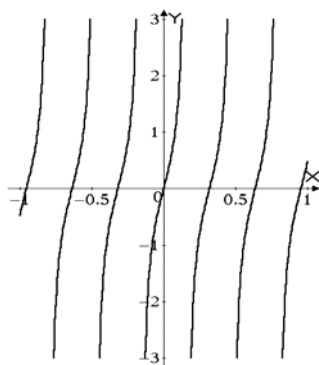
$$\text{убывает: } [8\pi n; 4\pi + 8\pi n];$$

$$\text{min: } x = 4\pi + 8\pi n;$$

$$\text{max: } x = 8\pi n.$$



в)



$$y = \operatorname{tg} \pi x ;$$

$$x \neq \frac{1}{2} + n, \quad y \in R ;$$

возрастает на обл. опр.;

нули: $x = n$;

экстремумов нет.

C-40

1.

$$\text{a) } \arccos \left(-\frac{\sqrt{2}}{2} \right) = \frac{3\pi}{4} ;$$

$$\text{б) } \arcsin \frac{1}{\sqrt{2}} - \operatorname{arctg}(-1) = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2} .$$

2.

$$\text{a) } \cos \left(x + \frac{3\pi}{7} \right) = -1 ; \quad x = \pi + 2\pi n - \frac{3\pi}{7} = \frac{4\pi}{7} + 2\pi n ;$$

$$\text{б) } \cos 2x = \cos x ; \quad 2 \cos^2 x - \cos x - 1 = 0 ; \quad D=1+8=9 ;$$

$$\cos x = 1, \quad x = 2\pi n ; \quad \cos x = -\frac{1}{2}, \quad x = \pm \frac{2\pi}{3} + 2\pi n .$$

3.

$$\text{a) } \sin 2x \geq \frac{1}{2}, \quad x \in \left[\frac{\pi}{12} + \pi n; \frac{5\pi}{12} + \pi n \right] ;$$

$$\text{б) } \operatorname{tg} \left(x + \frac{\pi}{4} \right) > 1, \quad x \in \left(\pi n; \frac{\pi}{4} + \pi n \right) .$$

C-41

$$\begin{cases} \sin x + \cos y = 0 \\ \cos^2 x + \sin^2 y = 1,5 \end{cases} ; \quad \begin{cases} \sin x = -\cos y \\ 1 - \cos^2 y + \sin^2 y = 1,5 \end{cases} ; \quad \begin{cases} \cos 2y = -\frac{1}{2} \\ \sin x = -\cos y \end{cases}$$

$$\begin{cases} y = \pm \frac{\pi}{3} + \pi n \\ \sin y = \pm \frac{1}{2} \end{cases} ; \quad \begin{cases} y = \pm \frac{\pi}{3} + 2\pi n \\ x = (-1)^{k+1} \frac{\pi}{6} + \pi k \end{cases} \quad \text{и} \quad \begin{cases} y = \pm \frac{2\pi}{3} + 2\pi n \\ x = (-1)^k \frac{\pi}{6} + \pi k \end{cases} .$$

C-42

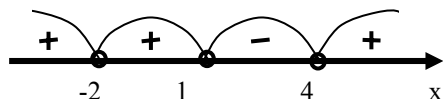
1.

a) $x^2 - 3x - 10 \leq 0$; $(x+2)(x-5) \leq 0$; $x \in [-2; 5]$;

б) $x^2 - 6x + 1 > 0$; $(x - (3 - 2\sqrt{2}))(x - (3 + 2\sqrt{2})) > 0$;
 $x \in (-\infty; 3 - 2\sqrt{2}) \cup (3 + 2\sqrt{2}; +\infty)$.

2.

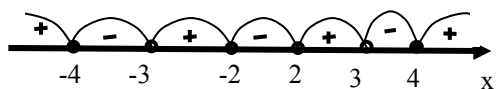
a) $(x-1)(x+2)^2(x-4) \leq 0$;
 $x \in \{-2\} \cup [1; 4]$;



б) $\frac{12}{x^2 - 4} - \frac{7}{x^2 - 9} \geq 0$;

$\frac{12x^2 - 108 - 7x^2 + 28}{(x^2 - 4)(x^2 - 9)} \geq 0$;

$\frac{x^2 - 16}{(x^2 - 4)(x^2 - 9)} \geq 0$;



$\frac{(x-4)(x+4)}{(x-2)(x+2)(x-3)(x+3)} \geq 0$;

$x \in (-\infty; -4] \cup (-3; -2) \cup (2; 3) \cup [4; +\infty)$.

C-43

a) $y = x^7 - 4\sqrt{x}$, $y' = 7x^6 - \frac{2}{\sqrt{x}}$;

б) $y = x \operatorname{tg} x$, $y' = \operatorname{tg} x + \frac{x}{\cos^2 x}$;

в) $y = \operatorname{ctg} \frac{x}{3}$, $y' = -\frac{1}{3 \sin^2 \frac{x}{3}}$;

г) $y = \sin x^3$, $y' = 2x \cos x^2$;

д) $y = \frac{1}{x^4} - \frac{1}{x^8}$, $y' = -\frac{4}{x^5} + \frac{8}{x^9}$.

C-44

1.

$f(x) = \sin(x - 3)$; $f'(x) = \cos(x - 3)$; $f'(3) = 1$ – тангенс угла наклона касательной.

2.

а) $\sqrt{0,9996} = 1 - \frac{0,0004}{2} = 0,9998$;

б) $\sin \frac{\pi}{100} \approx \sin 0,031416 \approx 0,031$.

C-45

1.

см.рис;

$$y = x^3 - 3x + 5;$$

$$x \in \mathbb{R}, y \in \mathbb{R};$$

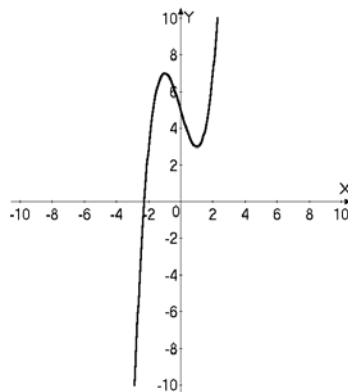
$$y' = 3x^2 - 3, y' = 0 \text{ при } x = \pm 1 -$$

критические точки.

возрастает: $x \in (-\infty; -1) \cup (1; +\infty)$;

убывает: $x \in (-1; 1)$;

$x = -1$ – min, $x = 1$ – max.



2.

$$y = x + \frac{4}{x}; y' = 1 - \frac{4}{x^2}; y' = 0 \text{ при } x = \pm 2;$$

$$y(1) = 5, y(2) = 2 + 2 = 4, y(4) = 5, y(-2) = -4;$$

max: $x = 1, x = 4$;

min: $x = -2$.

3.

$$S(t) = 3t + 2t^3;$$

$$S'(t) = 3 + 6t^2;$$

$$S''(t) = 12t;$$

$$F = ma = 12 \cdot 3 \cdot 4 = 144 \text{ Н.}$$

ВАРИАНТ 3

С-1

1. $64^\circ = \frac{\pi}{180} \cdot 64 = \frac{16\pi}{45}$; $160^\circ = \frac{\pi}{180} \cdot 160 = \frac{8\pi}{9}$.
2. $\frac{3\pi}{5} = 108^\circ$; $1\frac{3}{4}\pi = 135^\circ + 180^\circ = 315^\circ$.
3. $\alpha = \frac{180 \cdot (10 - 2)}{10} = 144^\circ = 0,8\pi$.
4. $\alpha = \frac{\pi}{2} - \frac{\pi}{5} = \frac{3\pi}{10}$; $\sin \alpha = \sin 54^\circ \approx 0,809$; $\operatorname{tg} \alpha \approx 1,3764$.

С-2

1. $\sin \alpha = -\frac{4}{5}$ $180^\circ < \alpha < 270^\circ$; $\cos \alpha = -\frac{3}{5}$; $\operatorname{ctg} \alpha = \frac{3}{4}$.
2. $16\sin^4 \alpha - (\sin^2 \alpha - 3\cos^2 \alpha)^2 = 24\sin^2 \alpha - 9$;
 $(4\sin^2 \alpha - \sin^2 \alpha + 3\cos^2 \alpha)(4\sin^2 \alpha + \sin^2 \alpha - 3\cos^2 \alpha) =$
 $= 15\sin^2 \alpha - 9\cos^2 \alpha = 24\sin^2 \alpha - 9$.
3. а) $\sin \frac{4\pi}{5} \operatorname{tg} \frac{\pi}{7} > 0$; б) $\sin 3 \cos 4 < 0$.

С-3

1. а) $\operatorname{tg}(-390^\circ) = -\operatorname{tg} 30^\circ = -\frac{1}{\sqrt{3}}$; б) $\cos \frac{11\pi}{4} = \cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$.
2. $\sin(180^\circ - \alpha) - \frac{\cos^2(180^\circ + \alpha)}{\cos(\alpha - 270^\circ)} = \sin \alpha + \frac{\cos^2 \alpha}{\sin \alpha} =$
 $= \sin \alpha + \frac{1}{\sin \alpha} - \sin \alpha = \frac{1}{\sin \alpha}$.
3. $\sin 105^\circ \cos 15^\circ + \sin 15^\circ \sin 165^\circ + \operatorname{tg} 225^\circ =$
 $= \cos^2 15^\circ + \sin^2 15^\circ + \operatorname{tg} 45^\circ = 2$.

C-4

1. $\sin \alpha = \frac{4}{5}$; $90^\circ < \alpha < 180^\circ$; $\cos \alpha = -\frac{3}{5}$; $\operatorname{tg} \alpha = -\frac{4}{3}$

а) $\sin 2\alpha = 2 \sin \alpha \cdot \cos \alpha = -\frac{24}{25}$;

б) $\sin (60^\circ - \alpha) = \frac{\sqrt{3}}{2} \cos \alpha - \frac{1}{2} \sin \alpha = -\frac{3\sqrt{3}}{10} - \frac{4}{10} = -\frac{4+3\sqrt{3}}{10}$;

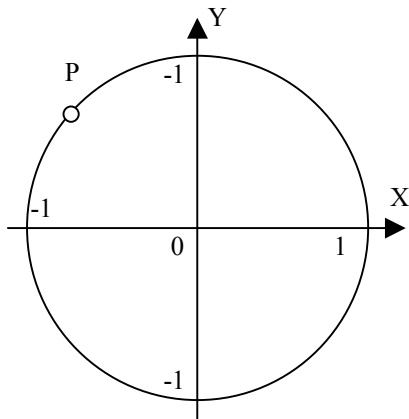
в) $\operatorname{tg} (45^\circ + \alpha) = \frac{1 + \operatorname{tg} \alpha}{1 - \operatorname{tg} \alpha} = -\frac{1}{3} \cdot \frac{3}{7} = -\frac{1}{7}$.

2. $\sin \left(\frac{\pi}{6} + x \right) \cos x - \cos \left(\frac{\pi}{6} + x \right) \sin x = \frac{1}{2}$; $\sin \left(\frac{\pi}{6} + x - x \right) = \frac{1}{2}$.

C-5

1.

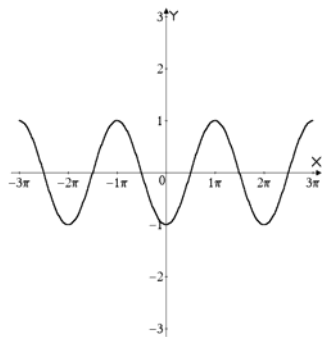
абсцисса : $\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$; ордината : $\sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$



2. а) II ; б) IV.

3.

$\sin \left(\frac{3\pi}{2} + x \right) = -\frac{1}{2}$; $\cos x = \frac{1}{2}$; $x = \pm \frac{\pi}{3} + 2\pi n$.



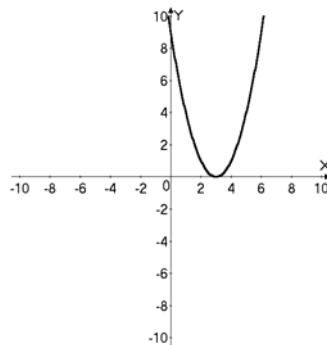
C-6

1. $f(x) = \frac{\sqrt{x}}{2x^2 - 5}$; ОДЗ: $\begin{cases} x \geq 0 \\ 2x^2 - 5 \neq 0 \end{cases}$; $x \in \left[0; \sqrt{\frac{5}{2}}\right) \cup \left(\frac{\sqrt{5}}{\sqrt{2}}; +\infty\right)$.

2. $f(x) = 2\sin 3x + 1$;

а) $f(0) = 1$; б) $f\left(\frac{\pi}{6}\right) = 3$; в) $f\left(-\frac{\pi}{4}\right) = 1 - \sqrt{2}$.

3.



C-7

а) $f(x) = \frac{3x^2}{4\cos x}$; $f(-x) = \frac{3(-x)^2}{4\cos(-x)} = \frac{3x^2}{4\cos x} = f(x)$. Четная

б) $\varphi(x) = 2x^5 + 3\operatorname{ctg} x$; $\varphi(-x) = 2(-x)^5 + 3\operatorname{ctg}(-x) = -2x^5 - 3\operatorname{ctg} x = -\varphi(x)$.
Нечетная.

C-8

1. а) $\sin(-1470^\circ) = -\sin 30^\circ = -\frac{1}{2}$;

б) $\cos(-690^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$;

в) $\operatorname{tg}(-1320^\circ) = -\operatorname{ctg} 30^\circ = -\sqrt{3}$.

2.

$$\frac{2 \cos\left(\frac{\pi}{2} - \alpha\right) \cos \alpha}{\cos(\pi + \alpha) \sin^3\left(\frac{3\pi}{2} + \alpha\right) - \sin(\pi - \alpha) \cos^3\left(\frac{3\pi}{2} + \alpha\right)} = \frac{\sin 2\alpha}{\cos^4 \alpha - \sin^4 \alpha} =$$

$$\frac{\sin 2\alpha}{(\cos^2 \alpha - \sin^2 \alpha)(\cos^2 \alpha + \sin^2 \alpha)} = \frac{\sin 2\alpha}{\cos^2 \alpha - \sin^2 \alpha} = \operatorname{tg} 2\alpha$$

3.

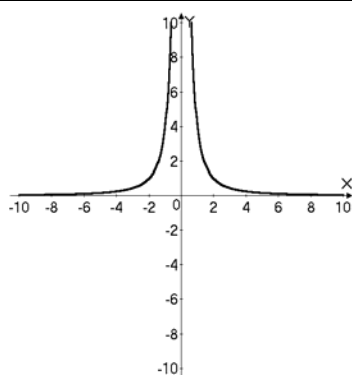
а) $f(x) = \cos\left(\frac{x}{3} + \frac{\pi}{4}\right)$; $T = 6\pi$; б) $f(x) = \operatorname{tg}\left(\frac{2x}{3} + \frac{\pi}{3}\right)$; $T = 1,5\pi$

C-9

1.

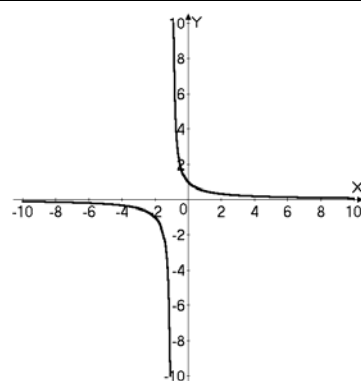
а)

возрастает при $x \in (-\infty; 0)$;
убывает при $x \in (0; +\infty)$;



б)

убывает на всей области
определения.



2.

$$x \in [-\pi; 0]; \quad x \in [\pi; 2\pi]; \quad x \in [3\pi; 4\pi].$$

3.

$$\cos 3 > \cos 6, \quad \cos 3 < 0, \quad \cos 6 > 0, \text{ значит, } \cos 6 > \cos 3.$$

С-10

1.

$$y = \frac{1}{2}x^2 - 2x - \frac{5}{2}$$

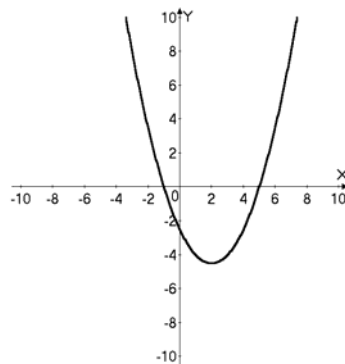
а) $x = 2$ — точка минимума;

$$y = -\frac{9}{2} \text{ — экстремум;}$$

б) см. рис.

$$\text{в) } x^2 - 4x - 5 \leq -5;$$

$$x(x-4) \leq 0; \quad x \in [0; 4].$$

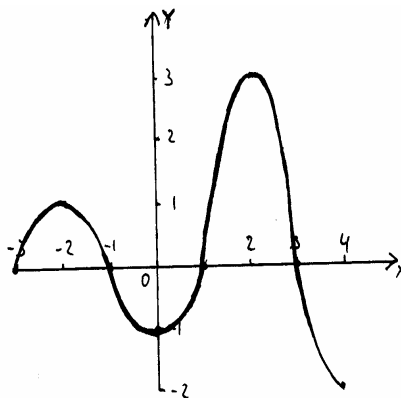


2.

$$y = 3\sin x + 2; \quad x_{\max} = \frac{\pi}{2} + 2\pi n;$$

$$x_{\min} = -\frac{\pi}{2} + 2\pi n; \text{ экстремумы: } y\left(\frac{\pi}{2} + 2\pi n\right) = 5; \quad y\left(-\frac{\pi}{2} + 2\pi n\right) = -1.$$

С-11



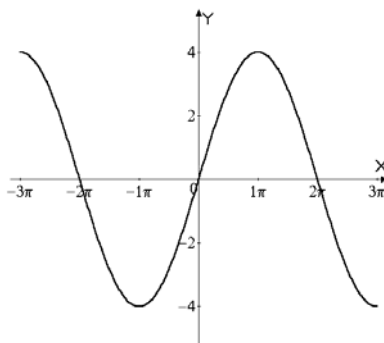
C-12

1.

$$f(x) = 1,5 \operatorname{tg} 1,5x; \quad \text{ОДЗ: } \cos 1,5x \neq 0; \quad \frac{3}{2}x \neq \frac{\pi}{2} + \pi n; \quad x \neq \frac{\pi}{3} + \frac{2\pi n}{3}.$$

2.

$$f(x) = 4 \sin \frac{1}{2} x;$$



- а) $x \in R$; б) $y \in [-4; 4]$; в) $x = 2\pi n$;
 г) $x_{\max} = \pi + 4\pi n$; $x_{\min} = -\pi + 4\pi n$; $y(\pi + 4\pi n) = 4$; $y(-\pi + 4\pi n) = -4$.

C-13

1.

а) $\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$; б) $\operatorname{arctg} \sqrt{3} = \frac{\pi}{3}$;

в) $\sin\left(\arccos\left(-\frac{\sqrt{2}}{2}\right)\right) = \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$;

г) $\operatorname{tg}\left(2 \arcsin\left(-\frac{\sqrt{3}}{2}\right)\right) = -\operatorname{tg} \frac{2\pi}{3} = \sqrt{3}$.

2.

а) $\arcsin(-0,7825) \approx -0,8987$; б) $\arccos(0,1524) \approx 1,4178$;

в) $\operatorname{arctg}\left(-\frac{\pi}{2}\right) \approx -1,0039$.

C-14

a) $\sin x = -1 \quad x = -\frac{\pi}{2} + 2\pi n;$

б) $\cos x = 1; \quad x = 2\pi n;$

в) $\operatorname{tg} 2x = -\sqrt{3}; \quad x = -\frac{\pi}{6} + \frac{\pi n}{2};$

г) $\sin 5x \cos x - \cos 5x \sin x = \frac{1}{2}; \quad \sin 4x = \frac{1}{2}; \quad x = (-1)^k \frac{\pi}{24} + \frac{\pi k}{4}.$

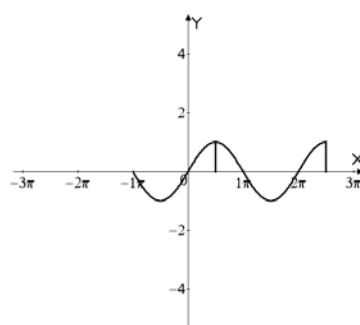
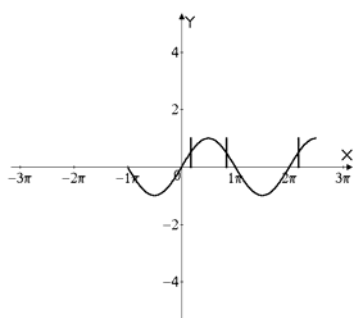
д) $\cos\left(2x + \frac{\pi}{4}\right) \cos x + \sin x \sin\left(2x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2};$

$\cos\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}; \quad x = \pm \frac{\pi}{4} - \frac{\pi}{4} + 2\pi n.$

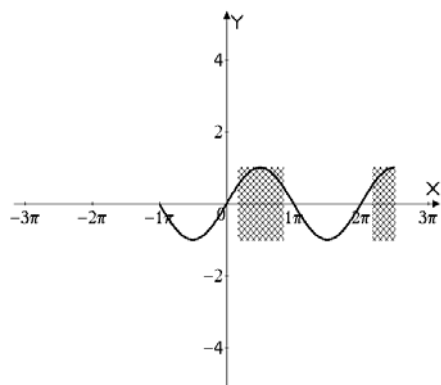
C-15

a) $\sin x = \frac{1}{2}; \quad x = (-1)^k \frac{\pi}{6} + \pi k;$

б) $\sin x = 1; \quad x = \frac{\pi}{2} + 2\pi n;$



в) $\sin x > \frac{1}{2} \quad x \in \left(\frac{\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n\right).$



C-16

а) $\sin x \geq \frac{\sqrt{2}}{2};$

$$x \in \left[\frac{\pi}{4} + 2\pi n; \frac{3\pi}{4} + 2\pi n \right].$$

б) $\cos 2x < -\frac{1}{2};$

$$x \in \left(\frac{\pi}{3} + \pi n; \frac{2\pi}{3} + \pi n \right).$$

в) $\operatorname{tg} x \geq -\sqrt{3};$

$$x \in \left[-\frac{\pi}{3} + \pi n; \frac{\pi}{2} + \pi n \right).$$

C-17

а) $4\sin^2 x - 1 = 0; \quad \sin x = \pm \frac{1}{2}; \quad x = \pm \frac{\pi}{6} + \pi k;$

б) $4\sin^2 x - 4\sin x + 1 = 0; \quad \sin x = \frac{1}{2}; \quad x = (-1)^k \frac{\pi}{6} + \pi k.$

в) $2\sin^2 x + 5\cos x + 1 = 0;$
 $2\cos^2 x - 5\cos x - 3 = 0;$

$\cos x = 3$ решений нет; $\cos x = -\frac{1}{2} \quad x = \pm \frac{2\pi}{3} + 2\pi n.$

C-18

$$\text{a) } \sin 2x + \cos 2x = 0; \quad \sin\left(2x + \frac{\pi}{4}\right) = 0; \quad x = -\frac{\pi}{8} + \frac{\pi n}{2}.$$

$$\begin{aligned} \text{б) } 1 - 2\sin 2x &= 6\cos^2 x; \\ \sin^2 x - 4\sin x \cos x - 5\cos^2 x &= 0; \quad \cos x \neq 0; \\ \operatorname{tg}^2 x - 4\operatorname{tg} x - 5 &= 0; \\ \operatorname{tg} x = 5; \quad x &= \operatorname{arctg} 5 + \pi n; \\ \operatorname{tg} x = -1; \quad x &= -\frac{\pi}{4} + \pi n. \end{aligned}$$

C-19

$$\begin{cases} x + y = \pi \\ \sin x + \sin y = \sqrt{3} \end{cases}; \quad \begin{cases} x = \pi - y \\ \sin(\pi - y) + \sin y = \sqrt{3} \end{cases}; \quad \begin{cases} x = \pi - y \\ \sin y = \frac{\sqrt{3}}{2} \end{cases};$$

$$\begin{cases} y = (-1)^k \frac{\pi}{3} + \pi k \\ x = \pi - (-1)^k \frac{\pi}{3} - \pi k \end{cases}.$$

C-20

$$\text{a) } \sqrt{3} \sin x + \cos x = \sqrt{2}; \quad \sin\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{2}}{2};$$

$$x = (-1)^k \frac{\pi}{4} - \frac{\pi}{6} + \pi k.$$

$$\begin{aligned} \text{б) } (\cos x + \sin x)^2 &= \cos 2x; \\ \cos^2 x + \sin^2 x + 2\sin x \cos x &= 1 - 2\sin^2 x; \quad \sin x(\cos x + \sin x) = 0; \\ \sin x = 0; \quad x &= \pi n; \\ \cos x + \sin x = 0; \quad x &= -\frac{\pi}{4} + \pi n. \end{aligned}$$

C-21

1.

$$f(x) = 3x + 2; \quad \frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{3(x + \Delta x) + 2 - 3x - 2}{\Delta x} = 3.$$

2.

$$f(1) = 1; \quad f(x_0 + \Delta x) = 2,56;$$

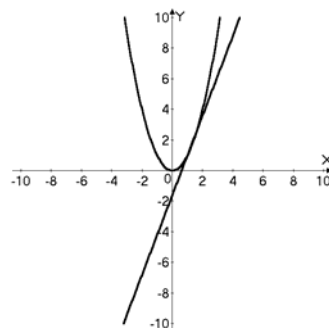
$$\begin{cases} 1 = k + b \\ 2,56 = 1,6k + b \end{cases};$$

$$0,6k = 1,56;$$

$$k = 2,6 - \text{угловой коэффициент};$$

$$b = -1,6;$$

$$y = 2,6x - 1,6 - \text{уравнение секущей}.$$



C-22

1. $x(t) = 2t^2 + 3; \quad v(t) = x'(t) = 4t; \quad v(2) = 8 \text{ м/с}.$

2. $f(x) = \frac{2}{x}; \quad f'(x) = -\frac{2}{x^2}.$

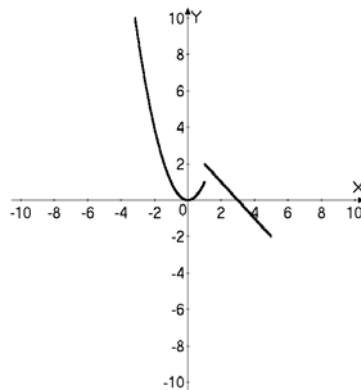
C-23

$$f(x) = \begin{cases} x^2, & x < 1; \\ -x + 3, & x \geq 1 \end{cases};$$

а) возрастает при $x \in (0; 1);$
убывает $x \in$ при $(-\infty; 0) \cup (1; +\infty)$

б) $\lim_{x \rightarrow -1} f(x) = 1;$

в) нет, не существует, т.к. в этой точке не существует производной.



C-24

1. $f(x) = 2x;$

а) (1,95; 2,05); **б)** (1,995; 2,005).

2.

$$\text{а) } \lim_{x \rightarrow 2} \left(\frac{1}{2} f(x) - 2g(x) \right) = \frac{1}{2} \lim_{x \rightarrow 2} f(x) - 2 \lim_{x \rightarrow 2} g(x) = 4 + 1 = 5;$$

$$\text{б) } \lim_{x \rightarrow 2} (3f(x)g(x)) = 3 \lim_{x \rightarrow 2} f(x) \cdot \lim_{x \rightarrow 2} g(x) = 3 \cdot 8 \cdot (-0,5) = -12;$$

$$\text{в) } \lim_{x \rightarrow 2} \frac{f(x) + 2}{4g(x) + 3} = \frac{\lim_{x \rightarrow 2} f(x) + 2}{4 \lim_{x \rightarrow 2} g(x) + 3} = \frac{8 + 2}{4 \cdot (-0,5) + 3} = 10.$$

C-25

$$1. \quad f(x) = x^3 + \frac{3}{2}x^2 - 1; \quad f'(x) = 3x^2 + 3x; \quad f'(x) = 0 \text{ при } x = 0 \text{ и } x = -1.$$

$$2. \quad f(x) = (3 + 2x)(2x - 3) = 4x^2 - 9; \quad f'(x) = 8x; \quad f'\left(\frac{1}{4}\right) = 2.$$

$$3. \quad \varphi(x) = \frac{2x}{1-x};$$

$$\text{а) } \varphi'(x) = \frac{2 - 2x + 2x}{(1-x)^2} = \frac{2}{(1-x)^2};$$

$$\text{б) } \varphi'(x) > 0, \quad \text{при } x \neq 1.$$

C-26

$$1. \quad f(x) = 10x^9 - 9x^{10}; \quad f'(x) = 90(x^8 - x^9); \quad f'(-1) = 180.$$

$$2. \quad y(x) = x^3 + 4x^2 - 3x; \quad y'(x) = 3x^2 + 8x - 3 \leq 0;$$

$$(x+3)\left(x - \frac{1}{3}\right) \leq 0;$$

$$x \in \left[-3; \frac{1}{3}\right].$$

$$3. \quad g(x) = (x-1)\sqrt{x+2}; \quad g'(x) = \sqrt{x+2} + \frac{x-1}{2\sqrt{x+2}};$$

$$g'(-1) = 1 + \frac{-2}{2} = 0.$$

C-27

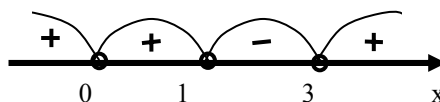
1. $y = \frac{\sqrt{16-x^2}}{x-2}$; ОДЗ: $\begin{cases} 16-x^2 \geq 0 \\ x \neq 2 \end{cases}$; $x \in [-4; 2) \cup (2; 4]$.
2. $\varphi(x) = (5+6x)^{10}$; $\varphi'(x) = 60(5+6x)^9$; $\varphi'(-1) = 60(5-6)^9 = -60$.
3. $f(x) = x+4$; $g(x) = x-4$.

C-28

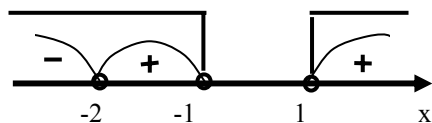
1.
 - а) $f(x) = 3\cos 2x$; $f'(x) = -6\sin 2x$; $f'\left(-\frac{2\pi}{3}\right) = -3\sqrt{3}$.
 - б) $\varphi(x) = 4\operatorname{tg} 3x$; $\varphi'(x) = \frac{12}{\cos^2 3x}$; $\varphi'\left(-\frac{\pi}{3}\right) = 12$.
2. $g(x) = \sin x + \frac{1}{2}\sin 2x$; $g'(x) = \cos x + \cos 2x$; $g'(x) = 0$ при $2\cos^2 x + \cos x - 1 = 0$;
 $\cos x = -1$; $x = \pi + 2\pi n$; $\cos x = \frac{1}{2}$; $x = \pm\frac{\pi}{3} + 2\pi n$.

C-29

а) $\frac{x^2(x-3)}{x-1} < 0$
 $x \in (1; 3)$



б) $(x+2)\sqrt{x^2-1} > 0$;



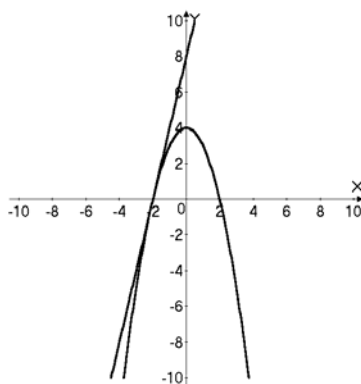
$x \in (-2; -1) \cup (1; +\infty)$;

C-30

$f(x) = 4 - x^2$;

а) $f(-2) = 0$; $f'(x) = -2x$; $f'(-2) = 4$; $y = 4x + 8$ – уравнение касательной.

б)



в) $S = \frac{1}{2} \cdot 2 \cdot 8 = 8$, так как касательная пересекает ось абсцисс в точке -2 , ось ординат в точке 8 .

C-31

а) $\sqrt{16,96} \approx 4,1183$; б) $\frac{2}{1,001^{10}} \approx \frac{2}{1+0,001 \cdot 10} = \frac{2}{1,01} \approx 1,98$.

C-32

1. $x(t) = 3t^3 + 9t^2 + 7$; $v(t) = x'(t) = 9t^2 + 18t$; $v(2) = 36 + 36 = 72$ м/с.
2. $s(t) = (2 + 5t)(2 + 6t)$; $v(t) = S'(t) = 10 + 30t + 12 + 30t = 22 + 60t$;
 $v(3) = 22 + 180 = 202$ см/с.

C-33

а) $f(x) = x^2 + 3x + 6$; $f'(x) = 2x + 3$; $f'(x) > 0$ при $x > -\frac{3}{2}$;

$f'(x) < 0$ при $x < -\frac{3}{2}$, значит,

возрастает при $x \in \left[-\frac{3}{2}; +\infty\right)$, убывает при $x \in \left(-\infty; -\frac{3}{2}\right]$

б) $\varphi(x) = x^3 + 2x - 1$; $\varphi'(x) = 3x^2 + 2$; $\varphi'(x) > 0$ при любых x , значит $\varphi(x)$ возрастает всюду на \mathbb{R} .

в) $g(x) = x^3 - 3x^2 + 5$; $g'(x) = 3x^2 - 6x$ $g'(x) > 0$ при $x \in (-\infty; 0) \cup (2; \infty)$;

$g'(x) < 0$ при $x \in (0; 2)$, значит, возрастает при $x \in (-\infty; 0) \cup (2; +\infty)$
убывает при $(0; 2)$.

C-34

а) $f(x) = x^4 - 8x^2$; $f'(x) = 4x^3 - 16x$; $f'(x) = 0$ при $x = 0$ и $x = \pm 2$;
 $x_{\max} = 0$; $x_{\min} = \pm 2$; $y(0) = 0$; $y(\pm 2) = -16$; $x_{\max} = 4$; $x_{\min} = -4$;

б) $\varphi(x) = \frac{x}{4} + \frac{4}{x}$; $\varphi'(x) = \frac{1}{4} - \frac{4}{x^2}$; $\varphi'(x) = 0$ при $x = \pm 4$;

$\varphi_{\max}(4) = 2$; $\varphi_{\min}(-4) = -2$.

C-35

$$f(x) = -x^2(x^2 - 4) = 4x^2 - x^4$$

$$f'(x) = 8x - 4x^3; f''(x) = 0 \text{ при } x = 0 \text{ и}$$

$$x = \pm \sqrt{2}$$

возрастает при $(-\infty; -\sqrt{2}] \cup [0; \sqrt{2}]$;

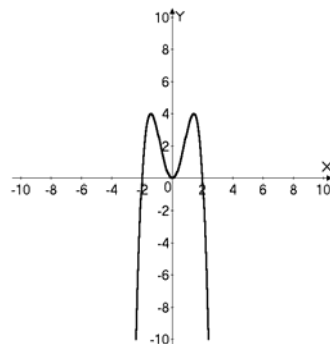
убывает при $[-\sqrt{2}; 0] \cup [\sqrt{2}; +\infty)$;

$$\min: y(0) = 0 \quad \max: y(\pm \sqrt{2}) = 4;$$

$$\text{нули: } x = 0, x = \pm 2;$$

$$y > 0 \text{ при } x \in (-2; 0) \cup (0; 2);$$

$$y < 0 \text{ при } x \in (-\infty; -2) \cup (2; +\infty).$$



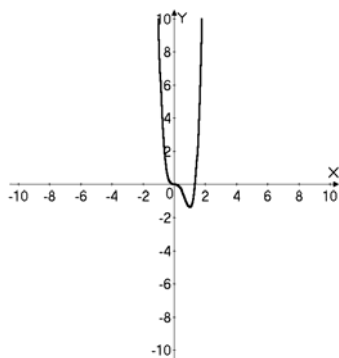
C-36.

$$f(x) = 4x^4 - \frac{16}{3}x^3;$$

$$f'(x) = 16x^3 - 16x^2; f''(x) = 0 \text{ при } x = 0 \text{ и } x = 1;$$

$$\min y(1) = 4 - \frac{16}{3} = -\frac{4}{3}$$

возрастает при $x \geq 1$, убывает при $x \leq 1$.



C-37

1.

$$f(x) = -\cos x - x, \quad x \in \left[-\frac{3}{2}\pi; \frac{5}{2}\pi\right];$$

$$f'(x) = \sin x - 1; \quad f''(x) = 0 \text{ при } x = \frac{\pi}{2} + 2\pi n;$$

$$f\left(\frac{\pi}{2} + 2\pi n\right) = -\frac{\pi}{2} - 2\pi n;$$

$$\max: y\left(-\frac{3}{2}\pi\right) = \frac{3}{2}\pi; \quad \min: y\left(\frac{5}{2}\pi\right) = -\frac{5}{2}\pi.$$

2.

$$\begin{cases} a + b = 15 \\ y = a^2 b \end{cases}; \quad \begin{cases} b = 15 - a \\ y = 15a^2 - a^3 \end{cases};$$

$y' = 30a - 3a^2$; $y'(x) = 0$ при $a = 0$ и $a = 10$; $a = 0$ не подходит, так как по условию $a > 0$, значит, искомая сумма. $10 + 5 = 15$.

C-38

1.

$$\frac{1 - \cos 2\alpha}{\cos^2 \alpha} \cdot \frac{1}{2} \operatorname{ctg} \alpha = \frac{\sin^2 \alpha}{\cos^2 \alpha} \operatorname{ctg} \alpha = \operatorname{tg} \alpha.$$

2.

$$\frac{\sin^4 \alpha + 2 \sin \alpha \cos \alpha - \cos^4 \alpha}{\operatorname{tg} 2\alpha - 1} = \cos 2\alpha;$$

$$\begin{aligned} \frac{\sin 2\alpha - (\sin^2 \alpha + \cos^2 \alpha)(\cos^2 \alpha - \sin^2 \alpha)}{\operatorname{tg} 2\alpha - 1} &= \\ &= \frac{\sin 2\alpha - \cos 2\alpha}{\operatorname{tg} 2\alpha - 1} = \frac{\sin 2\alpha - \cos 2\alpha}{\sin 2\alpha - \cos 2\alpha} \cdot \cos 2\alpha = \cos 2\alpha. \end{aligned}$$

3.

$$1 - \sin^4 22,5^\circ + \cos^4 22,5^\circ = 1 + \cos 45^\circ = \frac{2 + \sqrt{2}}{2}.$$

C-39

1.

$$y = 2\sin \frac{x}{2}$$

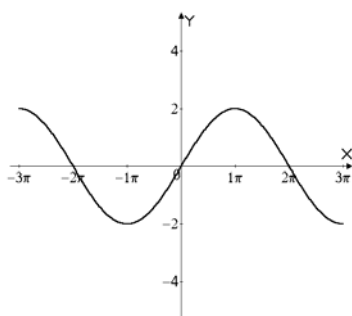
нули функции:

$$2\sin \frac{x}{2} = 0 \text{ при } x = 2\pi n$$

$$\max: x = \pi + 2\pi n; y(\pi + 2\pi n) = 2;$$

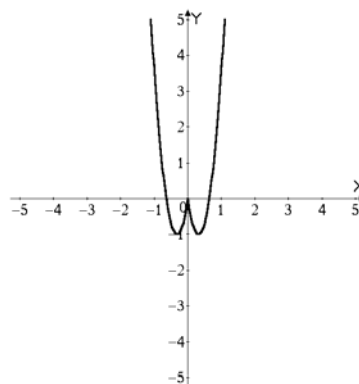
$$\min: x = -\pi + 2\pi n;$$

$$y(-\pi + 2\pi n) = -2.$$



2.

$$\begin{aligned} f(x) &= x^2 - 2|x| \\ f(-x) &= (-x)^2 - 2|-x| = \\ &= x^2 - 2|x| = f(x), \text{ значит,} \\ f(x) &\text{ четная.} \end{aligned}$$



C-40

$$1. \sin x \operatorname{tg} x + \sqrt{3} \sin x + \operatorname{tg} x + \sqrt{3} = 0; (\operatorname{tg} x + \sqrt{3})(\sin x + 1) = 0$$

$$\operatorname{tg} x = -\sqrt{3}; \quad x = -\frac{\pi}{3} + \pi n; \sin x = -1; \quad x = -\frac{\pi}{2} + 2\pi n.$$

$$2. 2\sin 2x + 1 \leq 0 \quad \sin 2x \leq -\frac{1}{2}; x \in \left[-\frac{5\pi}{12} + \pi n; -\frac{\pi}{12} + \pi n \right].$$

$$3. f(x) = 2x - \frac{1}{2} \sin 2x + \sin x; f'(x) = 2 - \cos 2x + \cos x; f'(x) = 0 \text{ при}$$

$$2\cos^2 x - \cos x - 3 = 0;$$

$$\cos x = \frac{3}{2} \text{ — не имеет решения; } \cos x = -1; x = \pi + 2\pi n.$$

C-41

$$\begin{cases} x - y = \frac{\pi}{2} \\ \sin 2x - \sin 2y = \sqrt{2} \end{cases}; \begin{cases} y = x - \frac{\pi}{2} \\ \sin 2x + \sin(\pi - 2x) = \sqrt{2} \end{cases}; \begin{cases} x = (-1)^k \frac{\pi}{8} + \frac{\pi k}{2} \\ y = (-1)^k \frac{\pi}{8} + \frac{\pi k}{2} - \frac{\pi}{2} \end{cases}.$$

C-42

а) $(2x^2 + x + 3)(x^2 - 3x) > 0$;,, поскольку $2x^2 + x + 3 > 0$ при любом x ,
имеем: $x(x - 3) > 0$; $x \in (-\infty; 0) \cup (3; +\infty)$;

б) $\frac{x^4(x^2 - 16)}{x^2 - 2x} \geq 0$; $\frac{x^4(x - 4)(x + 4)}{x(x - 2)} \geq 0$;



$x \in (-\infty; -4] \cup (0; 2) \cup [4; +\infty)$.

в) $(x - 5)\sqrt{x^2 - 4} \leq 0$; $x \in (-\infty; -2] \cup [2; 5]$.

C-43

1.

а) $y = \operatorname{tg} 3x$; $y' = \frac{3}{\cos^2 3x}$;

б) $y = \sqrt{x} \cos x$; $y' = \frac{\cos x}{2\sqrt{x}} - (\sin x) \sqrt{x}$;

в) $y = \sin^2 x$; $y' = 2 \sin x \cos x$;
г) $y = (\cos 3x + 6)^3$ $y' = -9 \sin 3x (\cos 3x + 6)^2$.

2.

$$f(x) = \frac{3x^2 + 4}{2x - 1} + 6 \cos \pi x$$

$$f'(x) = \frac{6x(2x - 1) - 6x^2 - 8}{(2x - 1)^2} - 6\pi \sin \pi x = \frac{6x^2 - 6x - 8}{(2x - 1)^2} - 6\pi \sin \pi x;$$

$$f'(1) = \frac{6 - 6 - 8}{(2 - 1)^2} - 6\pi \sin \pi = -8.$$

C-44

1. $y = x^2 - 3x + 2$; $y' = 2x - 3$; $y_1 = x_0^2 - 3x_0 + 2 + (2x_0 - 3)(x - x_0) -$
уравнение касательной. $2x_0 - 3 = -1$; $x_0 = 1$; $y_1 = 1 - x$.
2. $x(t) = 3\sin 7t$; $v(t) = 21\cos 7t$; $a(t) = -147\sin 7t$.

C-45

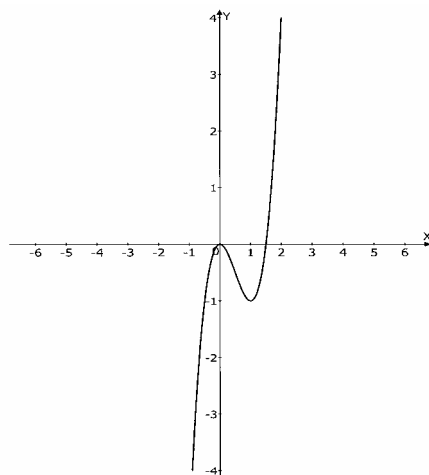
1.

$$\begin{cases} a + b = 8 \\ y = a^2 b^2 \end{cases} \quad \begin{cases} a = b - 8 \\ b^4 - 16b^3 + 64b^2 = y \end{cases}; \quad y' = 4b^3 - 48b^2 + 128b = 0;$$

$$b(b^2 - 12b + 32) = 0$$

$$\begin{cases} b = 0 \\ a = 8 \\ y = 0 \end{cases}; \quad \begin{cases} b = 8 \\ a = 0 \\ y = 0 \end{cases}; \quad \begin{cases} b = 4 \\ a = 4 \\ y = 256 \end{cases}, \text{ значит, } 4 + 4 = 8 - \text{ искомое разбиение.}$$

2.



$$f(x) = x^2(2x - 3) = 2x^3 - 3x^2;$$

$$f'(x) = 6x^2 - 6x = 0; \quad x = 0 \text{ и } x = 1;$$

$$f(0) = \max = 0; \quad f(1) = \min = -1;$$

$$\text{нули: } x = 0 \text{ и } x = \frac{3}{2};$$

$$f'(x) \text{ возрастает при } x \in (-\infty; 0] \cup [1; +\infty); \text{ убывает при } x \in [0; 1].$$

ВАРИАНТ 4

С-1

1. $56^\circ = \frac{\pi}{180} \cdot 56 = \frac{14}{45} \pi$, $170^\circ = \frac{\pi}{180} \cdot 170 = \frac{17}{18} \pi$.
2. $\frac{5\pi}{6} = 150^\circ$; $2\frac{1}{6}\pi = 390^\circ$.
3. $\frac{3\pi}{4}$; $\frac{\pi}{2}$.
4. $\pi - \frac{3\pi}{5} = \alpha$; $\alpha = \frac{2\pi}{5} = 72^\circ$; $\cos \alpha \approx 0,3090$; $\operatorname{tg} \alpha \approx 3,0777$.

С-2

1. $\cos \alpha = -\frac{24}{25}$, $90^\circ < \alpha < 180^\circ$; $\sin \alpha = \frac{7}{25}$, $\operatorname{tg} \alpha = -\frac{7}{24}$.
2. $(\operatorname{tg} \alpha - \sin \alpha) \left(\frac{\cos^2 \alpha}{\sin \alpha} + \operatorname{ctg} \alpha \right) = \sin^2 \alpha$;
 $\frac{1}{\cos \alpha} (\sin \alpha - \sin \alpha \cdot \cos \alpha) \left(\frac{\cos^2 \alpha + \cos \alpha}{\sin \alpha} \right) =$
 $= \cos \alpha - \cos^2 \alpha + 1 - \cos \alpha = \sin^2 \alpha$.
3.
а) $\cos \frac{3\pi}{5} \operatorname{tg} \frac{\pi}{9} < 0$; б) $\sin 4 \cos 5 < 0$.

С-3

1.
а) $\operatorname{ctg} (-420^\circ) = -\operatorname{ctg} 60^\circ = -\frac{\sqrt{3}}{3}$; б) $\sin \left(-\frac{21\pi}{4} \right) = \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$.
2. $\sin (90^\circ + \alpha) - \frac{\cos^2 (\alpha - 90^\circ)}{\sin (\alpha + 270^\circ)} = \cos \alpha + \frac{\sin^2 \alpha}{\cos \alpha} = \frac{1}{\cos \alpha}$.
3. $\sin 32^\circ \sin 148^\circ - \cos 32^\circ \sin 302^\circ + \operatorname{ctg} 225^\circ = 1 + \operatorname{ctg} 45^\circ = 2$.

C-4

1.

$$\cos \alpha = -\frac{4}{5}; \quad 180^\circ < \alpha < 270^\circ; \quad \sin \alpha = -\frac{3}{5}; \quad \operatorname{tg} \alpha = \frac{3}{4}$$

а) $\cos 2\alpha = \frac{7}{25};$

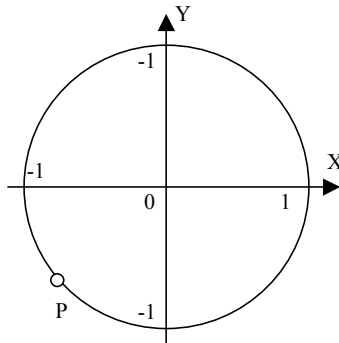
б) $\sin (30^\circ + \alpha) = \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha = -\frac{4}{10} - \frac{3\sqrt{3}}{10} = \frac{-4-3\sqrt{3}}{10};$

в) $\operatorname{tg} (45^\circ - \alpha) = \frac{1 - \frac{3}{4}}{1 + \frac{3}{4}} = \frac{1}{4} \cdot \frac{4}{7} = \frac{1}{7}.$

2. $\cos \left(\frac{\pi}{3} + x \right) \cos x + \sin \left(\frac{\pi}{3} + x \right) \sin x = \frac{1}{2}; \quad \cos \left(\frac{\pi}{3} \right) = \frac{1}{2}.$

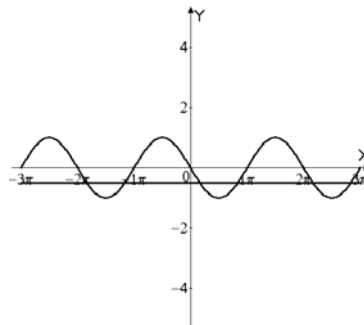
C-5

1. $\sin \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$ – ордината точки $P \frac{5\pi}{4};$
 $\cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$ – абсцисса точки $P \frac{5\pi}{4}.$



2. а) III; б) I.

3.



$$\cos\left(\frac{\pi}{2} + x\right) = -\frac{1}{2}; \quad \sin x = \frac{1}{2}; \quad x = (-1)^k \frac{\pi}{6} + 2\pi k.$$

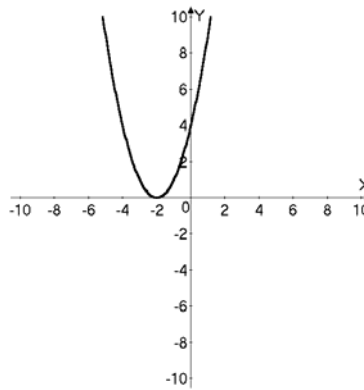
C-6

1. $f(x) = \frac{\sqrt{-x}}{3x^2 - 6}$; ОДЗ: $\begin{cases} x \leq 0 \\ x^2 - 3 \neq 0 \end{cases}$; $x \in (-\infty; -\sqrt{3}) \cup (-\sqrt{3}; 0]$.

2. $f(x) = 3\cos 2x - 1$;

a) $f(\pi) = 2$; б) $f\left(\frac{\pi}{4}\right) = -1$; в) $f\left(-\frac{\pi}{3}\right) = -\frac{5}{2}$.

3.



C-7

а) $f(x) = 2x^3 + \operatorname{tg} x$; $f(-x) = 2(-x)^3 + \operatorname{tg}(-x) = -2x^3 - \operatorname{tg} x = -f(x)$, значит, $f(x)$ нечетная;

б) $\varphi(x) = \frac{2x^4}{\cos x}$; $\varphi(-x) = \frac{2(-x)^4}{\cos(-x)} = \frac{2x^4}{\cos x} = \varphi(x)$, значит, $\varphi(x)$ четная.

C-8

1. **а)** $\sin(-1860^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$;

б) $\cos(-420^\circ) = \cos 60^\circ = \frac{1}{2}$; **в)** $\operatorname{ctg}(-930^\circ) = -\operatorname{ctg} 30^\circ = \sqrt{3}$.

2.
$$\frac{\cos\left(\frac{3\pi}{2} + \alpha\right) \sin^3(\pi - \alpha) - \cos(\pi + \alpha) \sin^3\left(\frac{3\pi}{2} - \alpha\right)}{2 \sin \alpha \sin\left(\frac{\pi}{2} - \alpha\right)} =$$

$$= \frac{\sin^4 \alpha - \cos^4 \alpha}{\sin 2\alpha} = -\operatorname{ctg} 2\alpha.$$

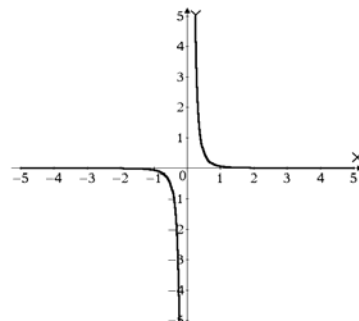
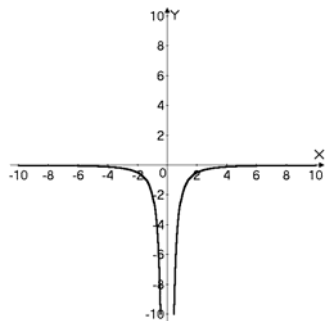
3. **а)** $f(x) = \sin\left(\frac{3x}{4} + \frac{\pi}{3}\right)$; $T = \frac{8\pi}{3}$; **б)** $\varphi(x) = \operatorname{tg}\left(\frac{3x}{5} - \frac{\pi}{6}\right)$; $T = \frac{5\pi}{3}$.

C-9

1.

а) убывает при $x \in (-\infty; 0)$
 возрастает при $x \in (0; +\infty)$

б) убывает на области
 определения.



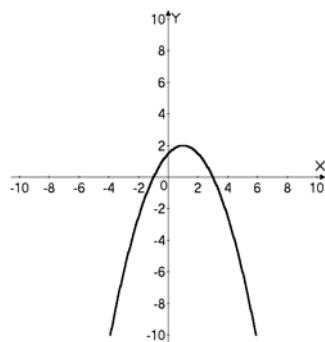
2. $y = 2\sin x - 1$; убывает при
 $x \in \left(\frac{\pi}{2}; \frac{3\pi}{2}\right) \cup \left(\frac{5\pi}{2}; \frac{7\pi}{2}\right) \cup \left(\frac{9\pi}{2}; \frac{11\pi}{2}\right).$

3. $\sin 2 > 0$, $\sin 4 < 0$, значит, $\sin 2 > \sin 4$.

С-10

1. $y = -\frac{1}{2}x^2 + x + \frac{3}{2}$ а) $x = 1$ – точка максимума;
 $y(1) = 2$ – экстремум функции;

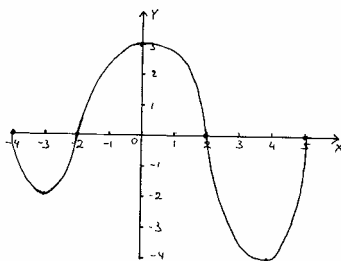
б)



в) $-x^2 + 2x + 3 \geq 3$; $x(x-2) \leq 0$ $x \in [0; 2]$

2. $y = 3\cos x - 2$ $x_{\max} = 2\pi n$; $x_{\min} = \pi + 2\pi n$;
 $y(2\pi n) = 1$; $y(\pi + 2\pi n) = -5$.

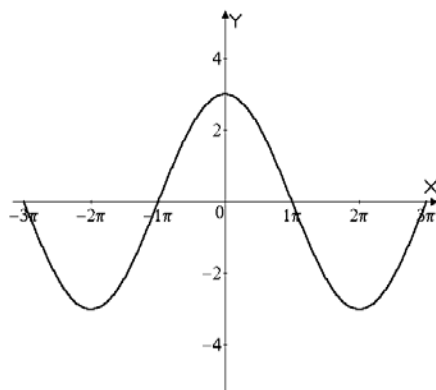
С-11



C-12

1. $f(x) = 2 - \operatorname{ctg} \frac{x}{2}$; ОДЗ: $\sin \frac{x}{2} \neq 0$; $x \neq 2\pi n$

2. $f(x) = 3\cos \frac{x}{2}$;



а) $x \in R$; б) $y \in [-3; 3]$; в) $\cos \frac{x}{2} = 0$ при $x = \pi + 2\pi n$;

г) $x_{\max} = 4\pi n$; $x_{\min} = 2\pi + 4\pi n$; $y(4\pi n) = 3$; $y(2\pi + 4\pi n) = -3$.

C-13

1.

а) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$; б) $\operatorname{arctg}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$;

в) $\operatorname{tg}\left(\arccos\left(-\frac{1}{2}\right)\right) = -\sqrt{3}$;

г) $\cos\left(2\arcsin\left(-\frac{\sqrt{3}}{2}\right)\right) = -\frac{1}{2}$.

2.

а) $\arcsin(-0,9317) = -1,1991$;

б) $\arccos(0,3745) = 1,1869$;

в) $\operatorname{arctg}\left(-\frac{3\pi}{2}\right) = -1,3617$.

C-14

a) $\cos x = -1$; $x = \pi + 2\pi n$; **б)** $\sin x = 1$; $x = \frac{\pi}{2} + 2\pi n$;

в) $\operatorname{tg} 3x = -\frac{\sqrt{3}}{3}$; $x = -\frac{\pi}{18} + \frac{\pi n}{3}$;

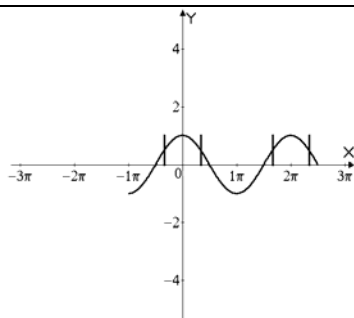
г) $\cos 5x \cos 2x + \sin 5x \sin 2x = \frac{1}{2}$; $\cos 3x = \frac{1}{2}$; $x = \pm \frac{\pi}{9} + \frac{2\pi n}{3}$;

д) $\sin\left(2x + \frac{\pi}{3}\right) \cos x - \cos\left(2x + \frac{\pi}{3}\right) \sin x = \frac{\sqrt{3}}{2}$; $\sin\left(x + \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$;

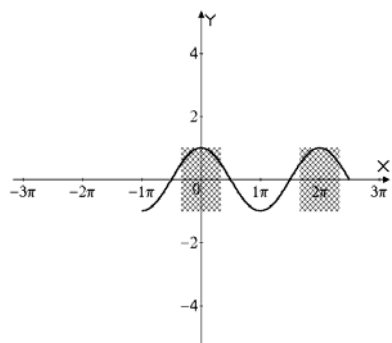
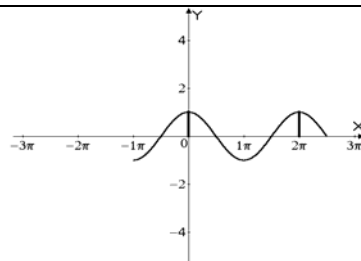
$x = -\frac{\pi}{3} + (-1)^k \frac{\pi}{3} + \pi k$.

C-15

a) $\cos x = \frac{1}{2}$ $x = \pm \frac{\pi}{3} + 2\pi n$;



б) $\cos x = 1$; $x = 2\pi n$;



в) $\cos x > \frac{1}{2}$;

$x \in \left(-\frac{\pi}{3} + 2\pi n, \frac{\pi}{3} + 2\pi n\right)$.

C-16

$$\begin{aligned} \text{а) } \cos x &\geq \frac{\sqrt{2}}{2}; & x &\in \left[-\frac{\pi}{4} + 2\pi n; \frac{\pi}{4} + 2\pi n \right]; \\ \text{б) } \sin 2x &< -\frac{1}{2}; & x &\in \left(-\frac{5\pi}{12} + \pi n; -\frac{\pi}{12} + \pi n \right); \\ \text{в) } \operatorname{tg} x &> -1; & x &\in \left(-\frac{\pi}{4} + \pi n; \frac{\pi}{2} + \pi n \right). \end{aligned}$$

C-17

$$\begin{aligned} \text{а) } 4\cos^2 x - 1 &= 0; \quad \cos x = \pm \frac{1}{2}; & x &= \pm \frac{\pi}{3} + 2\pi n \text{ и} \\ & & x &= \pm \frac{2\pi}{3} + 2\pi n; \\ \text{б) } 4\sin^2 x + 4\sin x + 1 &= 0; \quad \sin x = -\frac{1}{2}; & x &= (-1)^{k+1} \frac{\pi}{6} + \pi k; \\ \text{в) } 2\sin^2 x - 5\cos x + 1 &= 0; & 2\cos^2 x + 5\cos x - 3 &= 0; \\ \cos x &= \frac{-5-7}{4} = -3 \text{ нет решений;} \\ \cos x &= \frac{1}{2}; & x &= \pm \frac{\pi}{3} + 2\pi n. \end{aligned}$$

C-18

$$\begin{aligned} \text{а) } \sin 2x - \sqrt{3} \cos 2x &= 0; & \sin \left(2x - \frac{\pi}{3} \right) &= 0; & x &= \frac{\pi}{6} + \frac{\pi n}{2}; \\ \text{б) } 1 + 2\sin 2x + 2\cos^2 x &= 0; \\ \sin^2 x + 4\sin x \cos x + 3\cos^2 x &= 0; & \cos x &\neq 0; & \operatorname{tg}^2 x + 4\operatorname{tg} x + 3 &= 0; \\ \operatorname{tg} x &= -3; & x &= \operatorname{arctg}(-3) + \pi n; & \operatorname{tg} x &= -1; & x &= -\frac{\pi}{4} + \pi n. \end{aligned}$$

C-19

$$\begin{cases} x + y = \frac{\pi}{2} \\ \sin x + \sin y = -1 \end{cases}; \begin{cases} x = \frac{\pi}{2} - y \\ \sin y + \cos y = -1 \end{cases}; \begin{cases} \sin \left(y + \frac{\pi}{4} \right) = -\frac{\sqrt{2}}{2} \\ x = \frac{\pi}{2} - y \end{cases};$$

$$\begin{cases} y = -\frac{\pi}{4} + (-1)^{k+1} \frac{\pi}{4} + \pi k \\ x = \frac{3\pi}{4} + (-1)^{k+2} \frac{\pi}{4} - \pi k \end{cases}.$$

C-20

a) $\sqrt{3} \sin x - \cos x = 2$; $\sin\left(x - \frac{\pi}{6}\right) = 1$; $x = \frac{2\pi}{3} + 2\pi n$;

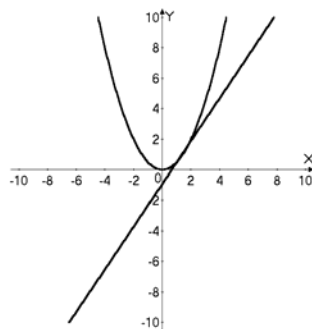
б) $(\cos x - \sin x)^2 = \cos 2x$;
 $(\cos x - \sin x)(\cos x - \sin x - \cos x - \sin x) = 0$;

$$\begin{cases} \cos x = \sin x \\ \sin x = 0 \end{cases}; \begin{cases} x = \frac{\pi}{4} + \pi n \\ x = \pi n \end{cases}.$$

C-21

1. $f(x) = 2x + 3$;

$$\frac{f(x + \Delta x) - f(x)}{\Delta x} = \frac{2(x + \Delta x) + 3 - 2x - 3}{\Delta x} = 2.$$



2. $f(x_0) = \frac{1}{2}$; $f(x_0 + \Delta x) = 1,62$;

$$\begin{cases} 1,62 = k \cdot 1,8 + b \\ \frac{1}{2} = k + b \end{cases}; \begin{cases} 1,12 = 0,8k \\ 0,5 = k + b \end{cases}; \begin{cases} k = 1,4 \\ b = -0,9 \end{cases}.$$

Ответ: $k = 1,4$ – угловой коэффициент;
 $y = 1,4x - 0,9$ – уравнение касательной.

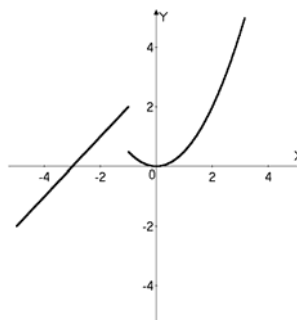
C-22

1. $x(t) = 3t^2 + 2$; $v(t) = 6t$; $v(3) = 18$ м/с.
2. $f(x) = 2\sqrt{x}$; $f'(x) = \frac{1}{\sqrt{x}}$.

C-23

$$f(x) = \begin{cases} 0,5x^2, & x \geq -1; \\ x+3, & x < -1; \end{cases}$$

- а)** возрастает при $x \in (-\infty; -1) \cup (0; +\infty)$;
убывает при $(-1; 0)$;
- б)** $\lim_{x \rightarrow 1} f(x) = \frac{1}{2}$;
- в)** нет, т.к. в точке $x = -1$ не существует производной.



C-24

1. **а)** $\frac{59}{30} < x < \frac{61}{30}$;
б) $1\frac{299}{300} < x < 2\frac{1}{300}$.
2. **а)** $\lim_{x \rightarrow 3} \left(\frac{1}{2} f(x) - 2g(x) \right) = 6$; **б)** $\lim_{x \rightarrow 3} (2f(x) g(x)) = -18$;
в) $\lim_{x \rightarrow 3} \frac{f(x) - 2}{2g(x) + 5} = \frac{4}{5 - 3} = 2$.

C-25

1. $f(x) = 2x^3 - 3x^2 + 1$; $f'(x) = 6(x^2 - x)$; $f''(x) = 0$ при $x = 0$ и $x = 1$.
2. $f(x) = (1 + 2x)(2x - 1) = 4x^2 - 1$; $f'(x) = 8x$; $f'\left(\frac{1}{2}\right) = 4$.

3.

$$\varphi(x) = \frac{6x}{x+1};$$

а) $\varphi'(x) = \frac{6x+6-6x}{(x+1)^2} = \frac{6}{(x+1)^2}$; **б)** $\varphi'(x) > 0$, при $x \neq -1$.

C-26

1. $f(x) = 8x^9 - 9x^8$; $f'(x) = 72(x^8 - x^7)$; $f'(-1) = 144$.

2. $y(x) = 2x^3 - 9x^2 + 12x + 7$; $y'(x) = 6x^2 - 18x + 12 \geq 0$;
 $x^2 - 3x + 2 \geq 0$; $x \in (-\infty; 1] \cup [2; +\infty)$.

3. $g(x) = \sqrt{x-3}(x+2)$; $g'(x) = \sqrt{x-3} + \frac{x+2}{2\sqrt{x-3}}$; $g'(4) = 1 + \frac{3}{1} = 4$.

C-27

1. $y = \frac{\sqrt{x^2 - 25}}{x+7}$; ОДЗ: $\begin{cases} x^2 - 25 \geq 0 \\ x \neq -7 \end{cases}$;
 $x \in (-\infty; -7) \cup (-7; -5] \cup [5; +\infty)$.

2. $\varphi(x) = (2x+3)^{12}$; $\varphi'(x) = 24(2x+3)^{11}$; $\varphi'(-2) = -24$.

3. $f(x) = x - 7$; $f(g(x)) = x$, значит, $g(x) = x + 7$.

C-28

1. **а)** $f(x) = 2\sin 5x$; $f'(x) = 10\cos 5x$; $f'\left(-\frac{\pi}{3}\right) = 5$;

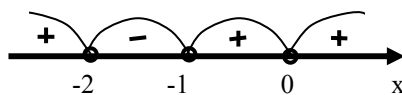
б) $\varphi(x) = 3\operatorname{ctg} 2x$; $\varphi'(x) = -\frac{6}{\sin^2 2x}$; $\varphi'\left(-\frac{\pi}{4}\right) = -6$.

2. $f(x) = \cos x - \frac{1}{4} \cos 2x$; $f'(x) = -\sin x + \frac{1}{2} \sin 2x$; $f'(x) = 0$ при $\sin x (\cos x - 1) = 0$; $x = \pi n$.

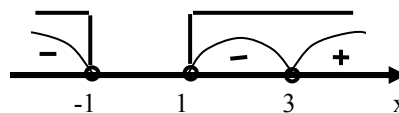
C-29

а) $\frac{x^2(x+2)}{x+1} < 0;$

$x \in (-2; -1);$



б) $(x-3)\sqrt{x^2-1} < 0;$



$x \in (-\infty; -1) \cup (1; 3).$

C-30

$f(x) = x^2 - 4; \quad f'(x) = 2x;$

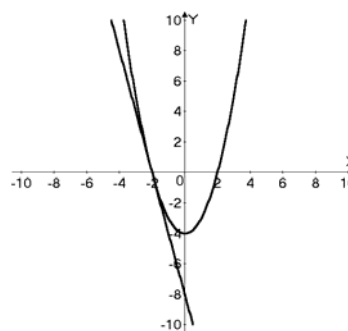
а) $f(-2) = 0; \quad f'(-2) = -4;$

$y = -4(x+2) = -4x - 8$ –

уравнение касательной;

б) см. рис;

в) $S = \frac{1}{2} \cdot 8 \cdot 2 = 8.$



C-31

а) $\sqrt{9,72} \approx 3,1177;$ б) $\frac{3}{1,002^{-20}} \approx 3 \cdot (1 + 0,002 \cdot 20) = 3,12.$

C-32

1. $x(t) = 4t^3 + 5t^2 + 4; \quad v(t) = 12t^2 + 10t; \quad v(3) = 138 \text{ м/с.}$

2. $R = 4 + 2t^2; \quad S(t) = \pi(16 + 4t^4 + 16t^2);$
 $S'(t) = 16\pi t^3 + 32\pi t; \quad S'(2) = 192\pi \text{ см/с.}$

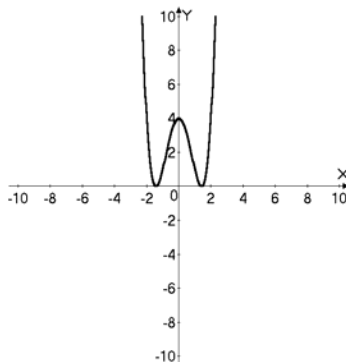
C-33

- а) $f(x) = -x^2 + 4x - 3$;
 возрастает при $x \in (-\infty; 2)$
 убывает при $x \in (2; +\infty)$
- б) $\varphi(x) = x^3 + 4x - 7$
 $\varphi'(x) = 3x^2 + 4 > 0$ при любых x , значит, $\varphi(x)$ возрастает на R ;
- в) $g(x) = 2x^3 - 3x^2 + 1$;
 $g'(x) = 6(x^2 - x)$;
 возрастает при $x \in (-\infty; 0) \cup (1; +\infty)$;
 убывает при $x \in (0; 1)$.

C-34

- а) $f(x) = 2x^4 - 4x^2 + 1$; $f'(x) = 8(x^3 - x)$; $f(x) = 0$ при $x = 0$ и $x = \pm 1$;
 $x_{\max} = 0$; $x_{\min} = \pm 1$; $y(0) = 1$; $y(\pm 1) = -1$;
- б) $\varphi(x) = \frac{x}{4} + \frac{9}{x}$; $\varphi'(x) = \frac{1}{4} - \frac{9}{x^2}$; $\varphi'(x) = 0$ при $x = \pm 6$; $x_{\max} = 6$; $x_{\min} = -6$;
 $\varphi(6) = \frac{3}{2} + \frac{3}{2} = 3$; $\varphi(-6) = -3$.

C-35



- $f(x) = (x^2 - 2)^2 = x^4 - 4x^2 + 4$; $f'(x) = 4(x^3 - 2x)$; $f'(x) = 0$ при
 $x = 0$ и $x = \pm \sqrt{2}$; $x_{\max} = 0$; $x_{\min} = \pm \sqrt{2}$;
 $f(0) = 4$; $f(\pm \sqrt{2}) = 0$;
 убывает при $x \in (-\infty; -\sqrt{2}) \cup (0; \sqrt{2})$
 возрастает при $x \in (-\sqrt{2}; 0) \cup (\sqrt{2}; +\infty)$

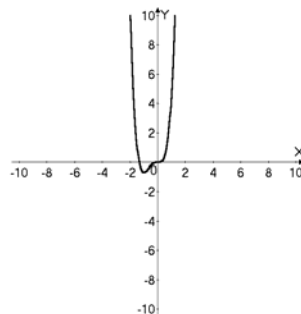
C-36

$$f(x) = 2x^4 + \frac{8}{3}x^3;$$

$$f'(x) = 8x^3 + 8x^2 = 8x^2(x+1); f'(x) = 0 \text{ при } x = 0, x = -1; x_{\min} = -1;$$

$$f(-1) = 2 - \frac{8}{3} = -\frac{2}{3};$$

возрастает при $x \in (-1; +\infty)$;
убывает при $x \in (-\infty; -1)$.



C-37

1.
 $f(x) = \sin x + x; x \in [-\pi; \pi]; f'(x) = \cos x + 1; f'(x) = 0 \text{ при } x = \pi + 2\pi n;$
наибольшее значение $f(\pi) = \pi$; наименьшее значение $f(-\pi) = -\pi$.

2.
 $\begin{cases} a + b = 20 \\ y = a^3 b \end{cases}; \begin{cases} b = 20 - a \\ y = 20a^3 - a^4 \end{cases}; y' = 60a^2 - 4a^3; y' = 0 \text{ при}$
 $\begin{cases} a = 0 \\ b = 20 \\ y = 0 \end{cases} \text{ и } \begin{cases} a = 15 \\ b = 5 \\ y = 16875 \end{cases}; \text{ Ответ: } 15 + 5 = 20.$

C-38

1. $\frac{1 + \cos 2\alpha}{2 \sin^2 \alpha} \operatorname{tg} \alpha = \operatorname{ctg}^2 \alpha \operatorname{tg} \alpha = \operatorname{ctg} \alpha.$
2. $\frac{2 \sin 2\alpha + \sin 4\alpha}{2(\cos \alpha + \cos 3\alpha)} = \operatorname{tg} 2\alpha \cos \alpha;$
 $\frac{\sin 2\alpha(1 + \cos 2\alpha)}{\cos \alpha + \cos 3\alpha} = \frac{\sin 2\alpha(1 + \cos 2\alpha)}{2 \cos 2\alpha \cos \alpha} = \operatorname{tg} 2\alpha \cos \alpha.$
3. $1 - \sin^4 15^\circ - \cos^4 15^\circ =$
 $= 1 - (\sin^2 15^\circ + \cos^2 15^\circ)^2 + 2 \sin^2 15^\circ \cos^2 15^\circ = \frac{1}{2} \sin^2 30^\circ = \frac{1}{8}.$

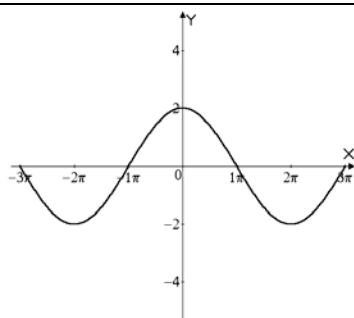
C-39

1. $y = 2\cos \frac{x}{2}$; $y = 0$ при

$$x = \pi + 2\pi n$$

$$x_{\max} = 4\pi n; \quad x_{\min} = 2\pi + 4\pi n;$$

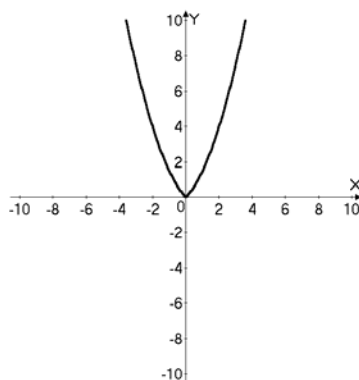
$$y(4\pi n) = 2; \quad y(2\pi + 4\pi n) = -2.$$



2. $f(x) = 0,5x^2 + |x|$;

$$f(-x) = \frac{1}{2}(-x)^2 + |-x| =$$

$$= \frac{1}{2}x^2 + |x| = f(x), \text{ значит, } f(x) \text{ четная}$$



C-40

1. $\sqrt{3} \operatorname{tg} x \sin x - \sqrt{3} \operatorname{tg} x + \sin x - 1 = 0$;

$$(\sqrt{3} \operatorname{tg} x + 1)(\sin x - 1) = 0;$$

$$\begin{cases} \operatorname{tg} x = -\frac{\sqrt{3}}{3}; \\ \sin x = 1 \end{cases} \begin{cases} x = -\frac{\pi}{6} + \pi n \\ x = \frac{\pi}{2} + 2\pi n \end{cases}.$$

2. $2\cos 3x + 1 \leq 0; \quad \cos 3x \leq -\frac{1}{2}; \quad x \in \left[\frac{2\pi}{9} + \frac{2\pi n}{3}; \frac{4\pi}{9} + \frac{2\pi n}{3} \right].$

3. $f(x) = \frac{1}{2} \sin 2x - \cos x + 2x$; $f'(x) = \cos 2x + \sin x + 2$; $f'(x) = 0$ при

$$2\sin^2 x - \sin x - 3 = 0; \quad \sin x = \frac{3}{2} \text{ — не имеет решений; } \sin x = -1,$$

$$\text{значит, } x = -\frac{\pi}{2} + 2\pi n.$$

C-41

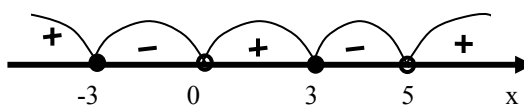
$$\begin{cases} x + y = \frac{\pi}{2} \\ \cos 2x - \cos 2y = -\sqrt{3} \end{cases}; \begin{cases} x = \frac{\pi}{2} - y \\ \cos(\pi - 2y) - \cos 2y = -\sqrt{3} \end{cases}$$

$$\begin{cases} \cos 2y = \frac{\sqrt{3}}{2} \\ x = \frac{\pi}{2} - y \end{cases}; \begin{cases} y = \pm \frac{\pi}{12} + \pi n \\ x = \frac{\pi}{2} \mp \frac{\pi}{12} - \pi n \end{cases}.$$

C-42

a) $(3x^2 + 2x + 5)(x^2 + 4x) < 0$; так как $3x^2 + 2x + 5 > 0$ при любом x , то $(x^2 + 4x) < 0$; $x \in (-4; 0)$;

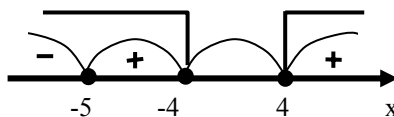
б) $\frac{x^4(x^2 - 9)}{x^2 - 5x} \leq 0$; ОДЗ: $x \neq 0, x \neq 5$;



$$\frac{x^4(x-3)(x+3)}{x(x-5)} \leq 0;$$

$$x \in [-3; 0) \cup [3; 5).$$

в) $(x+5)\sqrt{x^2 - 16} \geq 0$;



$$x \in [-5; -4] \cup [4; +\infty).$$

C-43

1.

a) $y = \operatorname{ctg} 2x, y' = \frac{-2}{\sin^2 2x}$; **б)** $y = \sqrt{x} \sin x, y' = \frac{\sin x}{2\sqrt{x}} + (\cos x) \sqrt{x}$;

в) $y = \cos^2 x; y' = -2\cos x \sin x$;

г) $y = (\sin 2x - 5)^3; y' = 3 \cdot 2\cos 2x(\sin 2x - 5)^2 = 6\cos 2x(\sin 2x - 5)^2$.

2.

$$f(x) = \frac{2x^2 - 3}{4x + 3} + 8 \sin \frac{\pi x}{2}; \quad f'(x) = \frac{16x^2 + 12x - 8x^2 + 12}{(4x + 3)^2} + 4\pi \cos \frac{\pi x}{2};$$

$$f'(-1) = \frac{16 - 8 - 12 + 12}{1} = 8.$$

С-44

1. $y = -x^2 + 3x - 2$, $y' = -2x + 3$; $-2x_0 + 3 = 1$, $x_0 = 1$, $y_0(1) = 0$,
значит в точке $(1, 0)$ касательная параллельна прямой $y = x$.

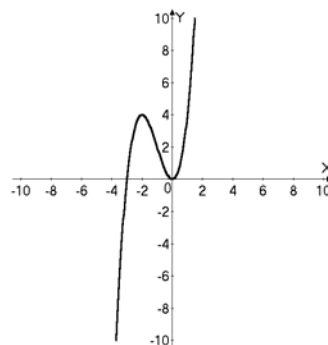
2. $x(t) = 2 \cos 4t$; $v(t) = -8 \sin 4t$; $a(t) = -32 \cos 4t > 0$ при
 $\cos 4t < 0$; $t \in \left[-\frac{3\pi}{8} + \frac{\pi n}{2}; -\frac{\pi}{8} + \frac{\pi n}{2} \right]$.

С-45

1. $\begin{cases} a + b = 12 \\ y = a^3 \cdot 3 \cdot b \end{cases}; \begin{cases} b = 12 - a \\ y = 36a^3 - 3a^4 \end{cases}; y' = 108a^2 - 12a^3; y' = 0$ при
 $\begin{cases} a = 0 \\ b = 12 \\ y = 0 \end{cases}$ и $\begin{cases} a = 9 \\ b = 3 \\ y = 6561 \end{cases}$.

Ответ: $9 + 3$.

2. $f(x) = x^2(x + 3) = x^3 + 3x^2$;
 $f'(x) = 3x^2 + 6x = 0$; $f'(x) = 0$ при
 $x = 0$ и $x = -2$; $x_{\min} = 0$; $x_{\max} = -2$;
 $f(0) = 0$; $f(-2) = -8 + 12 = 4$;
возрастает при $x \in (-\infty; 2) \cup (0; +\infty)$
убывает при $x \in (-2; 0)$
нули: $x = 0$ и $x = -3$.



ВАРИАНТ 5

С-1

1. $72^\circ = \frac{\pi}{180} \cdot 72 = \frac{2\pi}{5}$; $140^\circ = \frac{\pi}{18} \cdot 14 = \frac{7\pi}{9}$.
2. $\frac{11\pi}{12} = 165^\circ$; $\frac{23\pi}{8} = 517,5^\circ$.
3. $79^\circ = \frac{\pi}{180} \cdot 79 \approx 1,3781$; $\sin 79^\circ \approx 0,9816$, $\cos 79^\circ \approx 0,1908$;
 $38^\circ 22' \approx 0,6696$; $\sin 38^\circ 22' \approx 0,6187$, $\cos 38^\circ 22' \approx 0,7856$.
4. а) $0,7575 \approx 43^\circ 24'$; б) $2,0365 \approx 116^\circ 41'$.

С-2

1. $1 + \frac{\sin^4 \alpha + \sin^2 \alpha \cos^2 \alpha}{\cos^2 \alpha} = \frac{1}{\cos^2 \alpha}$; $1 + \frac{\sin^2 \alpha}{\cos^2 \alpha} = 1 + \operatorname{tg}^2 \alpha = \frac{1}{\cos^2 \alpha}$.
2. а) $\frac{\cos 200^\circ \operatorname{tg} 300^\circ}{\sin 400^\circ} > 0$; б) $\cos 2 \operatorname{tg} 4 < 0$.
3. $\cos \alpha = -\frac{2}{\sqrt{5}}$; $\alpha \in \text{III четверти}$; $\sin \alpha = -\frac{1}{\sqrt{5}}$; $\operatorname{tg} \alpha = \frac{1}{2}$.

С-3

1. а) $\sin 1050^\circ = -\sin 30^\circ = -\frac{1}{2}$; б) $\cos \frac{23\pi}{6} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$;
в) $\operatorname{tg} 2130^\circ = -\operatorname{tg} 30^\circ = -\frac{1}{\sqrt{3}}$.
2. $\frac{\sin^2\left(\frac{\pi}{2} + \alpha\right) - \cos^2\left(\alpha - \frac{3\pi}{2}\right)}{\operatorname{tg}^2\left(\frac{3\pi}{2} + \alpha\right) - \operatorname{ctg}^2\left(\alpha - \frac{\pi}{2}\right)} = \frac{\cos^2 \alpha - \sin^2 \alpha}{\operatorname{ctg}^2 \alpha - \operatorname{tg}^2 \alpha} = \sin^2 \alpha \cos^2 \alpha$.
3. $\frac{\cos(-\alpha)}{\sin\left(\frac{\pi}{2} + \alpha\right)} = \operatorname{tg}\left(\frac{3\pi}{2} - \alpha\right) \operatorname{ctg}\left(\frac{\pi}{2} - \alpha\right)$; $\frac{\cos \alpha}{\cos \alpha} = \operatorname{tg} \alpha \operatorname{ctg} \alpha = 1$.

C-4

- $$\frac{1 - \sin^2 22^\circ 30'}{2 \cos^2 15^\circ - 1} = \frac{\cos^2 22^\circ 30'}{\cos 30^\circ} = \frac{2}{\sqrt{3}} \cdot \frac{1}{2} (1 + \cos 45^\circ) = \frac{\sqrt{2} + 1}{\sqrt{6}}.$$
- $$\cos \alpha = -\frac{5}{13}; \quad \pi < \alpha < \frac{3\pi}{2}; \quad \sin \alpha = -\frac{12}{13}; \quad \operatorname{tg} \alpha = \frac{12}{5};$$

$$\cos 2\alpha = -\frac{119}{169}; \quad \operatorname{tg} 2\alpha = \frac{2 \operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha} = \frac{24}{5}; \quad \left(1 - \frac{144}{25}\right) =$$

$$= -\frac{25}{119} \cdot \frac{24}{5} = -\frac{120}{119}.$$
- $$\operatorname{ctg}^2 \alpha (1 - \cos 2\alpha)^2 - \cos^2 2\alpha = 4 \sin^4 \alpha \operatorname{ctg}^2 \alpha - \cos^2 2\alpha =$$

$$= \sin^2 2\alpha - \cos^2 2\alpha = -\cos 4\alpha.$$

C-5

- см. рис;

$$\text{абсцисса: } \cos \frac{23\pi}{6} = \cos \frac{\pi}{6} = \frac{\sqrt{3}}{2};$$

$$\text{ордината: } \sin \frac{23\pi}{6} = -\sin \frac{\pi}{6} = -\frac{1}{2}.$$

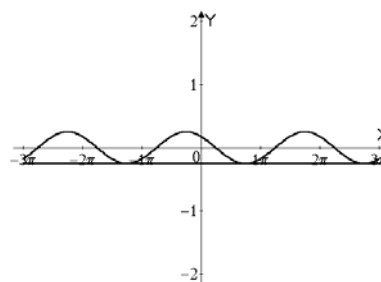
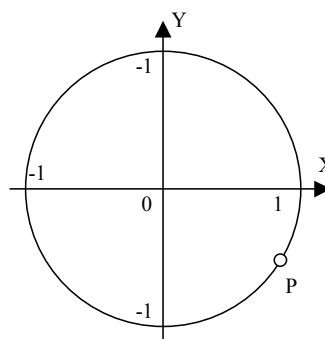
- а) I; б) IV.

-

$$\frac{1}{4} \cos \left(x + \frac{\pi}{4} \right) = -\frac{1}{4};$$

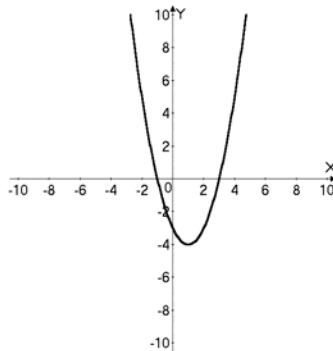
$$\cos \left(x + \frac{\pi}{4} \right) = -1;$$

$$x = \frac{3\pi}{4} + 2\pi n.$$



C-6

1. а) $f(x) = \frac{5x-4}{x^2-7x+6}$; ОДЗ: $x^2 - 7x + 6 \neq 0$; $x \neq 6$ и $x \neq 1$;
 б) $f(x) = \sqrt{\frac{1}{x^2-4}}$; ОДЗ: $x^2 - 4 > 0$; $x \in (-\infty; -2) \cup (2; \infty)$.
2. $f(x) = x^3 + 3x - 1$ $f(-2) = -8 - 6 - 1 = -15$;
 $f(x+1) = (x+1)(x^2 + 2x + 1 + 3) - 1 = x^3 + 3x^2 + 6x + 3$.
- 3.



C-7

1. $f(x) = \frac{x^3 \sin x}{\operatorname{tg}^2 x}$; $f(-x) = \frac{(-x)^3 \sin(-x)}{\operatorname{tg}^2(-x)} = \frac{x^3 \sin x}{\operatorname{tg}^2 x} = f(x)$, значит, $f(x)$ четная.
2. $g(x) = |x| \cos 2x \sin^3 3x$; $g(-x) = |-x| \cos(-2x) \sin^3(-3x) = -|x| \cos 2x \sin^3 3x = -f(x)$, значит, $g(x)$ нечетная.

C-8

1. а) $\cos 235^\circ 17' = -\sin 34^\circ 43'$; б) $\sin 5040^\circ = \sin 0^\circ = 0$;
 в) $\operatorname{tg} \frac{29\pi}{7} = \operatorname{tg} \frac{\pi}{7}$.
2. $\sin(-60^\circ) + \cos 690^\circ + \operatorname{tg}(-600^\circ) = -\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} - \sqrt{3} = -\sqrt{3}$.

3. а) $\operatorname{tg}\left(\frac{x}{2} - \frac{\pi}{5}\right)$, $T = 2\pi$; б) $y = \cos^2 2x - \sin 4x$;

$$y_1 = \cos^2 2x; \quad T_1 = \frac{\pi}{2}, \text{ значит, } T = \frac{\pi}{2}.$$

$$y_2 = \sin 4x; \quad T_2 = \frac{\pi}{2}$$

С-9

1.

а) $f(x) = \sqrt{x+1}$ возрастает на области определения, то есть при $x \in (-1; \infty)$;

б) $f(x) = -\frac{x-2}{x-1} = -1 + \frac{1}{x-1}$ убывает на области определения, то есть при $x \in (-\infty; 1) \cup (1; \infty)$.

2. $f(x) = \operatorname{tg}\left(2x - \frac{\pi}{4}\right)$; ОДЗ: $\cos\left(2x - \frac{\pi}{4}\right) \neq 0$; $x \neq \frac{3\pi}{8} + \frac{\pi n}{2}$;
возрастает на области определения.

3. $\sin 40^\circ$, $\cos 40^\circ$, $\sin 70^\circ$, $\cos 70^\circ$.
Ответ: $\sin 70^\circ$, $\cos 40^\circ$, $\sin 40^\circ$, $\cos 70^\circ$.

С-10

1. $y = 5x - 2x^2 - 2$; $x_{\max} = \frac{5}{4}$;

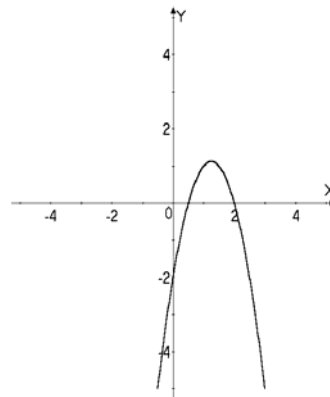
$$y \in \left(-\infty; \frac{9}{8}\right].$$

2. $f(x) = 3\cos\left(x - \frac{2\pi}{7}\right)$;

$$f'(x) = -3\sin\left(x - \frac{2\pi}{7}\right);$$

$$f'(x) = 0 \text{ при } x = \frac{2\pi}{7} + 2\pi n \text{ и}$$

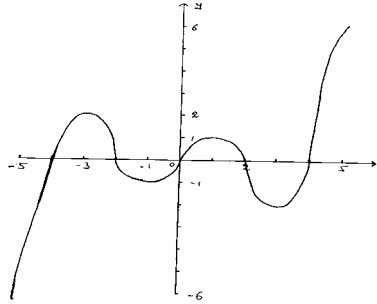
$$x = \frac{9\pi}{7} + 2\pi n;$$



$$x_{\max} = \frac{2\pi}{7} + 2\pi n; \quad x_{\min} = \frac{9\pi}{7} + 2\pi n;$$

$$\text{экстремумы: } f\left(\frac{2\pi}{7} + 2\pi n\right) = 3; \quad f\left(\frac{9\pi}{7} + 2\pi n\right) = -3.$$

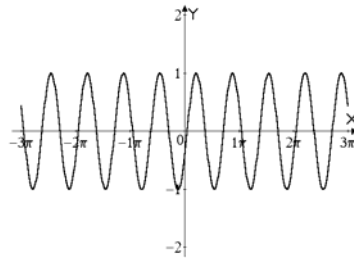
C-11



C-12

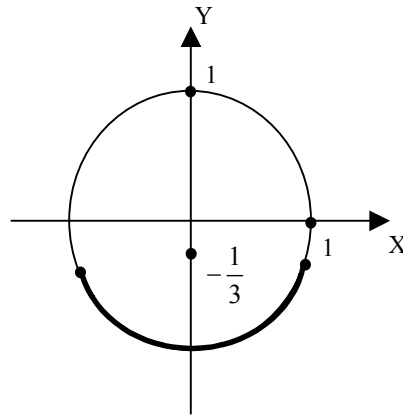
$$1. \quad f(x) = \frac{1}{\operatorname{tg}\left(x - \frac{\pi}{4}\right)}; \quad \text{ОДЗ: } \begin{cases} \sin\left(x - \frac{\pi}{4}\right) \neq 0 \\ \cos\left(x - \frac{\pi}{4}\right) \neq 0 \end{cases}; \quad \begin{cases} x \neq \frac{\pi}{4} + \pi n \\ x \neq \frac{3\pi}{4} + \pi n \end{cases}.$$

$$2. \quad f(x) = \sin\left(3x - \frac{\pi}{7}\right);$$



$$x_{\max} = \frac{3\pi}{14} + \frac{2\pi n}{3}; \quad x_{\min} = -\frac{5\pi}{42} + \frac{2\pi n}{3}.$$

3.



$$\sin t \leq -\frac{1}{3}; t \in [-\pi + \arcsin \frac{1}{3} + 2\pi n; -\arcsin \frac{1}{3} + 2\pi n].$$

C-13

1.

a) $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$; **б)** $\sin(\arcsin 0,1) = 0,1$;

в) $\operatorname{arctg}(-1) + \arccos(-1) = -\frac{\pi}{4} + \pi = \frac{3\pi}{4}$;

г) $\cos\left(3\operatorname{arctg} \frac{1}{\sqrt{3}}\right) = \cos \frac{\pi}{2} = 0$.

2.

a) $\arcsin(0,897) \approx 1,113$; **б)** $\arccos(-0,773) \approx 2,4544$;

в) $\operatorname{arctg}(-4) \approx -1,3258$.

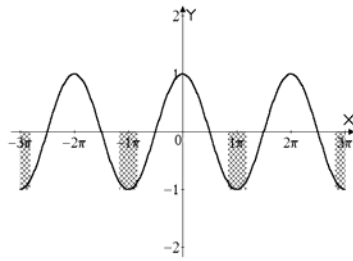
C-14

a) $\cos x = -\frac{\sqrt{3}}{2}$ $x = \pm \frac{5\pi}{6} + 2\pi n$; **б)** $\sin\left(x - \frac{\pi}{3}\right) = 1$; $x = \frac{5\pi}{6} + 2\pi n$;

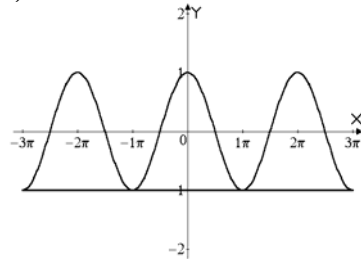
в) $\operatorname{tg}\left(3x + \frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$; $3x = \pi n$; $x = \frac{\pi n}{3}$.

C-15

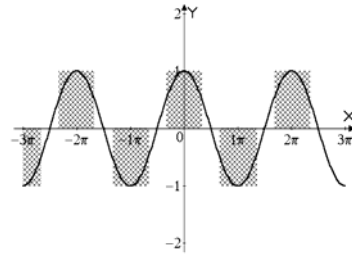
a)



б)



в)



$$\cos x \leq -\frac{\sqrt{3}}{2}; x \in \left[\frac{5\pi}{6} + 2\pi n; \frac{7\pi}{6} + 2\pi n \right].$$

C-16

a) $\cos 3x < \frac{1}{2}; \quad x \in \left(\frac{\pi}{9} + \frac{2\pi n}{3}; \frac{5\pi}{9} + \frac{2\pi n}{3} \right).$

б) $\operatorname{tg} \left(2x + \frac{\pi}{6} \right) \geq -\sqrt{3}; \quad 2x \in \left[\frac{\pi}{2} + \pi n; \frac{4\pi}{3} + \pi n \right)$
 $x \in \left[\frac{\pi}{4} + \frac{\pi n}{2}; \frac{2\pi}{3} + \frac{\pi n}{2} \right).$

C-17

a) $\operatorname{ctg} x = -4 - 3\operatorname{tg} x, \quad \operatorname{tg} x \neq 0;$
 $\operatorname{ctg}^2 x + 4\operatorname{ctg} x + 3 = 0; \quad \operatorname{ctg} x = -3 \quad x = -\operatorname{arctg} 3 + \pi n$
 $\operatorname{ctg} x = -1 \quad x = -\frac{\pi}{4} + \pi n.$

$$6) \quad 4\sin^4 x - 5\sin^2 x + 1 = 0;$$

$$\sin^2 x = 1; \quad x = \frac{\pi}{2} + \pi n; \quad \sin^2 x = \frac{1}{4}; \quad x = (-1)^k \frac{\pi}{6} + \pi k \text{ и}$$

$$x = (-1)^{z+1} \frac{\pi}{6} + \pi z.$$

C-18

$$a) \quad \sqrt{3} \sin\left(x - \frac{\pi}{3}\right) + 3\cos\left(x - \frac{\pi}{3}\right) = 0; \quad \cos\left(x - \frac{\pi}{3}\right) \neq 0;$$

$$\operatorname{tg}\left(x - \frac{\pi}{3}\right) = -\sqrt{3}; \quad x = \pi n;$$

$$6) \quad 2\sin^2 x + 2\sin x \cos x = 1; \quad \cos x \neq 0;$$

$$\operatorname{tg}^2 x + 2\operatorname{tg} x - 1 = 0;$$

$$\operatorname{tg} x = -1 \pm \sqrt{2}; \quad x = \operatorname{arctg}(\pm \sqrt{2}) + \pi n.$$

C-19

$$\begin{cases} \sin x \cos y = -\frac{1}{4}; \\ \cos x \sin y = \frac{3}{4} \end{cases}; \quad \begin{cases} \sin(x+y) + \sin(x-y) = -\frac{1}{2}; \\ \sin(x+y) - \sin(x-y) = \frac{3}{2} \end{cases};$$

$$\begin{cases} x+y = (-1)^k \frac{\pi}{6} + \pi k \\ x-y = -\frac{\pi}{2} + 2\pi n \end{cases}; \quad \begin{cases} x = (-1)^k \frac{\pi}{12} - \frac{\pi}{4} + \frac{\pi k}{2} + \pi n \\ y = (-1)^k \frac{\pi}{12} + \frac{\pi}{4} + \frac{\pi k}{2} - \pi n \end{cases}.$$

C-20

$$a) \quad \sin x + \sin 5x = \sin 3x + \sin 7x; \quad \sin 3x \cos 2x - \sin 5x \cos 2x = 0;$$

$$\cos 2x = 0 \quad x = \frac{\pi}{4} + \frac{\pi n}{2} \quad \text{или} \quad \sin x \cos 4x = 0;$$

$$\sin x = 0; \quad x = \pi n; \quad \cos 4x = 0; \quad x = \frac{\pi}{8} + \frac{\pi n}{4}.$$

$$6) \quad \sin x \sin 2x \cos 3x + \sin x \cos 2x \sin 3x = 0;$$

$$\sin x (\sin 2x \cos 3x + \cos 2x \sin 3x) = 0;$$

$$\sin x = 0; \quad \sin 5x = 0; \quad 5x = \pi n; \quad x = \frac{\pi n}{5}. \quad \text{Ответ: } \frac{\pi n}{5}.$$

$$x = \pi n;$$

C-21

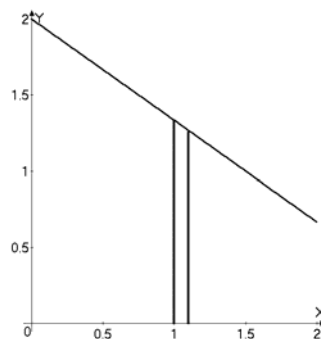
1.

$$f(x) = -\frac{2}{3}x + 2; \quad f(1) = \frac{4}{3};$$

$$f(1, 1) = \frac{-11}{15} + \frac{30}{15} = \frac{19}{15};$$

$$-f(x_0) + f(x_0 + \Delta x) = -\frac{1}{15};$$

$$f(x_0 + \Delta x) - f(x_0) = -\frac{2}{3}\Delta x.$$



2.

$$f(x) = 1 - 3x - 2x^2;$$

$$\frac{\Delta f(x_0)}{\Delta x} = \frac{1 - 3x_0 - 3\Delta x - 2\Delta x^2 - 2x_0^2 - 4\Delta x x_0 - 1 - 3x_0 - 2x_0}{\Delta x} =$$

$$= -3 - 4x_0 - 2\Delta x; \quad x_0 = 1, \quad \Delta x = 0,1; \quad \frac{\Delta f(x_0)}{\Delta x} = -7,2;$$

$$x_0 = 1, \quad \Delta x = 0,002 \quad \frac{\Delta f(x_0)}{\Delta x} = -7,004;$$

$$x_0 = 1, \quad \Delta x = 0,00001 \quad \frac{\Delta f(x_0)}{\Delta x} = -7,00002;$$

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta f(x_0)}{\Delta x} = -7 \quad (\text{при } x_0 = 1).$$

C-22

1. $x(t) = t^2 + 4; v(t) = 2t$. Импульс при $t = 4$, $m = 2$ равен $2 \cdot 4 \cdot 2 = 16$.

2. а) $f(x) = 6\sqrt{x}; \quad f'(x) = \frac{3}{\sqrt{x}}; \quad б) \quad f(x) = 4 - x^2; \quad f'(x) = -2x$.

C-23

1. а) $f(-2) = -1; \quad f(4) = 1; \quad б) \quad \lim_{x \rightarrow -2} f(x) = -1; \quad \lim_{x \rightarrow -4} f(x) = -2$

2. $f(x) = \frac{9-x^2}{x-3} = -x-3$, при $x \neq 3 \quad (3-x) < 0,001; \delta = 0,001$.

C-24

1. а) $y = f(x) - 2g^2(x)$; $\lim_{x \rightarrow 3} y = \lim_{x \rightarrow 3} f(x) - 2 \lim_{x \rightarrow 3} g^2(x) = 5 - 8 = -3$;

б) $y = \frac{f(x) - g(x)}{2f(x) - 5g(x)}$, предела не существует, т.к. знаменатель стремиться к 0.

2. а) $\lim_{x \rightarrow -2} (1 - 3x^3 + 4x^4) = 1 + 24 + 64 = 89$;

б) $\lim_{x \rightarrow 3} \frac{2x+9}{x^2-x-1} = \frac{15}{5} = 3$.

C-25

1.

а) $f(x) = x^9 - 3x^5 - \frac{3}{x^4} + 2$; $f'(x) = 9x^8 - 15x^4 + \frac{12}{x^5}$;

б) $f(x) = \frac{4-x^2}{3+2x}$; $f'(x) = \frac{-6x-4x^2-8+2x^2}{(3+2x)^2} = \frac{-2x^2-6x-8}{(3+2x)^2}$.

2. $f(x) = (x+1)\sqrt{x}$ $f'(x) = \sqrt{x} + \frac{x+1}{2\sqrt{x}}$;

$f(2) = \sqrt{2} + \frac{3}{2\sqrt{2}}$ $f'(4) = 2 + \frac{5}{4} = \frac{13}{4}$;

$f(x-2) = \sqrt{x-2} + \frac{x-1}{2\sqrt{x-2}}$.

3. $f(x) = 3x - x^3$; $f'(x) = 3 - 3x^2 \geq 0$ при $x \in [-1; 1]$.

C-26

1. $f(x) = \frac{\sqrt{x}-1}{\sqrt{x}+1}$; $f'(x) = 2(\sqrt{x}+1)^{-2} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$;

$f(t^2) = \frac{1}{t(t+1)^2}$.

2. а) $f(x) = 9x^3 + x$; $f'(x) = 27x^2 + 1 > 0$ всегда, значит, $f'(x) = 0$ и $f'(x) < 0$ не имеют решений;

$$\text{б) } f(x) = \frac{x^2 + 15}{x + 1}; f'(x) = \frac{2x^2 + 2x - x^2 - 15}{(x + 1)^2} = \frac{x^2 + 2x - 15}{(x + 1)^2} = 0; f'(x) = 0$$

$$\text{при } \frac{(x + 5)(x - 3)}{(x + 1)^2} = 0 \quad x = -5 \text{ и } x = 3;$$

$$f'(x) > 0 \text{ при } x \in (-\infty; 5) \cup (3; +\infty); \quad f'(x) < 0 \text{ при } x \in (-5; -1) \cup (-1; 3).$$

C-27

$$1. \quad \text{а) } f(x) = \sqrt{3\sqrt{x} - 1}; \quad \text{ОДЗ: } 3\sqrt{x} - 1 \geq 0; \quad x \geq \frac{1}{9};$$

$$\text{б) } f(x) = \frac{1}{\sqrt{x^2 - 6x + 9}}; \quad \text{ОДЗ: } x^2 - 6x + 9 \neq 0; \quad x \neq 3.$$

$$2. \quad f(x) = \frac{2 + 3x}{1 - x}; \quad g(x) = \sqrt{x}; \quad f(g(x)) = \frac{2 + 3\sqrt{x}}{1 - \sqrt{x}}; \quad g(f(x)) = \sqrt{\frac{2 + 3x}{1 - x}}.$$

$$3. \quad \text{а) } f(x) = (x^7 - 3x^4)^{120} \quad f'(x) = 120(7x^6 - 12x^3)(x^7 - 3x^4)^{119};$$

$$\text{б) } g(x) = \sqrt{x^2 - 1}; \quad g'(x) = \frac{x}{\sqrt{x^2 - 1}}.$$

C-28

$$\text{а) } f(x) = \operatorname{tg}\left(\frac{x}{3} + 10\right); \quad f'(x) = \frac{1}{3\cos^2\left(\frac{x}{3} + 10\right)};$$

$$\text{б) } f(x) = \cos(3 - 2x); \quad f'(x) = 2\sin(3 - 2x);$$

$$\text{в) } f(x) = \operatorname{tg} x \sin(2x + 5); \quad f'(x) = \frac{\sin(2x + 5)}{\cos^2 x} + 2\cos(2x + 5) \operatorname{tg} x.$$

C-29

$$1. \quad f(x) = \frac{x^2 - 4}{(x - 1)(x^2 - 3x - 4)};$$

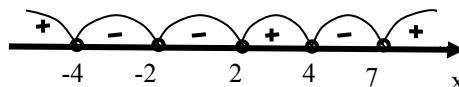
ОДЗ $(x - 1)(x^2 - 3x - 4) \neq 0$; $x \neq \pm 1$ и $x \neq 4$, значит, промежутки непрерывности: $x \in (-\infty; -1) \cup (-1; 1) \cup (1; 4) \cup (4; \infty)$.

2. а) $x^2 + 5x + 4 < 0$; $(x+1)(x+4) < 0$;



$x \in (-4; -1)$;

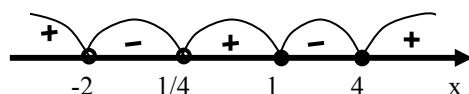
б) $\frac{(x-2)(x+2)^2(x-7)}{x^2-16} < 0$;



$x \in (-4; -2) \cup (-2; 2) \cup (4; 7)$;

в) $\frac{x-2}{x+2} \geq \frac{2x-3}{4x-1}$; $\frac{4x^2-9x+2-2x^2-x+6}{(x+2)(4x-1)} \geq 0$;

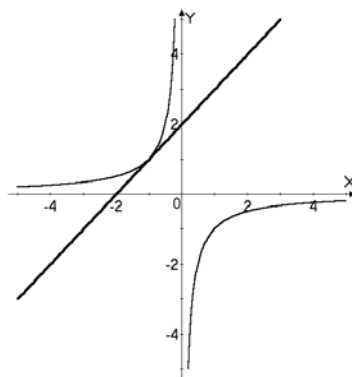
$\frac{2x^2-10x+8}{(x+2)(4x-1)} \geq 0$; $\frac{(x-1)(x-4)}{(x+2)(4x-1)} \geq 0$;



$x \in (-\infty; -2) \cup (\frac{1}{4}; 1] \cup [4; +\infty)$.

C-30

1. $y(x) = -\frac{1}{x}$; $y(-1) = 1$; $y'(x) = \frac{1}{x^2}$; $y'(-1) = 1$;



$Y_{\text{кас}} = 1 + x + 1 = x + 2$ – уравнение касательной.

2. $y = \cos \frac{x}{3}; \quad y(\pi) = \frac{1}{2}; \quad y' = -\frac{1}{3} \sin \frac{x}{3}; \quad y'(\pi) = -\frac{\sqrt{3}}{6};$
 $y_{\text{кас}} = \frac{1}{2} - \frac{\sqrt{3}}{6}(x - \pi) = -\frac{\sqrt{3}}{6}x + \frac{1}{2} + \frac{\sqrt{3}\pi}{6}$ – уравнение касательной.

C-31

1. $\sqrt{35,91} \approx 6(1 - 0,0025 \cdot \frac{1}{2}) = 5,9925.$
2. $1,00008^{1000} - 0,99996^{200} \approx 1 + 0,00008 \cdot 1000 - 1 + 0,00004 \cdot 200 =$
 $= 1,08 - 0,992 = 0,088.$

C-32

1. $s(t) = 17t - 2t^2 + \frac{1}{3}t^3; \quad v(t) = 17 - 4t + t^2;$
 $a(t) = -4 + 2t; \quad a(3) = 2; \quad F = ma = 3 \cdot 3 = 6 \text{ н.}$
2. $h(t) = h_0 + v_0t - \frac{gt^2}{2} = 2 + 4t - 5t^2; \quad v(t) = 4 - 10t;$
 $4 - 10t = \frac{4}{3}; \quad \frac{8}{3} = 10t; \quad t = \frac{8}{30} = \frac{4}{15};$
 $h\left(\frac{4}{15}\right) = 2 + \frac{16}{15} - 5 \cdot \frac{16}{225} = 2 + \frac{48-16}{45} = 2 \frac{32}{45} \text{ м.}$

C-33

1. $f(x) = 2x^3 - 3x^2 - 12x; \quad f'(x) = 6(x^2 - x - 2); f'(x) = 0$ при
 $x = 2$ и $x = -1;$
 $f(x)$ возрастает при $x \in (-\infty; -1) \cup (2; +\infty);$ убывает при $x \in (-1; 2).$
2. $f(x) = 2\sqrt{x} - x; f'(x) = \frac{1}{\sqrt{x}} - 1; f'(x) = 0$ при $x = 1; x = 1$ – точка max.

C-34

$f(x) = x^2(x - 6)^2 = x^4 - 12x^3 + 36x^2; f'(x) = 4(x^3 - 9x^2 + 18x); f'(x) = 0$ при $x = 0,$
 $x = 3$ и $x = 6; x_{\min} = 0; x_{\min} = 6; x_{\max} = 3;$
 $f(0) = 0; f(3) = 81; f(6) = 0;$
 $f(x)$ убывает при $x \in (-\infty; 0) \cup (3; 6];$ возрастает при $x \in (0; 3) \cup (6; +\infty).$

C-35

1.

$$y = -\frac{1}{2}x^2 + 2x + \frac{5}{2};$$

$$x_B = 2; \quad y_B = 4,5;$$

возрастает при $x \in (-\infty; 2)$;

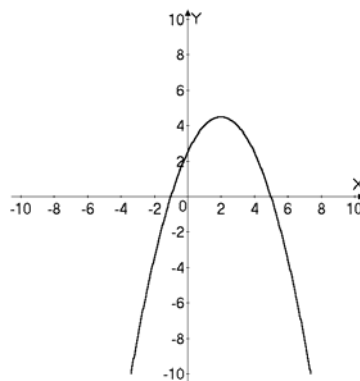
убывает при $x \in (2; \infty)$;

$$x \in R;$$

$$y \in (-\infty; 4,5];$$

$$\frac{1}{2}x^2 - 2x - \frac{5}{2} = 0;$$

$$\text{нули: } x = 5, \quad x = -1.$$

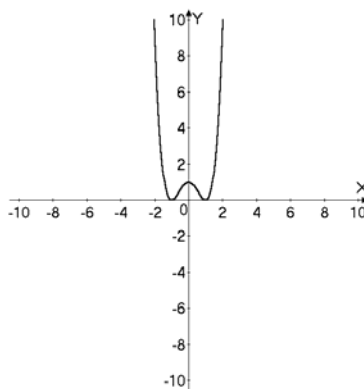


2. а) $3x^2 - 2x + 1 > 0 \quad \frac{D}{4} = 1 - 3 < 0$, значит, $x \in R$;

б) $9x^2 - 18x + 4 \leq 5x^2 - 6x + 11; \quad 4x^2 - 12x - 7 \leq 0;$

$$x_1 = \frac{7}{2}, \quad x_2 = -\frac{1}{2}; \quad x \in \left[-\frac{1}{2}; \frac{7}{2}\right].$$

C-36



$$y = x^4 - 2x^2 + 1 \quad y' = 4x(x^2 - 1); \quad y' = 0 \text{ при } x = 0 \text{ и } x = \pm 1$$

убывает при $x \in (-\infty; -1) \cup [0; 1]$; возрастает при $x \in [-1; 0] \cup [1; +\infty)$;

min: $(\pm 1; 0)$; max: $(0; 1)$.

C-37

1. $f(x) = 3x^5 - 5x^3 + 1$; $x \in [-2; 2]$;
 $f'(x) = 15x^2(x^2 - 1)$; $f(x) = 0$ при $x = 0$ и $x = \pm 1$;
 $f(0) = 1$; $f(1) = -1$; $f(-1) = 1$; $f(-2) = -55$; $f(2) = 57$;
наименьшее значение функции -55 ; наибольшее значение функции 57 .

2.

$$\begin{cases} a+b=6 \\ y=a^2b \end{cases}; \quad \begin{cases} b=6-a \\ y=6a^2-a^3 \end{cases}; \quad y' = 12a - 3a^2; y' = 0 \text{ при}$$
$$\begin{cases} a=0 \\ b=6 \text{ и} \\ y=0 \end{cases} \quad \begin{cases} a=4 \\ b=2 \\ y=32 \end{cases}.$$

Ответ: $4 + 2$.

C-38

1. $\sin \alpha = \frac{3}{5}$; $\frac{\pi}{2} < \alpha < \pi$;
 $\cos \beta = -\frac{4}{5}$; $\pi < \beta < \frac{3\pi}{2}$;
 $\cos \alpha = -\frac{4}{5}$; $\sin \beta = -\frac{3}{5}$;
 $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha = -\frac{12}{25} + \frac{12}{25} = 0$.
2. $\frac{\cos^2\left(\frac{3\pi}{2} - 2\alpha\right)}{\cos^2(\pi - \alpha)} + \left(2\cos^2 \frac{\alpha}{2} - 2\sin^2 \frac{\alpha}{2}\right)^2 = \frac{\sin^2 2\alpha}{\cos^2 \alpha} + 4\cos^2 \alpha = 4$.
3. $\sin 22^\circ 30' = \sqrt{\frac{1 - \sin 45^\circ}{2}} = \frac{\sqrt{2 - \sqrt{2}}}{2}$;
 $\cos 22^\circ 30' = \frac{\sqrt{2 + \sqrt{2}}}{2}$;
 $\operatorname{tg} 22^\circ 30' = \sqrt{\frac{2 - \sqrt{2}}{2 + \sqrt{2}}}$.

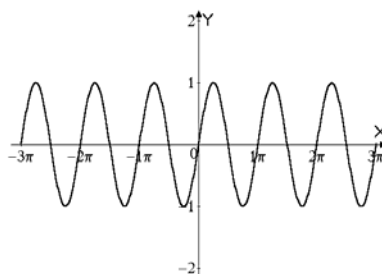
C-39

а) $y = \sin 2x$; $y = 0$ при $x = \frac{\pi n}{2}$ — нули; $x \in R$; $y \in [-1; 1]$

возрастает при $x \in \left(-\frac{\pi}{4} + \pi n; \frac{\pi}{4} + \pi n\right)$;

убывает при $x \in \left(\frac{\pi}{4} + \pi n; \frac{3\pi}{4} + \pi n\right)$

$$x = -\frac{\pi}{4} + \pi n - \min; \quad x = \frac{\pi}{4} + \pi n - \max.$$

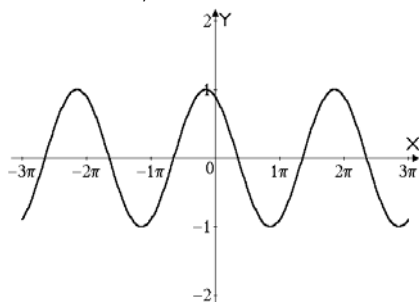


б) $y = \cos\left(x + \frac{\pi}{7}\right)$; $y = 0$ при $x = \frac{5\pi}{14} + \pi n$ — нули;

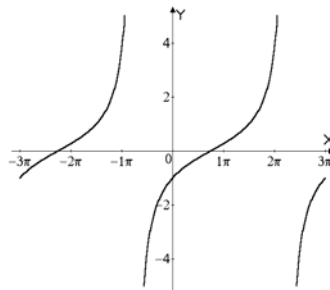
возрастает при $x \in \left[-\frac{8\pi}{7} + 2\pi n; -\frac{\pi}{7} + 2\pi n\right]$;

убывает при $x \in \left[-\frac{\pi}{7} + 2\pi n; \frac{6\pi}{7} + 2\pi n\right]$;

$$x = -\frac{\pi}{7} + 2\pi n - \max; \quad x = \frac{6\pi}{7} + 2\pi n - \min; \quad x \in R; \quad y \in [-1; 1].$$



$$\text{в) } y = \operatorname{tg}\left(\frac{x}{3} - \frac{\pi}{4}\right);$$



нули: $\operatorname{tg}\left(\frac{x}{3} - \frac{\pi}{4}\right) = 0$ при $x = \frac{\pi}{12} + 3\pi n$;

$y \in \mathbb{R}; x \neq \frac{9\pi}{4} + 3\pi n$; возрастает на области определения.

C-40

1.

а) $\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$; б) $\arcsin \frac{\sqrt{2}}{2} = \frac{\pi}{4}$; в) $\operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6}$.

2.

а) $2\cos\left(2x - \frac{\pi}{4}\right) = \sqrt{2}$; $2x - \frac{\pi}{4} = \pm \frac{\pi}{4} + 2\pi n$; $x = \pm \frac{\pi}{8} + \frac{\pi}{8} + \pi n$;

б) $\cos^2 x - \sin 2x = -\frac{1}{2}$; $\frac{1}{2} \cos 2x - \sin 2x = -1$;

$\sin 2x - \frac{1}{2} \cos 2x = 1$; $\sin(2x - \varphi) = \frac{2}{\sqrt{5}}$, $\varphi = \arccos \frac{2}{\sqrt{5}}$;

$x = \frac{1}{2}(-1)^k \arcsin \frac{2}{\sqrt{5}} + \frac{1}{2} \arccos \frac{2}{\sqrt{5}} + \frac{\pi k}{2}$.

3.

а) $\operatorname{tg} 2x < -1$; $x \in \left(-\frac{\pi}{4} + \frac{\pi n}{2}; -\frac{\pi}{8} + \frac{\pi n}{2}\right)$.

б) $\sin\left(x - \frac{\pi}{4}\right) > \frac{1}{2}$; $x \in \left(\frac{5\pi}{12} + 2\pi n; \frac{13\pi}{12} + 2\pi n\right)$.

C-41

$$\begin{cases} x + y = \frac{\pi}{3} \\ \sin^2 x + \sin^2 y = \frac{1}{2} \end{cases}; \quad \begin{cases} x = \frac{\pi}{3} - y \\ 1 - \cos\left(\frac{2\pi}{3} - 2y\right) + 1 - \cos 2y = 1 \end{cases};$$

$$\begin{cases} \cos \frac{\pi}{3} \cos\left(\frac{\pi}{3} - 2y\right) = \frac{1}{2} \\ x = \frac{\pi}{3} - y \end{cases}; \quad \begin{cases} y = \frac{\pi}{6} + \pi n \\ x = \frac{\pi}{6} - \pi n \end{cases}.$$

C-42

1.

a) $x^2 - 4x + 3 \leq 0$; $x \in [1; 3]$; **б)** $x^2 - 6x + 9 > 0$; $(x - 3)^2 > 0$; $x \neq 3$.

2.

a) $\frac{(x-1)(x-2)^2}{(x-3)^3} \leq 0$; $x \in [1; 3)$.

б) $\frac{2}{x-1} + \frac{3}{x-2} > 4$; $\frac{2x-4+3x-3-4x^2+12x-8}{(x-1)(x-2)} > 0$;

$\frac{4x^2-17x+15}{(x-1)(x-2)} < 0$;

$x_1 = \frac{17-7}{8} = \frac{5}{4}$, $x_2 = 3$; $x \in \left(1; \frac{5}{4}\right) \cup (2; 3)$.

C-43

a) $y = x^6 - 3x^4 + 2x^3 - 3$; $y' = 6x^5 - 12x^3 + 6x^2$;

б) $y = (3 - 2x)\sqrt{x}$; $y' = \frac{3-2x}{2\sqrt{x}} - 2\sqrt{x} = \frac{3-2x-4x}{2\sqrt{x}} = \frac{3-6x}{2\sqrt{x}}$;

в) $y = \sin 2x$; $y' = 2\cos 2x$;

г) $y = \operatorname{tg}\left(\frac{1}{3}x - 1\right)$; $y' = \frac{1}{3\cos^2\left(\frac{1}{3}x - 1\right)}$;

д) $y = (2x - 1)^{17}$; $y' = 34(2x - 1)^{16}$.

C-44

1.

$f(x) = 3x - x^2$; $f(1) = 2$; $f'(x) = 3 - 2x$; $f'(1) = 1$; $y = 2 + x - 1 = x + 1$ — уравнение касательной.

2. а) $\sqrt{0,998} \approx 1 - 0,002 \cdot \frac{1}{2} = 0,999$;

б) $(1,0003)^{50} \approx 1 + 0,0003 \cdot 50 = 1,015$.

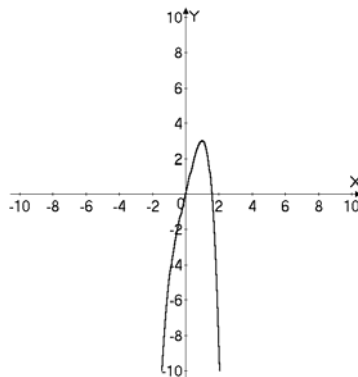
3.

$x(t) = t^3 - 2t^2 + 3t$; $v(t) = 3t^2 - 4t + 3$;
 $x(2) = 12 - 8 + 3 = 7$; $a(t) = 6t - 4$; $a(2) = 8$.

C-45

1.

$y = 4x - x^4$; $y' = 4 - 4x^3$; $y' = 0$ при $x = 1$
 y возрастает при $x \in (-\infty; 1)$; убывает при $x \in (1; +\infty)$;
 $x = 1$ — max; $y(1) = 3$;
нули: $x = 0$ и $x = \sqrt[3]{4}$.



2.

$f(x) = \frac{1}{x^2 + 1}$; $x \in [-1; \frac{1}{2}]$; $f'(x) = \frac{-2x}{(x^2 + 1)^2}$; $f'(x) = 0$ при $x = 0$;

$f(0) = 1$; $f(-1) = \frac{1}{2}$; $f(\frac{1}{2}) = \frac{4}{5}$;

наибольшее значение функции $f(0) = 1$

наименьшее значение функции $f(-1) = \frac{1}{2}$.

ВАРИАНТ 6

С-1

1. $42^\circ = \frac{\pi}{180} \cdot 42 = \frac{7\pi}{30}$; $130^\circ = \frac{\pi}{18} \cdot 13$.
2. $\frac{7\pi}{12} = 105^\circ$; $\frac{21\pi}{4} = 945^\circ$.
3. а) $57^\circ = \frac{\pi 57}{180}$; $\sin 57^\circ \approx 0,8387$; $\cos 57^\circ \approx 0,5446$;
б) $88^\circ 55' \approx 1,5519$; $\sin 88^\circ 55' \approx 0,9998$; $\cos 88^\circ 55' \approx 0,0192$.
4. а) $0,8796 \approx 50^\circ 24'$;
б) $2,3422 \approx 134^\circ 12'$.

С-2

1. $1 + \frac{\cos^4 \alpha + \sin^2 \alpha \cos^2 \alpha}{\sin^2 \alpha} = \frac{1}{\sin^2 \alpha}$; $1 + \operatorname{ctg}^2 \alpha = \frac{1}{\sin^2 \alpha}$.
2. а) $\frac{\sin 110^\circ \cos 220^\circ}{\operatorname{ctg} 330^\circ} > 0$; б) $\sin 2 \operatorname{ctg} 4 > 0$.
3. $\operatorname{tg} \alpha = 3$; $\alpha \in \text{I четверти}$;
 $\operatorname{ctg} \alpha = \frac{1}{3}$, $\frac{\sin \alpha}{\sqrt{1 - \sin^2 \alpha}} = 3$, $\sin^2 \alpha = 9 - 9\sin^2 \alpha$; $\sin \alpha = \frac{3}{\sqrt{10}}$.

С-3

1. а) $\sin 2280^\circ = \sin 120^\circ = \frac{\sqrt{3}}{2}$; б) $\cos \frac{43\pi}{6} = \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}$;
в) $\operatorname{tg} 1590^\circ = \operatorname{tg} 150^\circ = -\frac{1}{\sqrt{3}}$.

2.
$$\frac{\operatorname{ctg}(270^\circ - \alpha)}{1 - \operatorname{tg}^2(\alpha - 180^\circ)} \cdot \frac{\operatorname{ctg}^2(360^\circ - \alpha) - 1}{\operatorname{ctg}(180^\circ + \alpha)} = \frac{\operatorname{tg} \alpha}{1 - \operatorname{tg}^2 \alpha} \cdot \frac{\operatorname{ctg}^2 \alpha - 1}{\operatorname{ctg} \alpha} =$$

$$= \operatorname{tg} 2\alpha \operatorname{ctg} 2\alpha = 1.$$
3.
$$\frac{\sin(-\alpha)}{\cos\left(\frac{\pi}{2} + \alpha\right)} = \operatorname{ctg}\left(\frac{3\pi}{2} + \alpha\right) \operatorname{tg}\left(\frac{\pi}{2} + \alpha\right); \frac{\sin \alpha}{\sin \alpha} = \operatorname{tg} \alpha \operatorname{ctg} \alpha = 1.$$

C-4

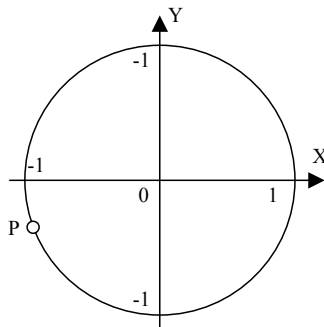
1.
$$\frac{1 - \sin^2 15^\circ}{2 \cos^2 \frac{\pi}{8} - 1} = \frac{\cos^2 15^\circ}{\cos \frac{\pi}{4}} = \frac{1 + \cos 30^\circ}{\sqrt{2}} = \frac{2 + \sqrt{2}}{2\sqrt{2}}.$$
2.
$$\sin \alpha = \frac{4}{5}; \quad \frac{\pi}{2} < \alpha < \pi;$$

$$\cos \alpha = -\frac{3}{5}; \cos 2\alpha = -\frac{7}{25}; \sin 2\alpha = -\frac{24}{25}; \operatorname{ctg} 2\alpha = \frac{7}{24}.$$
3.
$$\cos^2 2\alpha + (1 + \cos 2\alpha)^2 \operatorname{tg}^2 \alpha = \cos^2 2\alpha + 4 \cos^4 \alpha \operatorname{tg}^2 \alpha =$$

$$= \cos^2 2\alpha + \sin^2 2\alpha = 1.$$

C-5

1. абсцисса: $\cos \frac{43\pi}{6} = \cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2};$
 ордината: $\sin \frac{43\pi}{6} = -\sin \frac{5\pi}{6} = -\frac{1}{2}.$



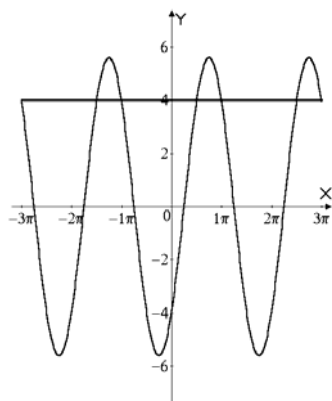
2. а) III; б) I.

3.

$$4\sqrt{2} \sin\left(x - \frac{\pi}{4}\right) = 4;$$

$$\sin\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2};$$

$$x = \frac{\pi}{4} + (-1)^k \frac{\pi}{4} + \pi k.$$



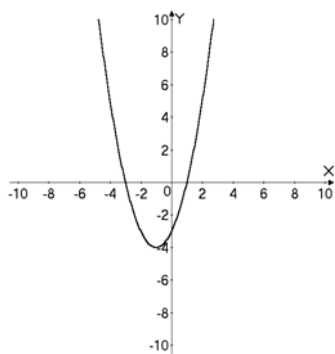
C-6

1. a) $f(x) = \frac{1-8x}{x^2-5x+6}$; ОДЗ: $x^2 - 5x + 6 \neq 0$; $x \neq 2$ и $x \neq 3$;

б) $f(x) = \sqrt{\frac{1}{16-x^2}}$; ОДЗ: $16 - x^2 > 0$; $x \in (-4; 4)$.

2. $f(x) = 2x^3 - x + 5$; $f(-1) = 4$;
 $f(x-1) = (x-1)(2x^2 - 4x + 2 - 1) + 5 = 2x^3 - 6x^2 + 5x + 4$.

3.



C-7

1. $f(x) = \frac{\sin x \cos^2 x \operatorname{tg} x}{x^2}$
 $f(-x) = \frac{\sin(-x) \cos^2(-x) \operatorname{tg}(-x)}{(-x)^2} = \frac{\sin x \cos^2 x \operatorname{tg} x}{x^2} = f(x)$, значит, $f(x)$ четная.
2. $g(x) = x |x| \sin 5x \operatorname{tg} 3x$; $g(-x) = |-x| (-x) \sin (-5x) \operatorname{tg} (-3x) =$
 $= -x |x| \sin 5x \operatorname{tg} 3x = -g(x)$, значит, $g(x)$ нечетная.

C-8

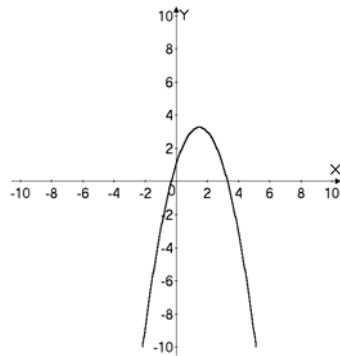
1. а) $\sin 312^\circ 19' = -\cos 42^\circ 19'$; б) $\cos 5042^\circ = \cos 2^\circ$;
 в) $\operatorname{ctg} \frac{33\pi}{8} = \operatorname{ctg} \frac{\pi}{8}$.
2. $\cos(-30^\circ) + \sin 660^\circ + \operatorname{ctg}(-510^\circ) = \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} + \sqrt{3} = \sqrt{3}$.
3. а) $y = \operatorname{tg}(1 - 3x)$; $T = \frac{\pi}{3}$;
 б) $y = \sin^4 x + \cos^4 x = 1 - \frac{1}{2} \sin^2 2x$; $T = \frac{\pi}{2}$.

C-9

1. а) $f(x) = \sqrt{1 - 2x}$; убывает на области определения, т.е. при $x \in \left(-\infty; -\frac{1}{2}\right]$;
 б) $f(x) = \frac{x+2}{x+1} = 1 + \frac{1}{x+1}$; убывает на области определения, т.е. при $x \in (-\infty; -1) \cup (-1; \infty)$.
2. $f(x) = \operatorname{tg}\left(\frac{\pi}{3} - \frac{x}{2}\right)$; ОДЗ: $\cos\left(\frac{x}{2} - \frac{\pi}{3}\right) \neq 0$; $x \neq \frac{5\pi}{3} + 2\pi n$;
 убывает на области определения.
3. $\cos 10^\circ$, $\cos 70^\circ$, $\cos(-20^\circ) = \cos 20^\circ$, $\sin 15^\circ$.
 Ответ: $\sin 15^\circ$, $\cos 70^\circ$, $\cos 20^\circ$, $\cos 10^\circ$.

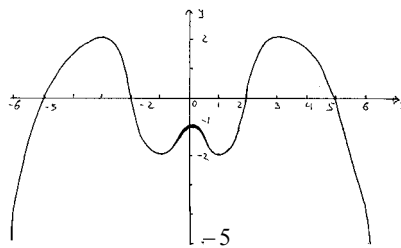
C-10

$$1. \quad y = 3x - x^2 + 1; \quad x_B = \frac{3}{2}; \quad y_B = \frac{9}{2} - \frac{9}{4} + 1 = \frac{11}{4}; \quad y \in (-\infty; \frac{13}{4}].$$



$$2. \quad f(x) = \sin\left(2x + \frac{\pi}{7}\right); \quad \min: \left(-\frac{9\pi}{28} + \pi n; -1\right); \quad \max: \left(\frac{5\pi}{28} + \pi n; 1\right).$$

C-11



C-12

$$1. \quad f(x) = \frac{1}{\operatorname{tg} 3x}; \quad \text{ОДЗ: } \begin{cases} \sin 3x \neq 0 \\ \cos 3x \neq 0 \end{cases}; \quad \begin{cases} x \neq \frac{\pi n}{3} \\ x \neq \frac{\pi}{6} + \frac{\pi n}{3} \end{cases}.$$

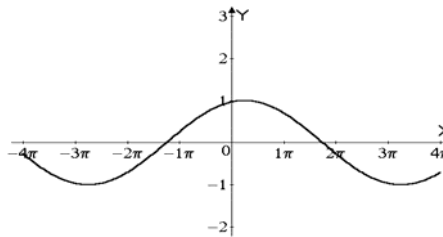
2. $f(x) = \cos\left(\frac{x}{3} - \frac{\pi}{12}\right)$

возрастает при

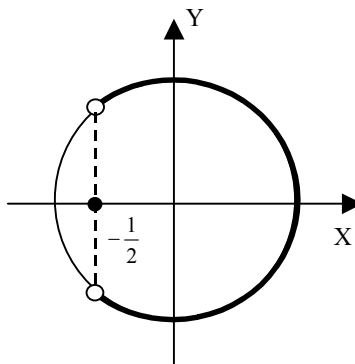
$$x \in \left[-\frac{11\pi}{4} + 6\pi n; \frac{\pi}{4} + 6\pi n\right];$$

убывает при

$$x \in \left[\frac{\pi}{4} + 6\pi n; \frac{13\pi}{4} + 6\pi n\right].$$



3. $\cos t > -\frac{1}{2}; t \in \left(-\frac{2\pi}{3} + 2\pi n; \frac{2\pi}{3} + 2\pi n\right)$



C-13

1.

а) $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$; **б)** $\cos(\arccos(-0,3)) = -0,3$;

в) $\operatorname{arctg}(-\sqrt{3}) + \operatorname{arctg}\frac{1}{\sqrt{3}} = -\frac{\pi}{3} + \frac{\pi}{6} = -\frac{\pi}{6}$.

г) $\sin(3\operatorname{arctg}\frac{1}{\sqrt{3}}) = \sin\pi = 0$.

2.

а) $\arcsin(-0,736) \approx -0,8271$; **б)** $\arccos(-0,997) \approx 3,0641$;

в) $\operatorname{arctg}(3,7) \approx 1,3068$.

C-14

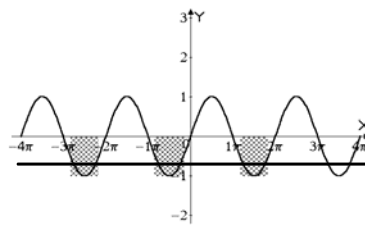
а) $\sin x = -\frac{\sqrt{2}}{2}; \quad x = (-1)^{k+1} \frac{\pi}{4} + \pi k.$

б) $\cos\left(x + \frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}; \quad x = -\frac{\pi}{6} \pm \frac{5\pi}{6} + 2\pi n;$

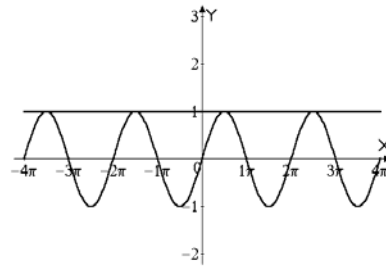
в) $\operatorname{tg}\left(2x - \frac{\pi}{3}\right) = \sqrt{3}; \quad x = \frac{\pi}{3} + \frac{\pi n}{2}.$

C-15

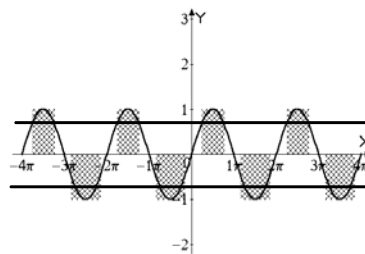
а)



б)



в)



$$\sin x \leq -\frac{1}{2}; \quad x \in \left[-\frac{5\pi}{6} + 2\pi n; -\frac{\pi}{6} + 2\pi n\right].$$

C-16

а) $\sin 2x > \frac{\sqrt{2}}{2}; \quad x \in \left(\frac{\pi}{8} + \pi n; \frac{3\pi}{8} + \pi n\right);$

$$\begin{aligned} \text{б) } \operatorname{tg}\left(3x - \frac{\pi}{4}\right) &< \frac{1}{\sqrt{3}}; & 3x &\in \left(-\frac{\pi}{4} + \pi n; \frac{5\pi}{12} + \pi n\right); \\ & & x &\in \left(-\frac{\pi}{12} + \frac{\pi n}{3}; \frac{5\pi}{36} + \frac{\pi n}{3}\right). \end{aligned}$$

C-17

$$\begin{aligned} \text{а) } \operatorname{tg} x + 3 \operatorname{ctg} x &= 4; & \operatorname{ctg} x &\neq 0; \\ \operatorname{tg}^2 x - 4 \operatorname{tg} x + 3 &= 0; & \operatorname{tg} x &= 3; & x &= \operatorname{arctg} 3 + \pi n; \\ & & \operatorname{tg} x &= 1; & x &= \frac{\pi}{4} + \pi n; \\ \text{б) } 2 \cos^4 x - 3 \cos^2 x + 1 &= 0; \\ \cos^2 x = 1; & x = \pi n; & \cos^2 x = \frac{1}{2}; & x = \frac{\pi}{4} + \frac{\pi n}{2}. \end{aligned}$$

C-18

$$\begin{aligned} \text{а) } \sin\left(x + \frac{\pi}{6}\right) + \cos\left(x + \frac{\pi}{6}\right) &= 0; & \cos\left(x + \frac{\pi}{6}\right) &\neq 0; \\ \operatorname{tg}\left(x + \frac{\pi}{6}\right) &= -1; & x &= -\frac{5\pi}{12} + \pi n. \\ \text{б) } \sin^2 x - \frac{5}{2} \sin 2x + 2 &= 0; & 5 \sin 2x + \cos 2x &= 5; \\ \sin(2x + \varphi) &= \frac{5}{\sqrt{26}}; \\ \varphi &= \arccos \frac{5}{\sqrt{26}}; \\ x &= -\frac{\varphi}{2} + \frac{1}{2}(-1)^k \arcsin \frac{5}{\sqrt{26}} + \frac{\pi k}{2}. \end{aligned}$$

C-19

$$\begin{cases} \cos x \cos y = \frac{1}{2} \\ \sin x \sin y = -\frac{1}{2} \end{cases}; \quad \begin{cases} \cos(x+y) + \cos(x-y) = 1 \\ \cos(x+y) - \cos(x-y) = -1 \end{cases};$$

$$\begin{cases} x + y = \frac{\pi}{2} + \pi n; \\ x - y = 2\pi k \end{cases} \quad \begin{cases} x = \frac{\pi}{4} + \frac{\pi n}{2} + \pi k \\ y = \frac{\pi}{4} + \frac{\pi n}{2} - \pi k \end{cases}.$$

C-20

a) $\cos x + \cos 5x = \cos 3x + \cos 7x;$

$\cos 3x \cos 2x - \cos 5x \cos 2x = 0; \cos 2x (\cos 3x - \cos 5x) = 0;$

$\cos 2x \sin 4x \sin x = 0;$

$\cos 2x = 0; \quad x = \frac{\pi}{4} + \frac{\pi n}{2}; \quad \sin 4x \sin x = 0; \sin 4x = 0; \quad x = \frac{\pi n}{4};$

б) $\cos x \cos 2x \cos 5x - \cos x \sin 2x \sin 5x + \sin x \sin 7x = 0$

$\cos x \cos 7x + \sin x \sin 7x = 0; \cos 6x = 0; \quad x = \frac{\pi}{12} + \frac{\pi n}{6}.$

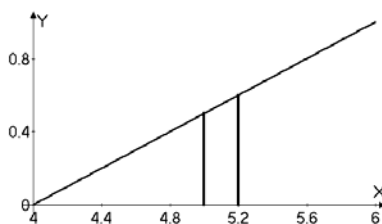
C-21

1.

$$f(x) = \frac{1}{2}x - 2;$$

$$f(x_0 + \Delta x) - f(x_0) = \frac{1}{2} \Delta x;$$

$x_0 = 5 \quad \Delta x = 0,2 \quad \Delta f(x_0) = 0,1.$



2.

$$f(x) = 2 + 3x - \frac{x^2}{2};$$

$$\frac{\Delta f(x_0)}{\Delta x} = \frac{2 + 3x_0 + 3\Delta x - \frac{x_0^2 + \Delta x^2 + 2x_0\Delta x}{2} - 2 - 3x_0 + \frac{x_0^2}{2}}{\Delta x} = 3 - \frac{\Delta x}{2} - x_0;$$

$x_0 = -1, \quad \Delta x = 0,1; \quad \frac{\Delta f(x_0)}{\Delta x} = 3,95;$

$x_0 = -1, \quad \Delta x = 0,002; \quad \frac{\Delta f(x_0)}{\Delta x} = 3,999;$

$x_0 = -1, \quad \Delta x = 0,00001; \quad \frac{\Delta f(x_0)}{\Delta x} = 3,999995;$

$x_0 = -1; \quad \lim_{\Delta x \rightarrow 0} \frac{\Delta f(x_0)}{\Delta x} = 4.$

C-22

1. $x(t) = 2t^2 - 1$; $v(t) = 4t$;
Импульс при $t = 2$ и $m = 3$ равен $4 \cdot 2 \cdot 3 = 24$ кг · м/с.
2. а) $f(x) = 4\sqrt{x}$, $f'(x) = \frac{2}{\sqrt{x}}$; б) $f(x) = x^2 + 3$; $f'(x) = 2x$.

C-23

1. а) $f(-3) = 1$; $f(2) = 2$; б) $\lim_{x \rightarrow -3} f(x) = 1$ $\lim_{x \rightarrow 2} f(x) = -1$.
2. $f(x) = \frac{x^2 - 4}{x - 2} = x + 2$, $x \neq 2$; $|x - 3| < 0,001$; $\delta = 0,001$.

C-24

1. а) $\lim_{x \rightarrow -1} y = \lim_{x \rightarrow -1} f^2(x) - 3 \lim_{x \rightarrow -1} g(x) = 4 - 9 = -5$;
б) $\lim_{x \rightarrow -1} y = \frac{\lim_{x \rightarrow -1} f(x) - \lim_{x \rightarrow -1} g(x)}{\lim_{x \rightarrow -1} f(x) + \lim_{x \rightarrow -1} g(x)} = \frac{2 - 3}{2 + 3} = -\frac{1}{5}$.
2. а) $\lim_{x \rightarrow 2} (1 - 3x^2 + 4x^4) = 1 - 12 + 64 = 53$;
б) $\lim_{x \rightarrow -3} \frac{3x - 5}{x^2 + x + 1} = \frac{-14}{7} = -2$.

C-25

1. а) $f(x) = x^7 + 2x^5 + \frac{4}{x^2} - 1$;
 $f'(x) = 7x^6 + 10x^4 - \frac{8}{x^3}$;
б) $f(x) = \frac{3 - x^2}{4 + 2x}$;
 $f'(x) = \frac{-8x - 4x^2 - 6 + 2x^2}{(4 + 2x)^2} = \frac{-2x^2 - 8x - 6}{(4 + 2x)^2}$.

2. $f(x) = x\sqrt{x+1}$; $f'(x) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$;
 $f'(0) = 1$; $f(3) = 2 + \frac{3}{2 \cdot 2} = 2\frac{3}{4}$; $f(x-1) = \sqrt{x} + \frac{x-1}{2\sqrt{x}}$.
3. $f(x) = x - 3x^3$; $f'(x) = 1 - 9x^2$; $f'(x) < 0$ при $x \in (-\infty; -\frac{1}{3}) \cup (\frac{1}{3}; +\infty)$.

C-26

1. $f(x) = \frac{\sqrt{x}+1}{\sqrt{x}-1}$; $f'(x) = \frac{\frac{1}{2} - \frac{1}{2\sqrt{x}} - \frac{1}{2} - \frac{1}{2\sqrt{x}}}{(\sqrt{x}-1)^2} = -\frac{1}{\sqrt{x}(\sqrt{x}-1)^2}$;
 $f(t^4) = \frac{1}{t^2(t^2-1)^2}$.
2. а) $f(x) = 3x^3 - x$; $f'(x) = 9x^2 - 1$; $f'(x) = 0$ при $x = \pm\frac{1}{3}$;
 $f'(x) > 0$ при $x \in (-\infty; -\frac{1}{3}) \cup (\frac{1}{3}; \infty)$; $f'(x) < 0$ при $x \in (-\frac{1}{3}; \frac{1}{3})$.
 б) $f(x) = \frac{x^2-8}{x+1}$; $f'(x) = \frac{x^2+2x+8}{(x+1)^2}$;
 $f'(x) > 0$ всегда, кроме $x = -1$.

C-27

1. а) $f(x) = \sqrt{4-2\sqrt{x}}$; ОДЗ: $\begin{cases} 4-2\sqrt{x} \geq 0 \\ x \geq 0 \end{cases}$; $x \in [0; 4]$;
 б) $f(x) = \frac{1}{\sqrt{x^2-3x+2}}$; ОДЗ: $x^2-3x+2 > 0$; $x \in (-\infty; 1) \cup (2; +\infty)$.
2. $f(x) = \frac{1+x}{1-2x}$; $g(x) = \sqrt{x}$;
 $f(g(x)) = \frac{1+\sqrt{x}}{1-2\sqrt{x}}$; $g(f(x)) = \sqrt{\frac{1+x}{1-2x}}$.

3. а) $f(x) = (x^5 - 2x^2)^{191}$; $f'(x) = 191(5x^4 - 4x)(x^5 - 2x^2)^{190}$;
 б) $g(x) = \sqrt{1-x^2}$; $g'(x) = \frac{-x}{\sqrt{1-x^2}}$.

C-28

а) $f(x) = \cos(3-4x)$; $f'(x) = 4\sin(3-4x)$;
 б) $f(x) = \operatorname{tg}(2x-7)$; $f'(x) = \frac{2}{\cos^2(2x-7)}$;
 в) $f(x) = \sin x \cos(2x-3)$; $f'(x) = \cos x \cos(2x-3) - 2\sin x \sin(2x-3)$.

C-29

1.

$$f(x) = \frac{x^2 - 4x}{(x+1)(x^2 - 4x + 3)}; \quad \text{ОДЗ: } \begin{cases} x+1 \neq 0 & x \neq -1 \\ x^2 - 4x + 3 \neq 0 & x \neq 3 \end{cases}; \text{ и } x \neq 3,$$

значит, $f(x)$ непрерывна при $x \in (-\infty; -1) \cup (-1; 3) \cup (3; \infty)$.

2.

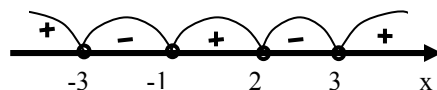
а) $x^2 - 3x + 2 > 0$ $(x-2)(x-1) > 0$;
 $x \in (-\infty; 1) \cup (2; +\infty)$.



б) $\frac{(x-3)(x+1)^2(x-2)^3}{x^2-9} < 0$;

$$\frac{(x-3)(x+1)^2(x-2)^3}{(x-3)(x+3)} < 0;$$

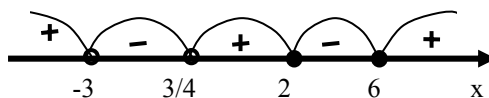
$$x \in (-3; -1) \cup (-1; 2).$$



в) $\frac{x-3}{x+3} \leq \frac{2x-5}{4x-3}$ $\frac{4x^2-15x+9-2x^2-x+15}{(x+3)(4x-3)} \leq 0$

$$\frac{2x^2-16x+24}{(x+3)(4x-3)} \leq 0; \quad \frac{(x-6)(x-2)}{(x+3)(4x-3)} \leq 0;$$

$$x \in (-3; \frac{3}{4}) \cup [2; 6].$$



C-30

1. $y = \sin 2x; \quad y\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}; \quad y' = 2\cos 2x; \quad y'\left(\frac{\pi}{6}\right) = 1;$

$y = \frac{\sqrt{3}}{2} + x - \frac{\pi}{6}$ – уравнение касательной.

2. $y = \frac{2}{x}; \quad y(-2) = -1; \quad y' = -\frac{2}{x^2}; \quad y'(-2) = -\frac{1}{2};$

$y = -1 - \frac{1}{2}(x + 2) = -\frac{1}{2}x - 2$ – уравнение касательной.

C-31

1. $\sqrt{49,07} \approx 7(1 + 0,0014 \cdot \frac{1}{2}) = 7,0049;$

2. $1,00006^{3000} - 0,99998^{6000} \approx 1 + 0,00006 \cdot 3000 - 1 +$
 $+ 0,00002 \cdot 6000 = 1,18 - 0,88 = 0,3.$

C-32

1. $s(t) = 4t + t^2 - \frac{1}{6}t^3; \quad v(t) = 4 + 2t - \frac{1}{2}t^2;$

$a(t) = 2 - t; \quad F = (2 - 2) \cdot 4 = 0 \text{ Н.}$

2. $h(t) = h_0 + v_0t - \frac{gt^2}{2} = 4 + 3t - 5t^2;$

$v(t) = 3 - 10t = \frac{3}{2}; \quad t = \frac{3}{20}; \quad h\left(\frac{3}{20}\right) = 4 + \frac{9}{20} - \frac{9}{80} = \frac{27}{80} + 4 = \frac{347}{80} \text{ м.}$

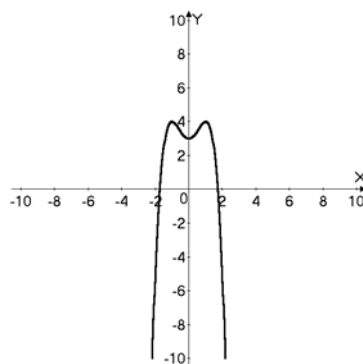
C-33

1. $f(x) = 2x^3 + 3x^2 - 12x; \quad f'(x) = 6(x^2 + x - 2); f'(x) = 0$ при $x = -2$ и $x = 1$, значит, $f(x)$ возрастает при $x \in (-\infty; -2) \cup (1; +\infty)$; убывает при $x \in (-2; 1)$.

2. $f(x) = 2x - \sqrt{x}; \quad f'(x) = 2 - \frac{1}{2\sqrt{x}}; f'(x) = 0$ при $x = \frac{1}{16}$ – точка min.

C-34

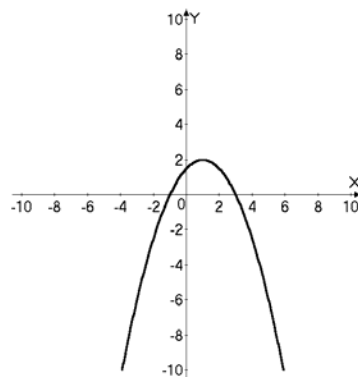
$f(x) = 2x^2 - x^4 + 3$; $f'(x) = 4(x - x^3)$; $f'(x) = 0$ при $x_{min} = 0$ и $x_{max} = \pm 1$;
 $y(\pm 1) = 4$; $y(0) = 3$;
 y возрастает при
 $x \in (-\infty; -1) \cup (0; 1)$;
 y убывает при
 $x \in (-1; 0) \cup (1; +\infty)$.



C-35

1.

$y = -\frac{1}{2}x^2 + x + \frac{3}{2}$; $x_B = 1$; $y_B = 2$;
 y возрастает при $x \in (-\infty; 1)$;
 y убывает при $x \in (1; \infty)$;
 нули $x^2 - 2x - 3 = 0$;
 $x = 3$, $x = -1$.



2.

а) $2x^2 - x + 1 < 0$ $D = 1 - 8 < 0$, значит, решений нет;

б) $16x^2 + 6x + 3 \geq 7x^2 - 6x - 1$;

$9x^2 + 12x + 4 \geq 0$; $(3x + 2)^2 \geq 0$, значит, $x \in R$.

С-36

$$y = 2x^3 - 6x^2 + 4;$$

$$y' = 6(x^2 - 2x) = 0; y' = 0 \text{ при}$$

$$x = 0, x = 2$$

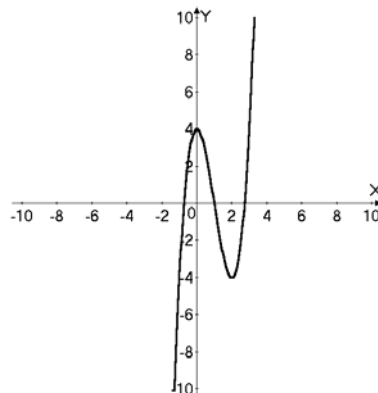
y возрастает при

$$x \in (-\infty; 0) \cup (2; +\infty);$$

убывает при $x \in (0; 2)$

$$x_{\max} = 0; y(0) = 4;$$

$$x_{\min} = 2; y(2) = -4.$$



С-37

1. $f(x) = x^5 + 20x^2 + 3; x \in [-1; 1]; f'(x) = 5(x^4 + 8x); f'(x) = 0$ при $x = 0$ и $x = -2; f(-1) = 22; f_{\min}(0) = 3; f_{\max}(1) = 24$, значит, наибольшее значение $f(1) = 24$; наименьшее значение $f(0) = 3$.

$$2. \quad \begin{cases} a + b = 8 \\ y = a^2 + b^3 \end{cases}; \quad \begin{cases} a = 8 - b \\ y = b^3 + b^2 - 16b + 64 \end{cases};$$

$$y' = 3b^2 + 2b - 16; y' = 0 \text{ при } b = -\frac{8}{3} \text{ не подходит; } \begin{cases} b = 2 \\ a = 6 \end{cases}, \text{ значит,}$$

$8 = 2 + 6$ – искомое разбиение.

С-38

$$1. \quad \cos \alpha = \frac{3}{5}; \quad \frac{3\pi}{2} < \alpha < 2\pi; \quad \cos \beta = -\frac{4}{5}; \quad \frac{\pi}{2} < \beta < \pi;$$

$$\sin \alpha = -\frac{4}{5}; \quad \sin \beta = \frac{3}{5};$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta = -\frac{12}{25} - \frac{12}{25} = -\frac{24}{25}.$$

$$\begin{aligned}
 2. \quad & \left(2\cos^2 \alpha - 2\sin^2 \alpha\right)^2 \sin^2 (\pi - 2\alpha) - \sin^2 \left(\frac{3\pi}{2} - 4\alpha\right) = \\
 & = 4\cos^2 2\alpha \sin^2 2\alpha - \cos^2 4\alpha = -\cos 8\alpha. \\
 3. \quad & \cos 15^\circ = \sqrt{\frac{1 + \cos 30^\circ}{2}} = \frac{\sqrt{\sqrt{3} + 2}}{2}; \\
 & \sin 15^\circ = \frac{\sqrt{2 - \sqrt{3}}}{2}; \quad \operatorname{tg} 15^\circ = \sqrt{\frac{2 - \sqrt{3}}{2 + \sqrt{3}}}.
 \end{aligned}$$

C-39

a)

$$y = \cos \frac{x}{2}; \quad x \in R; \quad y \in [-1; 1];$$

$$\text{нули: } x = \pi + 2\pi n;$$

$$x_{\max} = 4\pi n;$$

$$x_{\min} = 2\pi + 4\pi n;$$

$$y(4\pi n) = 1; \quad y(2\pi + 4\pi n) = -1;$$

y возрастает при

$$[-2\pi + 4\pi n; 4\pi n];$$

убывает при $[4\pi n; 2\pi + 4\pi n];$

$$\text{б) } y = \sin \left(x - \frac{2\pi}{5} \right);$$

$$x \in R; \quad y \in [-1; 1];$$

$$x_{\max} = \frac{9\pi}{10} + 2\pi n; \quad x_{\min} = -\frac{\pi}{10} + 2\pi n; \quad \text{нули: } x = \frac{2\pi}{5} + \pi n;$$

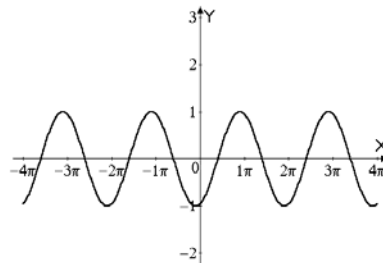
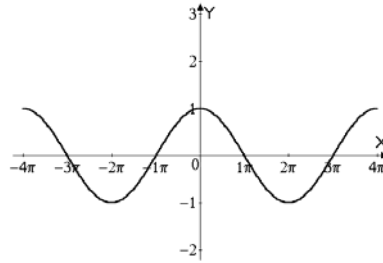
$$y\left(\frac{9\pi}{10} + 2\pi n\right) = 1; \quad y\left(-\frac{\pi}{10} + 2\pi n\right) = -1;$$

y возрастает при

$$\left(-\frac{\pi}{10} + 2\pi n; \frac{9\pi}{10} + 2\pi n\right);$$

убывает при

$$\left(\frac{9\pi}{10} + 2\pi n; \frac{19\pi}{10} + 2\pi n\right).$$



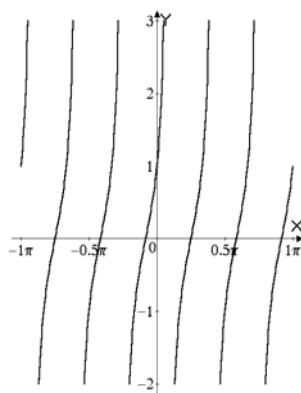
в)

$$y = \operatorname{tg}\left(3x + \frac{\pi}{4}\right); \text{ ОДЗ } \cos\left(3x + \frac{\pi}{4}\right) \neq 0;$$

$$x \neq \frac{\pi}{12} + \frac{\pi n}{3};$$

$$y \in R; \text{ нули: } x = -\frac{\pi}{12} + \frac{\pi n}{3};$$

возрастает на области определения.



C-40

1. а) $\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$; б) $\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6}$; в) $\operatorname{arctg}(-1) = -\frac{\pi}{4}$.

2. а) $2\sin\left(\frac{x}{2} + \frac{\pi}{3}\right) = 1$; $\frac{x}{2} = -\frac{\pi}{3} + (-1)^k \frac{\pi}{6} + \pi k$;

$$x = -\frac{2\pi}{3} + (-1)^k \frac{\pi}{3} + 2\pi k;$$

б) $\cos^2 x + \sin 2x = \frac{3}{2}$; $\cos 2x + 2\sin 2x = 2$;

$$\sin(2x + \varphi) = \frac{2}{\sqrt{5}}; \quad \varphi = \arccos \frac{2}{\sqrt{5}};$$

$$x = -\frac{\varphi}{2} + \frac{1}{2}(-1)^k \arcsin \frac{2}{\sqrt{5}} + \frac{\pi k}{2}.$$

3.

а) $\operatorname{tg} \frac{x}{2} > 1$; $x \in \left(\frac{\pi}{2} + 2\pi n; \pi + 2\pi n\right)$;

б) $\cos\left(x + \frac{\pi}{3}\right) < \frac{\sqrt{2}}{2}$; $x \in \left(-\frac{\pi}{12} + 2\pi n; \frac{17\pi}{12} + 2\pi n\right)$.

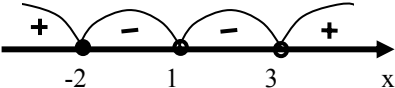
C-41

$$\begin{cases} x - y = \frac{2\pi}{3} \\ \cos x + \cos y = \frac{1}{2} \end{cases}; \quad \begin{cases} x = \frac{2\pi}{3} + y \\ 2\cos\left(\frac{\pi}{3} + y\right)\cos\frac{\pi}{3} = \frac{1}{2} \end{cases}; \quad \begin{cases} y = \pm\frac{\pi}{3} - \frac{\pi}{3} + 2\pi n \\ x = \frac{\pi}{3} \pm \frac{\pi}{3} + 2\pi n \end{cases}.$$

C-42

1. а) $x^2 - 6x + 8 > 0$; $x \in (-\infty; 2) \cup (4; +\infty)$;
 б) $x^2 - 12x + 36 \leq 0$; $D = 0$ $(x - 6)^2 \leq 0$; $x = 6$.

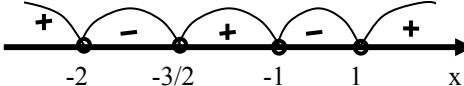
2. а) $\frac{(x-1)^2(x+2)^3}{x-3} \geq 0$;



$$x \in (-\infty; -2] \cup \{1\} \cup (3; +\infty);$$

б) $\frac{2}{x+1} + \frac{3}{x+2} < 2$; $\frac{2x+4+3x+3-2x^2-6x-4}{(x+1)(x+2)} < 0$;

$$\frac{2x^2+x-3}{(x+1)(x+2)} > 0$$

$$\frac{\left(x + \frac{3}{2}\right)(x-1)}{(x+1)(x+2)} > 0$$


$$x \in (-\infty; -2) \cup \left(-\frac{3}{2}; -1\right) \cup (1; +\infty).$$

C-43

а) $y = x^7 - 2x^5 + 3x - 3$; $y' = 7x^6 - 10x^4 + 3$;

б) $y = (1 + 3x)\sqrt{x}$; $y' = \frac{1+3x}{2\sqrt{x}} + 3\sqrt{x}$;

в) $y = \cos 5x$; $y' = -5\sin 5x$;

г) $y = \operatorname{ctg}\left(\frac{1}{2}x + 5\right)$; $y' = \frac{-1}{2\sin^2\left(\frac{x}{2} + 5\right)}$;

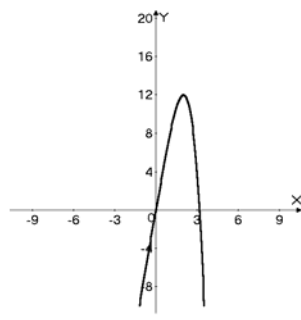
д) $y = \left(\frac{1}{3}x - 6\right)^{24}$; $y' = 8\left(\frac{1}{3}x - 6\right)^{23}$.

C-44

1. $f(x) = 3x + 2x^2$; $f(1) = 5$; $f'(x) = 3 + 4x$; $f'(1) = 7$;
 $y = 5 + 7(x - 1) = 7x - 2$ – уравнение касательной.
2. а) $\sqrt{1,002} \approx 1 + 0,001 = 1,001$;
б) $0,99997^{60} \approx 1 - 0,00003 \cdot 60 = 0,9982$.
3. $x(t) = t^3 + \frac{1}{2}t^2 - 7t$; $v(t) = 3t^2 + t - 7$;
 $v(3) = 23$; $a(t) = 6t + 1$; $a(3) = 19$.

C-45

1.



$$y = 8x - \frac{x^4}{4}; \quad x \in \mathbb{R}; \quad y \in (-\infty; 12];$$

$$y' = 8 - x^3; \quad y' = 0 \text{ при } x = 2, \text{ значит, } x_{\max} = 2; \quad y(2) = 12;$$

$$y \text{ возрастает при } x \in (-\infty; 2); \text{ убывает при } x \in (2; \infty);$$

$$\text{нули: } x = 0 \text{ и } x = \sqrt[3]{32}.$$

2. $f(x) = \frac{2x}{x^2 + 1}, \quad x \in [-2; 0,5];$

$$f'(x) = \frac{2x^2 + 2 - 4x^2}{(x^2 + 1)^2} = \frac{-2x^2 + 2}{(x^2 + 1)^2}; \quad f'(x) = 0 \text{ при}$$

$$x = \pm 1; \quad f(-2) = \frac{-4}{5}; \quad f\left(\frac{1}{2}\right) = \frac{4}{5}; \quad f(-1) = \frac{-2}{2} = -1, \text{ значит, наибольшее}$$

$$\text{значение функции } f\left(\frac{1}{2}\right) = \frac{4}{5}, \text{ наименьшее значение функции } f(-1) = -1.$$

ВАРИАНТ 7

С-1

1. $66^\circ = \frac{\pi}{180} \cdot 66 = \frac{11\pi}{30}$; $156^\circ = \frac{\pi}{180} \cdot 156 = \frac{13\pi}{15}$.
2. $\frac{5\pi}{18} = 50^\circ$; $\frac{29\pi}{3} = 1740^\circ$.
3. а) $71^\circ 4' \approx 1,2462$; $\sin 71^\circ 4' \approx 0,9494$;
 $\cos 71^\circ 4' \approx 0,314$;
б) $29^\circ 7' \approx 0,5111$; $\cos 29^\circ 17' \approx 0,8718$;
 $\sin 29^\circ 17' \approx 0,4898$.
4. а) $0,0367 \approx 2^\circ 6'$; б) $2,0033 \approx 114^\circ 47'$.

С-2

1. $\cos \alpha (1 + \cos^{-1} \alpha + \operatorname{tg} \alpha) (1 - \cos^{-1} \alpha + \operatorname{tg} \alpha) = 2 \sin \alpha$;
$$\frac{(\cos \alpha + 1 + \sin \alpha)(\cos \alpha - 1 + \sin \alpha)}{\cos \alpha} = \frac{1 + 2 \sin \alpha \cos \alpha - 1}{\cos \alpha} = 2 \sin \alpha.$$
2. а) $\frac{\sin 100^\circ \cos 100^\circ}{\operatorname{tg} 200^\circ \operatorname{ctg} 300^\circ} > 0$; б) $\sin 1 \cos 3 \operatorname{tg} 5 > 0$.
3. $\operatorname{tg} \alpha = -2$ $\cos \alpha > 0$, значит, $\alpha \in IV$ четверти;
$$\frac{\sin \alpha}{\sqrt{1 - \sin^2 \alpha}} = -2; \sin^2 \alpha = 4 - 4 \sin^2 \alpha; \sin \alpha = \frac{-2}{\sqrt{5}}; \cos \alpha = \frac{1}{\sqrt{5}}.$$

С-3

1. а) $\cos 1755^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$; б) $\sin 2160^\circ = \sin 0^\circ = 0$;
в) $\operatorname{ctg} \frac{39\pi}{4} = \operatorname{ctg} \frac{3\pi}{4} = -1$.
2. $(\sin 160^\circ + \sin 40^\circ)(\sin 140^\circ + \sin 20^\circ) + (\sin 50^\circ - \sin 70^\circ) \cdot$
 $\cdot (\sin 130^\circ - \sin 110^\circ) = 1 + 2 \sin 20^\circ \sin 40^\circ + 1 - 2 \sin 50^\circ \sin 70^\circ =$
 $= 2 - 2 \cos 60^\circ = 1.$

$$3. \quad \frac{\sin(\alpha + \pi)}{\sin\left(\alpha + \frac{3\pi}{2}\right)} + \frac{\cos(3\pi - \alpha)}{\cos\left(\frac{\pi}{2} + \alpha\right) - 1} = \frac{1}{\cos \alpha};$$

$$\operatorname{tg} \alpha + \frac{\cos \alpha}{\sin \alpha + 1} = \frac{\sin^2 \alpha + \sin \alpha + \cos^2 \alpha}{(\cos \alpha)(\sin \alpha + 1)} = \frac{1}{\cos \alpha}.$$

C-4

$$1. \quad \frac{1 - \sin^2 67^\circ 30'}{2 \cos^2 75^\circ - 1} = \frac{1 + \cos 135^\circ}{2 \cos 150^\circ} = -\frac{2 - \sqrt{2}}{4; \frac{1}{2} \cdot \sqrt{3}} = \frac{\sqrt{2} - 2}{2\sqrt{3}}.$$

$$2. \quad \sin \alpha = \frac{1}{3}; \quad \frac{\pi}{2} < \alpha < \pi; \quad \cos \alpha = -\frac{\sqrt{8}}{3}; \quad \sin 2\alpha = -\frac{4\sqrt{2}}{9};$$

$$\cos 2\alpha = \frac{7}{9}; \quad \sin 4\alpha = -\frac{8\sqrt{2} \cdot 7}{81} = -\frac{56\sqrt{2}}{81}; \quad \operatorname{tg} 2\alpha = -\frac{4\sqrt{2}}{7};$$

$$\operatorname{tg} 4\alpha = \frac{2\operatorname{tg} 2\alpha}{1 - \operatorname{tg}^2 2\alpha} = -\frac{8\sqrt{2}}{7} : \left(1 - \frac{32}{49}\right) = \frac{-8\sqrt{2}}{7} \cdot \frac{49}{17} = \frac{-56\sqrt{2}}{17}.$$

$$3. \quad \frac{1 + \operatorname{ctg} 2\alpha \operatorname{ctg} \alpha}{\operatorname{tg} \alpha + \operatorname{ctg} \alpha} = \left(1 + \frac{1 - \operatorname{tg}^2 \alpha}{2 \operatorname{tg} \alpha} \cdot \operatorname{ctg} \alpha\right) \sin \alpha \cos \alpha =$$

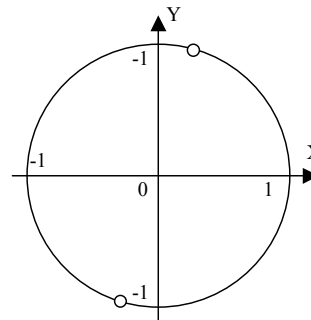
$$= \left(1 + \frac{1}{2} \operatorname{ctg}^2 \alpha - \frac{1}{2}\right) \frac{1}{2} \sin 2\alpha = \frac{\sin \alpha \cos \alpha}{2 \sin^2 \alpha} = \frac{1}{2} \operatorname{ctg} \alpha.$$

C-5

$$1. \quad \operatorname{tg} \alpha = 3; \quad \alpha = \operatorname{arctg} 3 + \pi n;$$

$$\sin^2 \alpha = 9 - 9 \sin^2 \alpha; \quad \sin \alpha = \pm \frac{3}{\sqrt{10}};$$

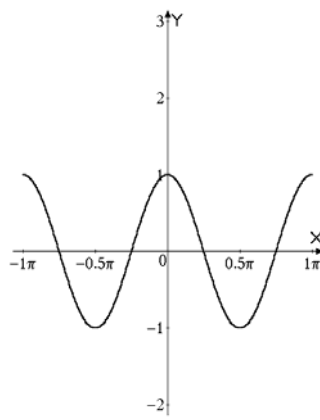
$$\cos \alpha = \pm \frac{1}{\sqrt{10}}; \quad \cos 2\alpha = -\frac{8}{10} = -\frac{4}{5}.$$



2. а) $\cos \alpha - \sin \alpha = -\frac{6}{5}$; $\sin\left(\alpha - \frac{\pi}{4}\right) = -\frac{6}{5\sqrt{2}}$; $\alpha \in \text{IV}$;

б) $\operatorname{tg} \frac{\alpha}{2} = 3$; $\alpha = 2\operatorname{arctg} 3 + 2\pi n$; $\alpha \in \text{II}$.

3. $y = \cos^4 x - \sin^4 x = (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x) = \cos 2x$.



C-6

1. а) $f(x) = \frac{\sqrt{3x-2}}{x^2-x-2}$;

ОДЗ: $\begin{cases} 3x-2 \geq 0 \\ x^2-x-2 \neq 0 \end{cases}$; $\begin{cases} x \neq 2 \\ x \geq \frac{2}{3} \end{cases}$ $x \neq -1$, значит, $x \in \left[\frac{2}{3}; 2\right) \cup (2; \infty)$;

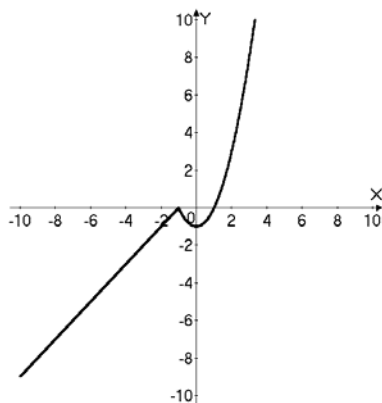
б) $f(x) = \sqrt{\frac{x-2}{5-2x}}$;

ОДЗ: $\frac{(x-2)}{5-2x} \geq 0$; $x \in [2; \frac{5}{2})$.

2. $f(x) = \begin{cases} x^2-1 & x \geq -1 \\ x+1 & x < -1 \end{cases}$;

а) $f(0) = -1$; $f(2) = 3$; $f(-1) = 0$; $f(-2) = -1$;

б)



C-7

а) $y = 2\sin x \cos 2x \operatorname{tg} 3x$; $y(-x) = 2\sin(-x) \cos(-2x) \operatorname{tg}(-2x) =$
 $= 2\sin x \cos 2x \operatorname{tg} 2x = y(x) \Rightarrow$ четная;

б) $y = x^2 \cos x \operatorname{ctg} 3x$; $y(-x) = (-x)^2 \cos(-x) \operatorname{ctg}(-3x) =$
 $= -x^2 \cos x \operatorname{ctg} 3x = -y(x) \Rightarrow$ нечетная;

в) $y = 2\cos\left(x + \frac{\pi}{6}\right) \sin x$ $y(-x) = 2\cos\left(\frac{\pi}{6} - x\right) \sin(-x)$, значит, y

ни четная, ни нечетная;

г) $y = 3x^2 + 2\sin 5x \cos x$; $y(-x) = 3(-x)^2 + 2\sin(-5x) \cos(-x) =$
 $= 3x^2 - 2\sin 5x \cos x$, значит, y ни четная, ни нечетная.

C-8

1. а) $\sin 311^\circ 17' = -\cos 41^\circ 43'$; б) $\sin 4160^\circ = -\cos 20^\circ$;

в) $\operatorname{tg} \frac{33\pi}{5} = -\operatorname{ctg} \frac{\pi}{10}$.

2. $\sin(-30^\circ) + \cos(660^\circ) + \operatorname{tg}(-510^\circ) = -\frac{1}{2} + \frac{1}{2} + \frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$.

3. а) $f(x) = \operatorname{tg}\left(2x - \frac{\pi}{7}\right)$; $T = \frac{\pi}{2}$;

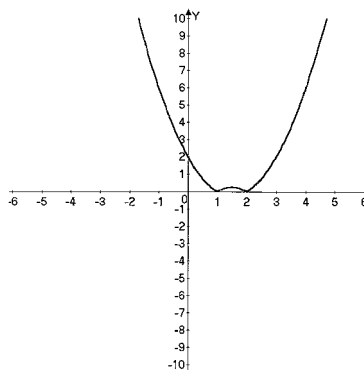
б) $f(x) = \sin^2 x + \operatorname{tg} x$; $T = \pi$; так как $f_1(x) = \sin^2 x$ $T = \pi$;
 $f_2(x) = \operatorname{tg} x$ $T = \pi$.

C-9

1. а) $f(x) = \sqrt{4 - x^2}$
 $f(x)$ возрастает при $x \in (-2; 0)$; убывает при $x \in (0; 2)$;
 б) $f(x) = \left| 1 - \frac{1}{x+1} \right|$;
 $f(x)$ возрастает при $x \in (-\infty; -1) \cup (0; +\infty)$; убывает при $x \in (-1; 0)$.
2. $f(x) = x^5 + x$; $f'(x) = 5x^4 + 1 > 0$ всегда.
3. $\sin 1, \sin 2, \sin 3, \sin 4$. Ответ: $\sin 4, \sin 3, \sin 1, \sin 2$.

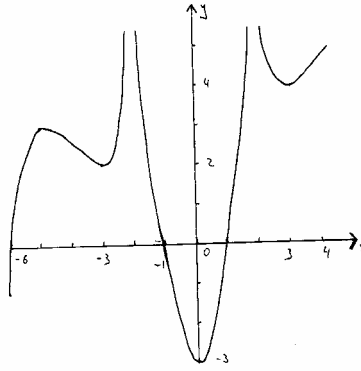
C-10

1. $f(x) = |x^2 - 3x + 2|$;
 $x^2 - 3x + 2 = 0$; $x = 2$ и $x = 1$; $x_B = \frac{3}{2}$; $y_B = \left| \frac{9}{4} - \frac{9}{2} + 2 \right| = \left| -\frac{1}{4} \right| = \frac{1}{4}$;
 $x_{\max} = \frac{3}{2}$; $x_{\min} = 2$; $x_{\min} = 1$; $f\left(\frac{3}{2}\right) = \frac{1}{4}$; $f(2) = 0$; $f(1) = 0$;
 $|x^2 - 3x + 2| \geq 1$; $\left[\begin{array}{l} x^2 - 3x + 1 \geq 0 \\ x^2 - 3x + 3 \leq 0 \end{array} \right.$; $x \in \left(-\infty; \frac{3-2\sqrt{5}}{2} \right] \cup \left[\frac{3+2\sqrt{5}}{2}; +\infty \right)$.



2. $f(x) = \sqrt{3} \sin 2x - \cos 2x - 1 = 2\sin\left(2x - \frac{\pi}{6}\right) - 1$
 $f(x) \in [-3; 1]$ $x_{\max} = \frac{\pi}{3} + \pi n$ $x_{\min} = -\frac{\pi}{3} + \pi n$

C-11



C-12

$$1. \quad f(x) = \operatorname{tg} \frac{x}{2} + \frac{1}{\operatorname{tg} \left(2x - \frac{\pi}{6} \right)}; \text{ ОДЗ: } \begin{cases} \cos \left(2x - \frac{\pi}{6} \right) \neq 0 \\ \cos \frac{x}{2} \neq 0 \\ \sin \left(2x - \frac{\pi}{6} \right) \neq 0 \end{cases}; \begin{cases} x \neq \frac{\pi}{3} + \pi n \\ x \neq \pi + 2\pi n \\ x \neq \frac{\pi}{12} + \frac{\pi n}{2} \end{cases}.$$

$$2. \quad y = \cos \left(\frac{x}{2} - \frac{\pi}{12} \right);$$

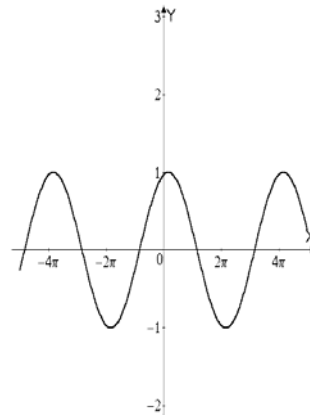
$$\cos \left(\frac{x}{2} - \frac{\pi}{12} \right) = 1; \quad x_{\max} = \frac{\pi}{6} + 4\pi n;$$

$$\cos \left(\frac{x}{2} - \frac{\pi}{12} \right) = -1; \quad x_{\min} = \frac{13\pi}{6} + 4\pi n;$$

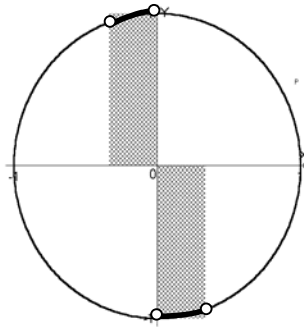
y возрастает при

$$x \in \left(-\frac{11\pi}{12} + 4\pi n; \frac{\pi}{6} + 4\pi n \right)$$

убывает при $x \in \left(\frac{\pi}{6} + 4\pi n; \frac{13\pi}{6} + 4\pi n \right).$



3.



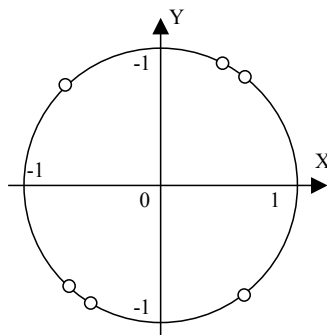
C-13

1.
 - a) $\arccos \frac{1}{\sqrt{2}} - \arcsin 1 = \frac{\pi}{4} - \frac{\pi}{2} = -\frac{\pi}{4}$;
 - б) $\arcsin (\sin 110^\circ) = \arcsin (\sin 70^\circ) = 70^\circ$;
 - в) $\cos (2\arccos \frac{1}{3}) = 2 \cdot \frac{1}{9} - 1 = -\frac{7}{9}$.
2. $\arcsin (-1) < \arctg (-1)$.
3.
 - a) $\arcsin (-0,3217) \approx -0,3275$;
 - б) $\arccos (-0,7991) \approx -2,4966$;
 - в) $\arctg (3,257) \approx 1,2729$.

C-14

$$\begin{aligned} \text{a) } \operatorname{tg} x &= -\frac{1}{\sqrt{3}}; & x &= -\frac{\pi}{6} + \pi n; \\ \text{б) } \sin \left(x + \frac{\pi}{5} \right) &= \frac{\sqrt{2}}{2}; \\ x &= -\frac{\pi}{5} + (-1)^k \frac{\pi}{4} + \pi k; \\ \text{в) } \cos \left(3x - \frac{\pi}{6} \right) &= -1; \\ 3x - \frac{\pi}{6} &= \pi + 2\pi n; & x &= \frac{7\pi}{18} + \frac{2\pi n}{3}. \end{aligned}$$

C-15



$$\cos 2t(\sin t - \sqrt{3} \cos t) = 0;$$

$$t = \frac{\pi}{4} + \frac{\pi n}{2}; t = \frac{\pi}{3} + \pi k;$$

$$\cos 2t(\sin t - \sqrt{3} \cos t) > 0;$$

$$x \in \left(\frac{\pi}{4} + 2\pi n; \frac{\pi}{3} + 2\pi n \right) \cup \left(\frac{3\pi}{4} + 2\pi n; \frac{5\pi}{4} + 2\pi n \right) \cup \left(\frac{4\pi}{3} + 2\pi n; \frac{7\pi}{4} + 2\pi n \right).$$

C-16

$$\text{a) } \sin \frac{x}{2} \leq -\frac{\sqrt{2}}{2}; \quad x \in \left[-\frac{3\pi}{2} + 4\pi n; -\frac{\pi}{2} + 4\pi n \right].$$

$$\text{б) } \operatorname{tg} \left(\frac{x}{3} - 1 \right) \leq -1; \quad \frac{x}{3} \in \left(-\frac{\pi}{2} + 1 + \pi n; -\frac{\pi}{4} + 1 + \pi n \right);$$

$$x \in \left(-\frac{3\pi}{2} + 3 + 3\pi n; -\frac{3\pi}{4} + 3 + 3\pi n \right).$$

C-17

$$\text{a) } \cos^2 x - 3\sin x - 3 = 0; \quad \sin^2 x + 3\sin x + 2 = 0;$$

$$\sin x = -2 - \text{решений нет}; \quad \sin x = -1; \quad x = -\frac{\pi}{2} + 2\pi n;$$

$$\text{б) } \sin 2x = 2\sqrt{3} \sin^2 x; \quad \sin x = 0; \quad x = \pi n;$$

$$\cos x = \sqrt{3} \sin x; \quad \operatorname{ctg} x = \sqrt{3};$$

$$x = \frac{\pi}{6} + \pi n.$$

C-18

$$\begin{aligned} \text{a)} \quad \frac{\cos 2\alpha}{1 + \sin 2\alpha} &= \frac{1 - \operatorname{tg} \alpha}{1 + \operatorname{tg} \alpha}; \\ \frac{(\cos \alpha - \sin \alpha)(\cos \alpha + \sin \alpha)}{(\sin \alpha + \cos \alpha)^2} &= \frac{(\cos \alpha - \sin \alpha) \cos \alpha}{(\cos \alpha + \sin \alpha) \cos \alpha} = \frac{1 - \operatorname{tg} \alpha}{1 + \operatorname{tg} \alpha}; \\ \text{б)} \quad \frac{2 \sin \frac{\alpha}{2} + \sin \alpha}{2 \sin \frac{\alpha}{2} - \sin \alpha} &= \operatorname{ctg}^2 \frac{\alpha}{4}; \quad \frac{1 + \cos \frac{\alpha}{2}}{1 - \cos \frac{\alpha}{2}} = \frac{2 \cos^2 \frac{\alpha}{4}}{2 \sin^2 \frac{\alpha}{4}} = \operatorname{ctg}^2 \frac{\alpha}{4}. \end{aligned}$$

C-19

$$\begin{aligned} &\begin{cases} \cos(x+y) = -\frac{1}{2}; \\ \sin x + \sin \alpha = \sqrt{3} \end{cases}; \quad \begin{cases} x+y = \pm \frac{2\pi}{3} + 2\pi n; \\ \sin x + \sin y = \sqrt{3} \end{cases}; \\ 1. \quad &\begin{cases} x = \frac{2\pi}{3} - y + 2\pi n \\ \sin\left(\frac{2\pi}{3} - y\right) + \sin y = \sqrt{3} \end{cases}; \quad \sin \frac{\pi}{3} \cos\left(\frac{\pi}{3} - y\right) = \frac{\sqrt{3}}{2}; \\ &\begin{cases} y = \frac{\pi}{3} - 2\pi k \\ x = \frac{\pi}{3} + 2\pi k + 2\pi n \end{cases}; \\ 2. \quad &\begin{cases} x = -\frac{2\pi}{3} - y + 2\pi n & -\sin \frac{\pi}{3} \cos\left(\frac{\pi}{3} + y\right) = \frac{\sqrt{3}}{2}; \\ \sin y - \sin\left(\frac{2\pi}{3} + y\right) = \sqrt{3} & \cos\left(\frac{\pi}{3} + y\right) = -1 \end{cases}; \\ &\begin{cases} y = \frac{2\pi}{3} + 2\pi k \\ x = -2\pi k + 2\pi n \end{cases}. \end{aligned}$$

C-20

$$\text{a) } \operatorname{tg} x = \operatorname{tg} 3x; \quad \frac{\sin 3x \cos x - \sin x \cos 3x}{\cos x \cos 3x} = 0; \quad \text{ОДЗ: } x \neq \frac{\pi}{6} + \frac{\pi n}{3};$$

$$\sin 2x = 0 \quad x = \frac{\pi n}{2}, \quad \text{но} \quad x \neq \frac{\pi}{6} + \frac{\pi n}{3}, \quad \text{значит, } x = \pi n;$$

$$\text{б) } \operatorname{tg} x + \frac{\cos x}{1 + \sin x} = 1; \quad \text{ОДЗ: } x \neq \frac{\pi}{2} + \pi n;$$

$$\frac{\sin x + \sin^2 x + \cos^2 x}{\cos x(1 + \sin x)} = 1; \quad \frac{1}{\cos x} = 1;$$

$$\cos x = 1; \quad x = 2\pi n;$$

$$\text{в) } \sin 3x = \cos x; \quad \sin 3x - \sin\left(\frac{\pi}{2} - x\right) = 0;$$

$$\sin\left(2x - \frac{\pi}{4}\right) \cos\left(x + \frac{\pi}{4}\right) = 0; \quad x = \frac{\pi}{8} + \frac{\pi n}{2}; \quad x = \frac{\pi}{4} + \pi k.$$

C-21

$$\begin{aligned} 1. \quad f(x) &= x^2 - 3x; \quad f(x_0 + \Delta x) - f(x_0) = \\ &= x_0^2 + \Delta x^2 + 2x_0\Delta x - 3x_0 - 3\Delta x - x_0^2 + 3x_0 = (\Delta x)^2 + 2x_0\Delta x - 3\Delta x; \end{aligned}$$

$$\text{a) } x_0 = 3; \quad \Delta x = -\frac{1}{2}; \quad \Delta f = \frac{1}{4} - 3 + \frac{3}{2} = -\frac{5}{4};$$

$$\text{б) } x_0 = -2; \quad \Delta x = 1; \quad \Delta f = 1 - 2 \cdot 2 - 3 = -6.$$

$$\begin{aligned} 2. \quad f(x) &= x^3 - 5x \quad \Delta f = (x_0 + \Delta x)(x_0^2 + (\Delta x)^2 + 2x_0\Delta x - 5) - \\ &- x_0^3 + 5x_0 = x_0(\Delta x)^2 + 2x_0^2\Delta x + \Delta x x_0^2 + (\Delta x)^3 + 2x_0(\Delta x)^2 - 5\Delta x = \\ &= \Delta x^3 + 3x_0(\Delta x)^2 + 3x_0^2\Delta x - 5\Delta x; \end{aligned}$$

$$\frac{\Delta f}{\Delta x} = (\Delta x)^2 + 3x_0\Delta x + 3x_0^2 - 5.$$

C-22

$$1. \quad x(t) = 3 - 2t + t^2; \quad v(t) = -2 + 2t;$$

$$v(4) = 6; \quad E = \frac{3 \cdot 36}{2} = 54 \text{ Дж.}$$

$$2. \quad \text{a) } f(x) = 7 - 5x; \quad f'(x) = -5;$$

$$\text{б) } f(x) = x^2 - 4x - 7; \quad f'(x) = 2x - 4.$$

C-23

1. **a)** $f(-1) = -\frac{1}{2}$; $f(1) = -\frac{1}{2}$; **б)** $\lim_{x \rightarrow -1} f(x) = \frac{1}{2}$; $\lim_{x \rightarrow 1} f(x) = -1,5$.
2. $f(x) = \frac{x^2 - 6x + 5}{2(x - 5)} = \frac{x - 1}{2}$, $x \neq 5$; $\left| \frac{x - 1}{2} - 2 \right| < 0,001$;
 $|x - 5| < 0,002$; $\delta = 0,002$.

C-24

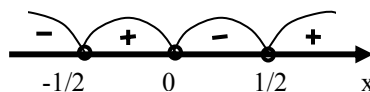
1. $\lim_{x \rightarrow 3} f(x) = \frac{1}{2}$; $\lim_{x \rightarrow 3} g(x) = -\frac{1}{3}$;
a) $\lim_{x \rightarrow 3} y = \frac{\lim_{x \rightarrow 3} f(x)}{\lim_{x \rightarrow 3} g(x)} - \lim_{x \rightarrow 3} f(x) \cdot \lim_{x \rightarrow 3} g(x) = -\frac{3}{2} + \frac{1}{6} = -\frac{3}{4}$;
б) $\lim_{x \rightarrow 3} y = \frac{3 \lim_{x \rightarrow 3} f(x) - \lim_{x \rightarrow 3} g^2(x)}{2 \lim_{x \rightarrow 3} f(x)} = \frac{\frac{3}{2} - \frac{1}{9}}{1} = \frac{25}{18}$.
2. **a)** $\lim_{x \rightarrow 3} \left(1 - x + \frac{1}{2}x^2 - \frac{1}{3}x^3 \right) = 1 - 3 + \frac{9}{2} - 9 = -11 + \frac{9}{2} = -\frac{13}{2}$;
б) $\lim_{x \rightarrow -2} \frac{4x + 8}{2x^2 + x - 1} = 0$.

C-25

1. **a)** $f(x) = x^7 - 3x^5 + \frac{1}{\sqrt{x}} - 2$; $f'(x) = 7x^6 - 15x^4 - \frac{1}{2x^{3/2}}$;
б) $f(x) = (x + 5)\sqrt{x}$; $f'(x) = \sqrt{x} + \frac{x + 5}{2\sqrt{x}}$.
2. $f(x) = \frac{3 - 2x}{x + 5}$; $f'(x) = \frac{-2x - 10 - 3 + 2x}{(x + 5)^2} = -\frac{13}{(x + 5)^2}$;
 $f'(-4) = -13$; $f'(8) = -\frac{1}{13}$; $f'(x^2 - 5) = -\frac{13}{x^4}$.
3. $f(x) = x + \frac{1}{x}$; $f'(x) = 1 - \frac{1}{x^2} \geq 0$ при $x \in (-\infty; -1] \cup [1; +\infty)$.

C-26

1. $f(x) = 100(\sqrt{x})^{10} - 10(\sqrt{x})^{100} = 100x^5 - 10x^{50};$
 $f'(x) = 500(x^4 - x^{49}); \quad f'(1) = 0.$
2. а) $f(x) = 2x^4 - x^2; \quad f'(x) = 2x(4x^2 - 1); f'(x) = 0$ при $x = 0$ и $x = \pm \frac{1}{2};$



$$f'(x) > 0 \text{ при } x \in \left(-\frac{1}{2}; 0\right) \cup \left(\frac{1}{2}; +\infty\right);$$

$$f'(x) < 0 \text{ при } x \in \left(-\infty; -\frac{1}{2}\right) \cup \left(0; \frac{1}{2}\right);$$

$$\text{б) } f(x) = \frac{x^2 - 12}{x - 2}; \quad f'(x) = \frac{2x^2 - 4x - x^2 + 12}{(x - 2)^2} = \frac{x^2 - 4x + 12}{(x - 2)^2}$$

$$f'(x) > 0 \text{ всегда, кроме } x = 2.$$

C-27

$$1. \text{ а) } f(x) = \frac{1}{\sqrt{x-3}-1}; \text{ ОДЗ: } \begin{cases} x \geq 3 \\ \sqrt{x-3} \neq 1 \end{cases} \begin{cases} x \geq 3 \\ x \neq 4 \end{cases}, \text{ значит, } x \in [3; 4) \cup (4; \infty);$$

$$\text{б) } f(x) = \frac{1}{\sqrt{2-\sqrt{x}}}; \quad \text{ОДЗ: } \begin{cases} x \geq 0 \\ 2 - \sqrt{x} > 0 \end{cases}; \quad x \in [0; 4).$$

$$2. f(x) = x^3 + 2x; g(x) = \sin x; f(g(x)) = \sin^3 x + 2\sin x; g(f(x)) = \sin(x^3 + 2x).$$

$$3. \text{ а) } f(x) = (5x^4 - 4x^5)^{101}; \quad f'(x) = 101(20x^3 - 20x^4)(5x^4 - 4x^5)^{100};$$

$$\text{б) } g(x) = \sqrt{3x^2 - 6x}; \quad g'(x) = \frac{3x - 3}{\sqrt{3x^2 - 6x}}.$$

C-28

$$\text{а) } f(x) = \cos\left(\frac{2x}{3} - 1\right); \quad f'(x) = -\frac{2}{3} \sin\left(\frac{2x}{3} - 1\right);$$

$$\text{б) } f(x) = \sin x \cos 2x + \cos x \sin 2x = \sin 3x; \quad f'(x) = 3\cos 3x;$$

$$\text{в) } f(x) = \cos x \cos 2x - \sin x \sin 2x;$$

$$f'(x) = -\sin x \cos 2x - 2\sin 2x \cos x - \frac{3}{\cos^2 3x}.$$

C-29

1. $f(x) = \frac{3x-8}{x^3-7x^2+6x}$; ОДЗ: $\frac{x(x^2-7x+6) \neq 0}{x \neq 0 \quad x \neq 6 \quad x \neq 1}$, значит, $f(x)$ непрерывна на $x \in (-\infty; 0) \cup (0; 1) \cup (1; 6) \cup (6; \infty)$

2. а) $\frac{(x-2)(x+8)(x-9)}{x^2-4} \geq 0$;

$$\frac{(x-2)(x+8)(x-9)}{(x-2)(x+2)} \geq 0;$$

$$x \in [-8; -2) \cup [9; +\infty);$$

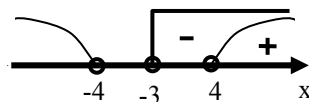


б) $(x^2-16)\sqrt{x+3} < 0$;

$$(x-4)(x+4)\sqrt{x+3} < 0;$$

ОДЗ $x \geq -3$;

$$x \in (-3; 4).$$



C-30

1. $y = \sin \frac{x}{2}; \quad y\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2};$

$$y' = \frac{1}{2} \cos \frac{x}{2}; \quad y'\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{4};$$

$$y_{\text{кас}} = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{4} \left(x - \frac{\pi}{2}\right) = \frac{\sqrt{2}}{4}x + \frac{4\sqrt{2} - \sqrt{2}\pi}{8} - \text{уравнение касательной.}$$

2. $y = x^2 - 2x; \quad x_0 = 2; \quad y(2) = 0; \quad y' = 2x - 2; \quad y'(2) = 2; \quad y_{\text{кас}} = 2x - 4 -$
уравнение касательной.

C-31.

1. $\sqrt{16,08} \approx 4(1 + 0,005 \cdot \frac{1}{2}) = 4,01.$

2. $1,00004^{100} + 0,99996^{100} \approx 1 + 0,00004 \cdot 100 + 1 - 0,00004 \cdot 100 =$
 $= 1,004 + 0,996 = 2.$

С-32

1. $s(t) = 2t + \sqrt{t}$; $v(t) = 2 + \frac{1}{2\sqrt{t}}$; $a(t) = -\frac{1}{4t^{3/2}}$; $F = -\frac{1}{4 \cdot 8} \cdot 5 = -\frac{5}{32}$ Н.
2. $\varphi = 3t - 0,01t^2$; $\varphi'(t) = 3 - 0,02t$;
а) $\varphi'(7) = 2,86$; б) $3 - 0,02t = 0$; $t = 150$.

С-33

1. $y = 3x^3 - x^2 - 7x$; $y' = 9x^2 - 2x - 7$; $y' = 0$ при $x_1 = 1$ и $x_2 = -\frac{7}{9}$, значит,
 y возрастает при $x \in (-\infty; -\frac{7}{9}) \cup (1; +\infty)$; убывает при $x \in (-\frac{7}{9}; 1)$.

2. $f(x) = \frac{x^2}{9} + \frac{4}{x^2}$;
 $f'(x) = \frac{2x}{9} - \frac{8}{x^3}$; $f'(x) = 0$ при
 $2x^4 = 72$; $x^4 = 36$; $x = \pm\sqrt{6}$;
 $x = \pm\sqrt{6}$ – точки минимума.

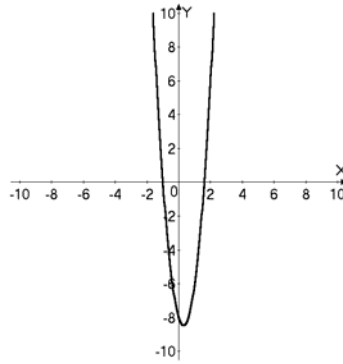
С-34

$$f(x) = -\frac{1}{(x-1)^2}; f'(x) = \frac{2}{(x-1)^3} > 0 \text{ при } x > 1; f'(x) < 0 \text{ при } x < 1,$$

значит, $f(x)$ возрастает при $x \in (1; \infty)$; убывает при $x \in (-\infty; 1)$;
экстремумов нет.

С-35

1. $f(x) = 5x^2 - 3x - 8$; $x_{\text{в}} = x_{\text{мин}} = 0,3$;
 $f_{\text{в}} = f_{\text{мин}} = 0,45 - 0,9 - 8 = -8,45$,
 $x \in R$, $f(x) \in [-8,45; \infty)$;
 $f(x)$ возрастает при $x \in (0,3; \infty)$; убывает при $x \in (-\infty; 0,3)$.
 $5x^2 - 3x - 8 = 0$;
нули: $x = -1$ и $x = 1,6$.



2. а) $2x^2 + 5x + 2 < 0$; $x \in (-2; -\frac{1}{2})$;
 б) $x^2 - 12x + 36 \leq 0$; $(x - 6)^2 \leq 0$, значит, $x = 6$.

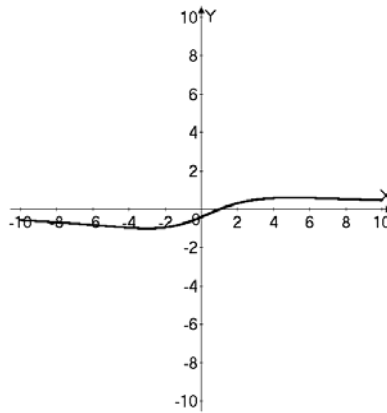
C-36

$$= \frac{6(x-1)}{x^2+15}; \quad f'(x) = \frac{6x^2+90-12x^2+12x}{(x^2+15)^2} = \frac{-6(x^2-2x-15)}{(x^2+15)^2};$$

$$f'(x) = 0 \text{ при } x_{\max} = 5 \text{ и } x_{\min} = -3; \quad f(5) = \frac{24}{40};$$

$$f(-3) = \frac{-24}{24} = -1; f'(x) \text{ возрастает при } x \in (-3; 5);$$

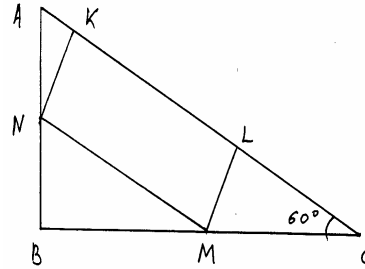
убывает при $x \in (-\infty; -3) \cup (5; \infty)$; нули: $x = 1$.



C-37

1. $f(x) = x^3 - 2x^2 + 8x - 2$; $x \in [-4; 2]$;
 $f'(x) = 3x^2 - 4x + 8$; $3x^2 - 4x + 8 = 0$; $D = 16 - 96 = -80 < 0$, значит,
 экстремумов нет;
 наибольшее значение $-f(2) = 14$; наименьшее значение $-f(-4) = -130$.

2.
 $BC = 8$ см;
 $AB = \sqrt{256 - 64} = 8\sqrt{3}$ см;
 Пусть $KL = x$, тогда $NM = x$
 $BM = \frac{x}{2}$; $CM = 8 - \frac{x}{2}$;



$$LC = 4 - \frac{x}{4};$$

$$\sin 60^\circ = \frac{LM}{MC};$$

$$LM = 4\sqrt{3} - \frac{x\sqrt{3}}{4}; S = 4\sqrt{3}x - \frac{\sqrt{3}}{4}x^2; S' = 4\sqrt{3} - \frac{\sqrt{3}}{2}x = 0; S' = 0 \text{ при}$$

$$x = 8, \quad LM = 4\sqrt{3} - 2\sqrt{3} = 2\sqrt{3} \text{ см}; \quad KL = 8 \text{ см}.$$

C-38

1. $\cos \alpha = \frac{5}{13}$; $\sin \beta = \frac{12}{13}$; $0 < \alpha < \frac{\pi}{2}$; $\frac{\pi}{2} < \beta < \pi$;
 $\sin \alpha = \frac{12}{13}$; $\cos \beta = -\frac{5}{13}$; $\cos(\alpha + \beta) = -\frac{25}{169} - \frac{144}{169} = -1$.

2.
 $8\sin^2(\pi - \alpha) \sin^2\left(\frac{3\pi}{2} + \alpha\right) - 1 = 8\sin^2\alpha \cos^2\alpha - 1 = 2\sin^2 2\alpha - 1 = -\cos 4\alpha$.

3. $\cos \alpha = -\frac{1}{3}$ $0 < \alpha < \pi \Rightarrow \alpha \in \text{II четверти}; \sin \alpha = \frac{\sqrt{8}}{3}$;

$$\sin \frac{\alpha}{2} = \sqrt{\frac{1 + 1/3}{2}} = \sqrt{\frac{2}{3}}; \quad \operatorname{tg} \frac{\alpha}{2} = \frac{\sin \frac{\alpha}{2}}{\sqrt{1 - \sin^2 \frac{\alpha}{2}}} = \frac{\sqrt{2}}{\sqrt{3}} \sqrt{3} = \sqrt{2}.$$

C-39

$$\text{а) } f(x) = \cos\left(2x - \frac{\pi}{3}\right);$$

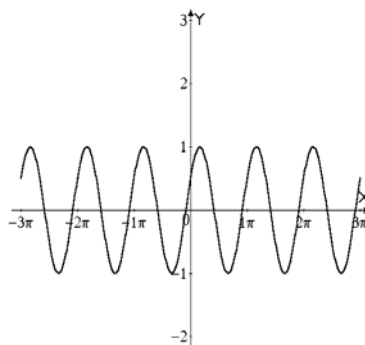
$$x \in R \quad y \in [-1; 1];$$

$$\cos\left(2x - \frac{\pi}{3}\right) = 0;$$

$$\text{нули: } x = \frac{5\pi}{12} + \frac{\pi n}{2};$$

$$x_{\max} \frac{\pi}{6} + \pi n; \quad x_{\min} \frac{2\pi}{3} + \pi n;$$

$$f\left(\frac{\pi}{6} + \pi n\right) = 1; \quad f\left(\frac{2\pi}{3} + \pi n\right) = -1;$$



$$f(x) \text{ возрастает при } x \in \left(-\frac{\pi}{3} + \pi n; \frac{\pi}{6} + \pi n\right);$$

$$\text{убывает при } x \in \left(\frac{\pi}{6} + \pi n; \frac{2\pi}{3} + \pi n\right).$$

б)

$$y = \frac{1}{2} + \sin \frac{x}{2};$$

$$x \in R; \quad y \in \left[-\frac{1}{2}; \frac{3}{2}\right];$$

$$\text{нули: } \sin \frac{x}{2} = -\frac{1}{2};$$

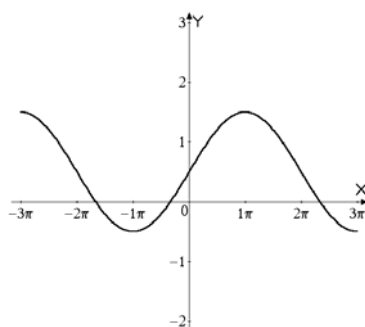
$$x = (-1)^{k+1} \frac{\pi}{3} + 2\pi k;$$

возрастает:

$$[-\pi + 4\pi n; \pi + 4\pi n];$$

$$\text{убывает: } [\pi + 4\pi n; 3\pi + 4\pi n]$$

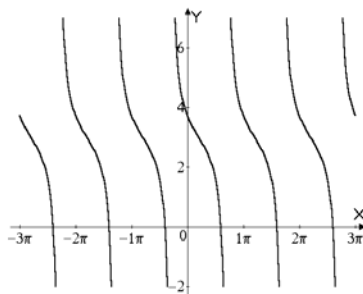
$$\max: \left(\pi + 4\pi n; \frac{3}{2}\right) \quad \min: \left(-\pi + 4\pi n; -\frac{1}{2}\right)$$



$$\text{в) } 3 - \operatorname{tg}\left(x - \frac{\pi}{5}\right) = f(x) \quad \text{ОДЗ: } \cos\left(x - \frac{\pi}{5}\right) \neq 0 \quad x \neq \frac{7\pi}{10} + \pi n$$

$$y \in R \quad x \neq \frac{7\pi}{10} + \pi n \quad \text{нули: } \operatorname{tg}\left(x - \frac{\pi}{5}\right) = 3$$

$x = \frac{\pi}{5} + \operatorname{arctg} 3 + \pi n$ возрастает на всей области определения.



C-40

1. **a)** $\arccos(-1) = \pi$; **б)** $\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$; **в)** $\operatorname{arctg} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$.
2. **a)** $\sin^2\left(\frac{x}{3} + \pi\right) = \frac{1}{2}$; $\sin \frac{x}{3} = \pm \frac{\sqrt{2}}{2}$; $x = \frac{3\pi}{4} + \frac{3\pi n}{2}$;
б) $8\cos^2 x - 2\sin x = 5$; $8\sin^2 x + 2\sin x - 3 = 0$;
 $\sin x = -\frac{3}{4}$; $x = (-1)^{k+1} \arcsin \frac{3}{4} + \pi k$;
 $\sin x = \frac{1}{2}$; $x = (-1)^n \frac{\pi}{6} + \pi n$.
3. **a)** $\operatorname{tg} 2x > -\frac{1}{\sqrt{3}}$; $x \in \left(-\frac{\pi}{12} + \frac{\pi n}{2}; \frac{\pi}{4} + \frac{\pi n}{2}\right)$;
б) $\cos\left(2x + \frac{\pi}{4}\right) < \frac{1}{2}$; $2x \in \left(-\frac{5\pi}{3} - \frac{\pi}{4} + 2\pi n; -\frac{\pi}{3} - \frac{\pi}{4} + 2\pi n\right)$;
 $x \in \left(-\frac{23\pi}{24} + \pi n; -\frac{7\pi}{24} + \pi n\right)$.

C-41

$$\begin{cases} \cos x \sin y = \frac{1}{2} \\ \sin 2x + \sin 2y = 0 \end{cases}; \begin{cases} \sin(x+y)\cos(x-y) = 0 \\ \cos x \sin y = \frac{1}{2} \end{cases}; \begin{cases} \sin(x+y)\cos(x-y) = 0 \\ \sin(x+y) - \sin(x-y) = 1 \end{cases};$$

1. $\begin{cases} \sin(x+y) = 0 \\ \sin(x-y) = -1 \end{cases}; \begin{cases} x+y = \pi n \\ x-y = -\frac{\pi}{2} + \pi k \end{cases}; \begin{cases} x = -\frac{\pi}{4} + \frac{\pi k}{2} + \frac{\pi n}{2} \\ y = \frac{\pi}{4} + \frac{\pi n}{2} - \frac{\pi k}{2} \end{cases};$
2. $\begin{cases} \cos(x-y) = 0 \\ \sin(x+y) = 0 \end{cases}$ то же самое.

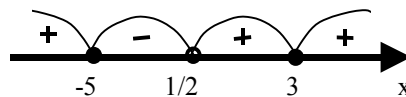
C-42

1. а) $x^2 - 3x - 11 > 0; \left(x - \frac{3 - \sqrt{53}}{2}\right)\left(x - \frac{3 + \sqrt{53}}{2}\right) > 0;$

$$x \in \left(-\infty; \frac{3 - \sqrt{53}}{2}\right) \cup \left(\frac{3 + \sqrt{53}}{2}; +\infty\right)$$

б) $x^2 + 7x + 12 \leq 0; x_1 = -4, x_2 = -3; (x+4)(x+3) \leq 0;$
 $x \in [-4; -3].$

2. а) $\frac{(x-3)^4(x+5)^5}{2x-1} \leq 0; x \in [-5; \frac{1}{2}) \cup \{3\}.$

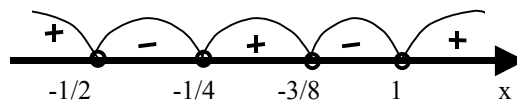


б) $\frac{3}{2x+1} + \frac{5}{4x+1} < 2 \quad \frac{12x+3+10x+5-16x^2-12x-2}{(2x+1)(4x+1)} < 0;$

$$\frac{-16x^2+10x+6}{(2x+1)(4x+1)} > 0 \quad \frac{8x^2-5x-3}{(2x+1)(4x+1)} > 0;$$

$$\frac{(x-1)\left(x+\frac{3}{8}\right)}{(2x+1)(4x+1)} > 0;$$

$$x \in \left(-\infty; -\frac{1}{2}\right) \cup \left(-\frac{1}{4}; -\frac{3}{8}\right) \cup (1; +\infty).$$



C-43

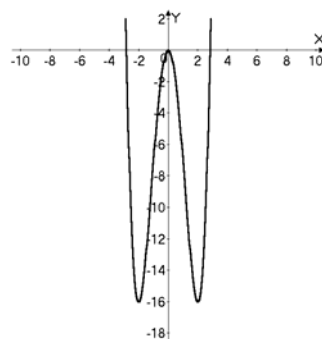
$$\begin{aligned} \text{a)} \quad y &= x^8 - 3x^6 + 2x^3 - 7; & y' &= 8x^7 - 18x^5 + 6x^2; \\ \text{б)} \quad y &= x\sqrt{3+x}; & y' &= \sqrt{3+x} + \frac{x}{2\sqrt{3+x}}; \\ \text{в)} \quad y &= \sin \frac{x}{5}; & y' &= \frac{1}{5} \cos \frac{x}{5}; \\ \text{г)} \quad y &= \operatorname{tg} \left(2x - \frac{\pi}{4} \right); & y' &= \frac{2}{\cos^2 \left(2x - \frac{\pi}{4} \right)}; \\ \text{д)} \quad y &= \left(\frac{1}{7} - 3x^2 \right)^{35}; & y' &= -210x \left(\frac{1}{7} - 3x^2 \right)^{34}. \end{aligned}$$

C-44

$$\begin{aligned} 1. \quad & f(x) = x^2 - 2x + 3; f(0) = 3; f'(x) = 2x - 2; f'(0) = -2; y = 3 - 2x \\ 2. \quad & \text{a)} \quad \sqrt{\sqrt{1,00004}} \approx (1 + 0,00002)^{1/2} \approx 1,00001; \\ & \text{б)} \quad 1,00003^{500} \approx 1 + 0,00003 \cdot 500 = 1,015. \\ 3. \quad & x(t) = \frac{1+t}{2+t} = 1 - \frac{1}{t+2}; \quad v(t) = \frac{1}{(t+2)^2}; \\ & a(t) = -\frac{2}{(t+2)^3}; \quad v(2) = \frac{1}{16}; \quad a(2) = -\frac{1}{32}. \end{aligned}$$

C-45

$$\begin{aligned} 1. \quad & f(x) = x^4 - 8x^2 \\ & f'(x) = 4x(x^2 - 4); \\ & \text{нули: } x = 0 \quad x = \pm\sqrt{8} \\ & f'(x) = 0 \quad x = 0 \quad x = \pm 2; \\ & \text{max: } (0; 0) \quad \text{min: } (\pm 2; -16) \\ & x \in R \quad y \geq -16 \\ & \text{убывает: } x \in (-\infty; -2] \cup [0; 2]; \\ & \text{возрастает: } x \in [-2; 0] \cup [2; +\infty] \end{aligned}$$



$$\begin{aligned}
 2. \quad & f(x) = \sin^2 x \cos x \quad x \in [0; \frac{\pi}{2}] \quad f'(x) = 2\sin x \cos^2 x - \sin^3 x = 0 \\
 & 2\sin x - 3\sin^3 x = 0 \quad \sin x = 0 \quad x = \pi n \\
 & \sin^2 x = \frac{2}{3}; \sin x = \sqrt{\frac{2}{3}}; \text{ (т.к. } x \in [0; \frac{\pi}{2}]); \cos x = \frac{1}{\sqrt{3}} \\
 & f(x_{\max}) = f\left(\arccos \frac{1}{\sqrt{3}}\right) = \frac{2}{3\sqrt{3}} \quad f(x_{\min}) = f(0) = f\left(\frac{\pi}{2}\right) = 0
 \end{aligned}$$

ВАРИАНТ 8

С-1

$$\begin{aligned}
 1. \quad & 48^\circ = \frac{\pi}{180} \cdot 48 = \frac{4\pi}{15}; \quad 188^\circ = \frac{\pi}{180} \cdot 188 = \frac{47\pi}{45}. \\
 2. \quad & \frac{3\pi}{16} = 33^\circ 45'; \quad \frac{22\pi}{9} = 440^\circ. \\
 3. \quad & \text{а) } 23^\circ 6' \approx 0,4119; \sin 23^\circ 6' \approx 0,4003; \cos 23^\circ 6' \approx 0,9164; \\
 & \text{б) } 83^\circ 53' \approx 1,4640; \sin 83^\circ 53' \approx 0,9943; \cos 83^\circ 53' \approx 0,1063. \\
 4. \quad & \text{а) } 0,0995 = 5^\circ 42'; \quad \text{б) } 3,1012 = 177^\circ 41'.
 \end{aligned}$$

С-2

$$\begin{aligned}
 1. \quad & \sin^2 \alpha (1 + \sin^{-1} \alpha + \operatorname{ctg} \alpha) (1 - \sin^{-1} \alpha + \operatorname{ctg} \alpha) = 2\sin \alpha \cos \alpha; \\
 & (\sin \alpha + 1 + \cos \alpha) \cdot (\sin \alpha - 1 + \cos \alpha) = (\sin \alpha + \cos \alpha)^2 - 1 = 2\sin \alpha \cos \alpha. \\
 2. \quad & \text{а) } \frac{\sin 200^\circ \cos 20^\circ}{\operatorname{tg} 300^\circ \operatorname{ctg} 100^\circ} < 0; \text{ б) } \cos 1 \sin 3 \operatorname{tg} 5 < 0. \\
 3. \quad & \operatorname{tg} \alpha = 3 \quad \alpha \in \text{III четверти} \quad \sin^2 \alpha = 9 - 9\sin^2 \alpha; \\
 & \sin \alpha = -\frac{3}{\sqrt{10}}; \quad \cos \alpha = -\frac{1}{\sqrt{10}}.
 \end{aligned}$$

С-3

$$\begin{aligned}
 1. \quad & \text{а) } \sin 1935^\circ = \sin 135^\circ = \frac{\sqrt{2}}{2}; \text{ б) } \operatorname{tg} 1395^\circ = \operatorname{tg} 45^\circ = 1; \\
 & \text{в) } \cos \frac{71\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}.
 \end{aligned}$$

2.
 $(\cos 70^\circ + \cos 50^\circ)(\cos 310^\circ + \cos 290^\circ) + (\cos 40^\circ + \cos 160^\circ) \cdot$
 $\cdot (\cos 320^\circ - \cos 380^\circ) = 1 + 2\cos 70^\circ \cos 50^\circ + 1 - 2\cos 40^\circ \cos 20^\circ =$
 $= 2 + 2(\sin 20^\circ \sin 40^\circ - \cos 40^\circ \cos 20^\circ) = 2 - 2\cos 60^\circ = 1.$
3.
 $\operatorname{tg}(\pi - \alpha) \left(1 + \operatorname{tg} \left(\frac{3\pi}{2} + \alpha \right) \operatorname{ctg} \left(\frac{\pi}{2} + 2\alpha \right) \right) = \operatorname{tg}(2\pi - \alpha) - \operatorname{ctg} \left(\frac{\pi}{2} - 2\alpha \right);$
 $-\operatorname{tg} \alpha (1 + \operatorname{ctg} \alpha \operatorname{tg} 2\alpha) = -\operatorname{tg} \alpha - \operatorname{tg} 2\alpha.$

C-4

1. $\frac{1 - 2\cos^2 \frac{5\pi}{8}}{\sin^2 75^\circ - 1} = \frac{-\cos \frac{5\pi}{4}}{-\cos^2 75^\circ} = -\frac{\sqrt{2}}{2} \cdot \frac{2}{1 + \cos 150^\circ} = -\frac{2\sqrt{2}}{2 - \sqrt{3}}.$
2. $\cos \alpha = \frac{1}{3}; \quad \sin \alpha < 0, \text{ значит, } \alpha \in \text{IV четверти};$
 $\sin \alpha = -\frac{\sqrt{8}}{3};$
 $\sin 2\alpha = -\frac{2\sqrt{8}}{9}; \quad \cos 2\alpha = -\frac{7}{9};$
 $\sin 4\alpha = \frac{56\sqrt{2}}{81};$
 $\operatorname{tg} 2\alpha = \frac{2\sqrt{8}}{7};$
 $\operatorname{ctg} 4\alpha = \frac{1 - \operatorname{tg}^2 2\alpha}{2\operatorname{tg} 2\alpha} = \frac{49 - 32}{49} \cdot \frac{7}{4\sqrt{8}} = \frac{17}{56\sqrt{2}}.$
3. $\frac{1 - \operatorname{ctg} 2\alpha \operatorname{tg} \alpha}{\operatorname{tg} \alpha + \operatorname{ctg} \alpha} = \frac{1 - \frac{(\cos^2 \alpha - \sin^2 \alpha) \cdot \sin \alpha}{2 \sin \alpha \cos \alpha} \cdot \frac{\sin \alpha}{\cos \alpha}}{\frac{\sin \alpha}{\cos \alpha} + \frac{\cos \alpha}{\sin \alpha}} =$
 $= \frac{\frac{2 \cos^2 \alpha - \cos^2 \alpha + \sin^2 \alpha}{2 \cos^2 \alpha}}{\frac{\sin^2 \alpha + \cos^2 \alpha}{\cos \alpha \cdot \sin \alpha}} = \frac{\cos \alpha \sin \alpha}{2 \cos^2 \alpha} = \frac{1}{2} \operatorname{tg} \alpha.$

C-5

1.

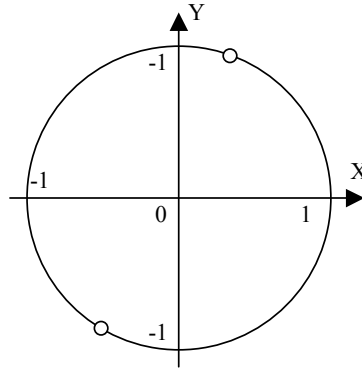
$$\operatorname{ctg} \alpha = \frac{1}{2};$$

$$\cos^2 \alpha = \frac{1}{4} - \frac{1}{4} \cos^2 \alpha;$$

$$\cos \alpha = \pm \frac{1}{\sqrt{5}};$$

$$\sin \alpha = \pm \frac{2}{\sqrt{5}};$$

$$\sin 2\alpha = \frac{4}{5}.$$



2.

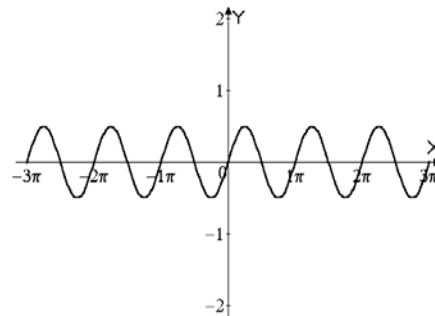
a) $\sin \alpha + \cos \alpha = -1,3;$

$$\sin\left(\alpha + \frac{\pi}{4}\right) = -\frac{13}{10\sqrt{2}}; \quad \text{IV четверть};$$

б) $\operatorname{ctg} \frac{\alpha}{2} = \frac{1}{2};$

$$\alpha = 2\operatorname{arctg} \frac{1}{2} + 2\pi n; \quad \text{I четверть}.$$

3.



$$\begin{aligned} y &= \sin^3 x \cos x + \sin x \cos^3 x = \sin x \cos x (\sin^2 x + \cos^2 x) = \\ &= \sin x \cos x = \frac{1}{2} \sin 2x. \end{aligned}$$

C-6

1.

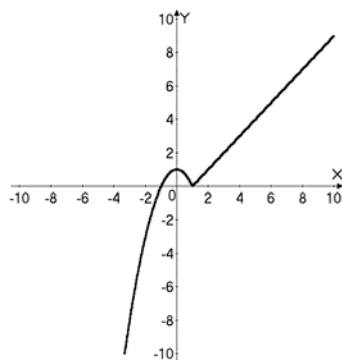
а) $f(x) = \frac{\sqrt{x+2}}{x^2+5x+4}$; ОДЗ: $x \geq -2$ $x \geq -2, x \neq -1$, значит,
 $x^2+5x+4 \neq 0$
 $x \in [-2; -1) \cup (-1; \infty)$;

б) $f(x) = \sqrt{\frac{3x-7}{x+2}}$; ОДЗ: $\frac{3x-7}{x+2} \geq 0$; $x \in (-\infty; -2) \cup [\frac{7}{3}; +\infty)$.

2. $f(x) = \begin{cases} 1-x^2 & x < 1; \\ x-1 & x \geq 1; \end{cases}$

а) $f(0) = 1$; $f(1) = 0$; $f(-1) = 0$; $f(2) = 1$;

б)



C-7

а) $y = 2\sin x \cos 3x \operatorname{tg} 5x$; $y(-x) = 2\sin(-x) \cos(-3x) \operatorname{tg}(-5x) =$
 $= 2\sin x \cos 3x \operatorname{tg} 5x = y(x) \Rightarrow$ четная;

б) $y = x^3 \sin(x + |x|)$; $y(-x) = (-x)^3 \sin(-x + |-x|) =$
 $= -x^3 \sin(|x| - x)$, значит, y ни четная, ни нечетная;

в) $y = \operatorname{tg}\left(x - \frac{\pi}{3}\right)$; $y(-x) = \operatorname{tg}\left(-x - \frac{\pi}{3}\right)$, значит, y ни четная,

ни нечетная;

г) $y = \operatorname{ctg} x + x \cos^2 x$ $y(-x) = \operatorname{ctg}(-x) + (-x) \cos^2(-x) =$
 $= -\operatorname{ctg} x - x \cos^2 x = -y(x)$, значит, y нечетная.

C-8

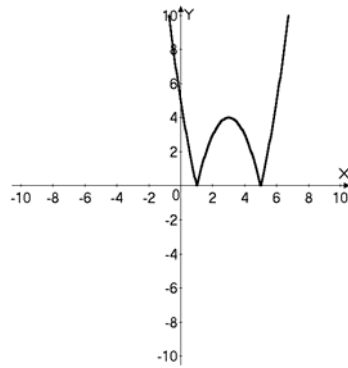
1. а) $\cos 393^\circ 17' = \cos 33^\circ 17'$; б) $\operatorname{tg} 4020^\circ = \operatorname{tg} 60^\circ = \sqrt{3}$;
в) $\cos \frac{63\pi}{11} = \cos \frac{3\pi}{11}$.
2. $\cos(-60^\circ) + \sin(690^\circ) + \operatorname{tg}(-600^\circ) = \frac{1}{2} - \frac{1}{2} - \sqrt{3} = -\sqrt{3}$.
3. а) $f(x) = \cos\left(\frac{x}{3} + \frac{\pi}{9}\right)$; $T = 6\pi$;
б) $f(x) = \cos^2 x - \operatorname{ctg} x$; $f_1(x) = \cos^2 x$ $T = \pi \Rightarrow T = \pi$.
 $f_2(x) = \operatorname{ctg} x$ $T = \pi$

C-9

1. а) $f(x) = \sqrt{x^2 - 1}$; возрастает: $x \geq 1$ убывает: $x \leq -1$;
б) $f(x) = \left|1 + \frac{1}{x-1}\right|$ убывает: $(-\infty; 0] \cup (1; +\infty)$;
возрастает: $[0; 1)$.
2. $f(x) = 3 - 3x - 2x^3$ $f'(x) = -3 - 6x^2 < 0$ всегда.
3. $\sin \frac{1}{2}$, $\sin \frac{3}{2}$, $\sin 3$, $\sin 4,5$;
Ответ: $\sin \frac{3}{2}$, $\sin \frac{1}{2}$, $\sin 3$, $\sin 4,5$.

C-10

1. $y = |x^2 - 6x + 5| = 0$; $y = 0$ при $x = 5$ и $x = 1$; $x_{\text{в}} = 3$; значит, $x_{\text{max}} = 3$;
 $x_{\text{min}} = 5$; $x_{\text{min}} = 1$.
 $\begin{cases} x^2 - 6x + 5 \leq 3 \\ x^2 - 6x + 5 \geq -3 \end{cases}$; $\begin{cases} x^2 - 6x + 2 \leq 0 \\ x^2 - 6x + 8 \geq 0 \end{cases}$; $\begin{cases} x \in (3 - \sqrt{7}; 3 + \sqrt{7}) \\ x \leq 2, x \geq 4 \end{cases}$.



Итого: $x \in (3 - \sqrt{7}; 2] \cup [4; 3 + \sqrt{7})$.

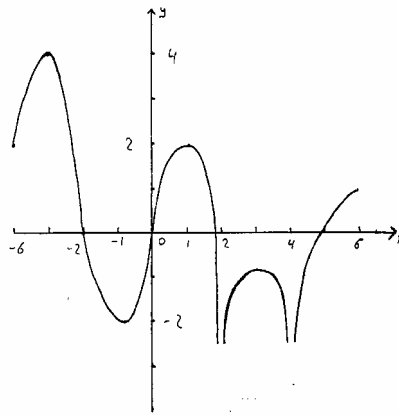
2.

$$f(x) = \sqrt{3} \sin 3x + \cos 3x + 5 = 2 \sin \left(3x + \frac{\pi}{6} \right) + 5;$$

$$y \left(\frac{\pi}{9} + \frac{2\pi n}{3} \right) = 7; \quad y \left(-\frac{2\pi}{9} + \frac{2\pi n}{3} \right) = 3; \quad y \in [3; 7]$$

$$x_{\max} = \frac{\pi}{9} + \frac{2\pi n}{3} \quad x_{\min} = -\frac{2\pi}{9} + \frac{2\pi n}{3}$$

C-11



C-12

1. $f(x) = \operatorname{ctg} 2x + \frac{1}{\operatorname{ctg}\left(\frac{x}{2} - \frac{\pi}{3}\right)};$

ОДЗ:
$$\begin{cases} \sin 2x \neq 0 \\ \cos\left(\frac{x}{2} - \frac{\pi}{3}\right) \neq 0; \\ \sin\left(\frac{x}{2} - \frac{\pi}{3}\right) \neq 0 \end{cases} \quad \begin{cases} x \neq \frac{\pi n}{2} \\ x \neq \frac{5\pi}{3} + 2\pi n. \\ x \neq \frac{2\pi}{3} + 2\pi n \end{cases}$$

2. $y = \cos\left(\frac{x}{4} + \frac{\pi}{5}\right);$

y возрастает при

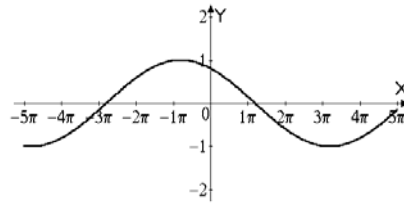
$$x \in \left[-\frac{24\pi}{5} + 8\pi n; -\frac{4\pi}{5} + 8\pi n\right];$$

y убывает при

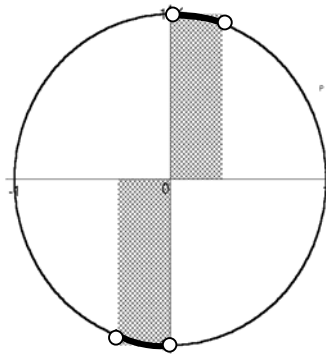
$$x \in \left[-\frac{4\pi}{5} + 8\pi n; \frac{16\pi}{5} + 8\pi n\right];$$

$$x_{\max} = -\frac{4\pi}{5} + 8\pi n \quad x_{\min} = -\frac{24\pi}{5} + 8\pi n \quad y\left(-\frac{4\pi}{5} + 8\pi n\right) = 1$$

$$y\left(-\frac{24\pi}{5} + 8\pi n; -1\right) = -1$$



3.



C-13

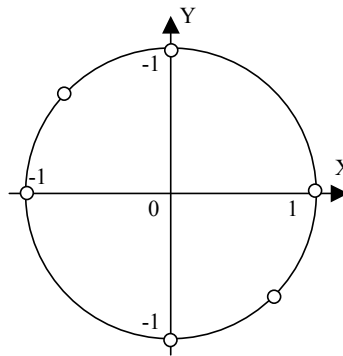
1. **a)** $\operatorname{arctg} 1 - \arccos \frac{1}{\sqrt{2}} = \frac{\pi}{4} - \frac{\pi}{4} = 0$.
 б) $\arccos (\cos (-12^\circ)) = \arccos (\cos (12^\circ)) = 12^\circ$;
 в) $\cos (2\arcsin \frac{1}{4}) = 1 - 2 \cdot \frac{1}{16} = \frac{7}{8}$.
2. $\arccos 1 = 0 < \frac{\pi}{4} = \operatorname{arctg} 1$.
3. **a)** $\arcsin (0,9898) \approx 1,4279$; **б)** $\arccos (-0,3737) \approx 1,9538$;
 в) $\operatorname{arctg} (-5,72) \approx -1,3977$.

C-14

- a)** $\operatorname{tg} x = -\sqrt{3}$; $x = -\frac{\pi}{3} + \pi n$;
- б)** $\cos \left(\frac{\pi}{3} - x \right) = -1$; $x = -\frac{2\pi}{3} + 2\pi n$;
- в)** $\sin \left(\frac{x}{2} + \frac{\pi}{5} \right) = \frac{\sqrt{3}}{2}$; $\frac{x}{2} = -\frac{\pi}{5} + (-1)^k \frac{\pi}{3} + \pi k$; $x = -\frac{2\pi}{5} + (-1)^k \frac{2\pi}{3} + 2\pi k$.

C-15

$$\begin{aligned} \sin 2t(\sqrt{3} \sin t + \cos t) &= 0; \\ \sin 2t &= 0; \quad t = \frac{\pi n}{2}; \\ \sqrt{3} \sin t + \cos t &= 0; \\ \operatorname{ctg} t &= -\sqrt{3}; \quad t = \frac{5\pi}{6} + \pi n; \\ \sin 2t(\sqrt{3} \sin t + \cos t) &\leq 0; \end{aligned}$$



$$x \in \left[-\frac{\pi}{6} + 2\pi n; 2\pi n \right] \cup \left[\frac{\pi}{2} + 2\pi n; \frac{5\pi}{6} + 2\pi n \right] \cup \left[\pi + 2\pi n; \frac{3\pi}{2} + 2\pi n \right].$$

C-16

$$\begin{aligned} \text{а) } \operatorname{tg} 3x < 1; & \quad x \in \left(-\frac{\pi}{6} + \frac{\pi n}{3}; \frac{\pi}{12} + \frac{\pi n}{3} \right); \\ \text{б) } \cos \left(2x - \frac{\pi}{6} \right) \geq -\frac{\sqrt{2}}{2}; & \quad 2x \in \left[-\frac{7\pi}{12} + 2\pi n; \frac{11\pi}{12} + 2\pi n \right]; \\ & \quad x \in \left[-\frac{7\pi}{24} + \pi n; \frac{11\pi}{24} + \pi n \right]. \end{aligned}$$

C-17

$$\begin{aligned} \text{а) } \sin^2 x - 3\cos x - 3 &= 0; \quad \cos^2 x + 3\cos x + 2 = 0; \\ \cos x &= -2; \quad \text{решений нет}; \quad \cos x = -1 \quad x = \pi + 2\pi n; \\ \text{б) } 2\sin^2 x - \sqrt{3} \sin 2x &= 0; \quad \sqrt{3} \sin 2x + \cos 2x = 1; \\ \sin \left(2x + \frac{\pi}{6} \right) &= \frac{1}{2}; \quad x = -\frac{\pi}{12} + (-1)^k \frac{\pi}{12} + \frac{\pi n}{2}. \end{aligned}$$

C-18

$$\begin{aligned} \text{а) } \frac{\cos 2\alpha}{1 - \sin 2\alpha} &= \frac{1 + \operatorname{tg} \alpha}{1 - \operatorname{tg} \alpha}; \\ \frac{(\cos \alpha + \sin \alpha)(\cos \alpha - \sin \alpha)}{(\cos \alpha - \sin \alpha)^2} &= \frac{(\cos \alpha + \sin \alpha) \cos \alpha}{(\cos \alpha - \sin \alpha) \cos \alpha} = \frac{1 + \operatorname{tg} \alpha}{1 - \operatorname{tg} \alpha}; \\ \text{б) } \frac{\sin 2\alpha - 2 \sin \alpha}{\sin 2\alpha + 2 \sin \alpha} &= -\operatorname{tg}^2 \frac{\alpha}{2}; \quad \frac{\cos \alpha - 1}{\cos \alpha + 1} = \frac{-\sin^2 \frac{\alpha}{2}}{\cos^2 \frac{\alpha}{2}} = -\operatorname{tg}^2 \frac{\alpha}{2}. \end{aligned}$$

C-19

$$\left\{ \begin{array}{l} \sin(x+y) = 1 \\ \sin^2 x + \cos^2 y = 1 \end{array} \right\}; \quad \left\{ \begin{array}{l} x = \frac{\pi}{2} + \pi n - y \\ \cos y = \pm \frac{1}{\sqrt{2}} \end{array} \right\}; \quad \left\{ \begin{array}{l} y = \frac{\pi}{4} + \frac{\pi k}{2} \\ x = \frac{\pi}{4} + \pi n - \frac{\pi k}{2} \end{array} \right\}.$$

C-20

$$\text{а) } \operatorname{tg} 3x = \operatorname{tg} 5x; \quad \frac{\sin 3x \cos 5x - \cos 5x \sin 3x}{\cos 3x \cos 5x} = 0; \quad \sin 2x = 0; \quad x = \frac{\pi n}{2},$$

но $\frac{\pi n}{2} + \pi n$ не проходит через ОДЗ, значит, $x = \pi n$;

$$\text{б) } \sin^4 x + \cos^4 x = \sin 2x; \quad 1 - \frac{1}{2} \sin^2 2x - \sin 2x = 0;$$

$$\sin^2 2x - 2\sin 2x - 2 = 0; \quad \sin 2x = \frac{-2 \pm 2\sqrt{3}}{2}; \quad \sin 2x = -1 - \sqrt{3} -$$

посторонний корень; $\sin 2x = -1 + \sqrt{3}$

$$x = (-1)^k \frac{\arcsin(\sqrt{3}-1)}{2} + \frac{\pi k}{2}.$$

$$\text{в) } \cos 3x = \sin x \quad \sin x - \sin\left(\frac{\pi}{2} - 3x\right) = 0;$$

$$\sin\left(2x - \frac{\pi}{4}\right) \cos\left(x - \frac{\pi}{4}\right) = 0; \quad x = \frac{\pi}{8} + \frac{\pi n}{2} \text{ и } x = \frac{3\pi}{4} + \pi n.$$

C-21

$$\begin{aligned} 1. f(x) &= x^2 + 2x; \quad \Delta f = x_0^2 + 2x_0\Delta x + (\Delta x)^2 + 2x_0 + 2\Delta x - x_0^2 - 2x_0 = \\ &= 2x_0\Delta x + (\Delta x)^2 + 2\Delta x; \end{aligned}$$

$$\text{а) } x_0 = 2; \quad \Delta x = -1; \quad \Delta f = -5; \quad \text{б) } x_0 = -3; \quad \Delta x = \frac{1}{2}; \quad \Delta f = -1 \frac{3}{4}$$

$$\begin{aligned} 2. \quad f(x) &= x^3 + 4x; \\ \Delta f &= (\Delta x + x_0)((\Delta x)^2 + x_0^2 + 2x_0\Delta x + 4) - x_0^3 - 4x_0 = \\ &= (\Delta x)^3 + 2x_0(\Delta x)^2 + \Delta x x_0^2 + 4\Delta x + x_0(\Delta x)^2 + 2\Delta x x_0^2 = \\ &= (\Delta x)^3 + 3x_0(\Delta x)^2 + 3\Delta x(x_0)^2 + 4\Delta x; \quad \frac{\Delta f}{\Delta x} = (\Delta x)^2 + 3x_0\Delta x + 3x_0^2 + 4. \end{aligned}$$

C-22

$$1. \quad x(t) = 2 - 4t + 3t^2; \quad v(t) = -4 + 6t; \quad v(1) = 2; \quad E = \frac{2 \cdot 2^2}{2} = 4 \text{ Дж.}$$

$$2. \quad \text{а) } f(x) = 2 - 7x; \quad f'(x) = -7; \quad \text{б) } f(x) = x^2 + 3x - 2; \quad f'(x) = 2x + 3.$$

C-23

$$1. \quad \text{а) } f(-3) = 0; \quad f(0) - \text{не определено};$$

$$\text{б) } \lim_{x \rightarrow -3} f(x) = 1; \quad \lim_{x \rightarrow 0} f(x) = 1.$$

$$2. \quad f(x) = \frac{x^2 + 8x + 7}{3(x+1)} = (x+7) \frac{1}{3}, \quad x \neq -1; \quad \left| \frac{x+7}{3} - 2 \right| < 0,002;$$

$$|x+1| < 0,006; \quad \delta = 0,006.$$

C-24

$$1. \quad \text{a) } y = \frac{f(x)}{g(x)} + 2f(x)g(x) = -\frac{1}{6} - 3 = -3\frac{1}{6};$$

$$\lim_{x \rightarrow 2} y = \frac{\lim_{x \rightarrow 2} f(x)}{\lim_{x \rightarrow 2} g(x)} + 2 \lim_{x \rightarrow 2} f(x) \cdot \lim_{x \rightarrow 2} g(x) = -\frac{1}{6} - 3 = -3\frac{1}{6};$$

$$\text{б) } \lim_{x \rightarrow 2} y = \frac{2 \lim_{x \rightarrow 2} f(x) - \lim_{x \rightarrow 2} g(x)}{6 \lim_{x \rightarrow 2} f(x) + \lim_{x \rightarrow 2} g(x)} \text{ не существует.}$$

$$2. \quad \text{a) } \lim_{x \rightarrow 4} (1 - x + 2x^2 - 3x^3) = 1 - 4 + 32 - 192 = -163;$$

$$\text{б) } \lim_{x \rightarrow -3} \frac{3x-9}{2x^2-x+1} = \frac{-18}{18+3+1} = -\frac{9}{11}.$$

C-25

$$1. \quad \text{a) } f(x) = x^8 - 2x^6 - \sqrt{x^5} + 9; \quad f'(x) = 8x^7 - 12x^5 - \frac{5}{2x^{7/2}};$$

$$\text{б) } g(x) = x\sqrt{x+1}; \quad g'(x) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}.$$

$$2. \quad f(x) = \frac{2x+3}{3x+5}; \quad f'(x) = \frac{10-9}{(3x+5)^2} = \frac{1}{(3x+5)^2};$$

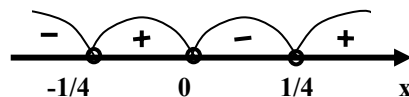
$$f'(-3) = \frac{1}{16}; \quad f'(6) = \frac{1}{529}; \quad f'(x^2-1) = \frac{1}{(3x^2+2)^2}.$$

$$3. \quad f(x) = 2x + \frac{4}{x}; \quad f'(x) = 2 - \frac{4}{x^2} < 0 \text{ при}$$

$$x^2 < 2; \quad x \in (-\sqrt{2}; 0) \cup (0; \sqrt{2}).$$

C-26

1. $f(x) = 40\left(\sqrt[4]{x}\right)^8 - 8\left(\sqrt[4]{x}\right)^{40}; f'(x) = 80x - 80x^9; f'(1) = 0;$
 $f'(\sqrt{x}) = 80\sqrt{x} - 80(\sqrt{x})^9.$
2. а) $f(x) = 8x^4 - x^2; f'(x) = 2x(16x^2 - 1); f'(x) = 0$ при $x = 0$ и $x = \pm \frac{1}{4};$



$$f'(x) > 0 \text{ при } x \in \left(-\frac{1}{4}; 0\right) \cup \left(\frac{1}{4}; +\infty\right);$$

$$f'(x) < 0 \text{ при } x \in \left(-\infty; -\frac{1}{4}\right) \cup \left(0; \frac{1}{4}\right);$$

$$\text{б) } f(x) = \frac{x^2 + 21}{x - 2} \quad f'(x) = \frac{2x^2 - 4x - x^2 - 21}{(x - 2)^2} = \frac{x^2 - 4x - 21}{(x - 2)^2};$$

$$f'(x) = 0 \text{ при } x = 7 \text{ и } x = -3$$

$$f'(x) > 0, x \in (-\infty; -3) \cup (7; +\infty); \quad f'(x) < 0, x \in (-3; 2) \cup (2; 7).$$

C-27

1. а) $f(x) = \frac{1}{\sqrt{x+2}-4};$ ОДЗ: $\begin{cases} x \geq -2 & x \geq -2, \quad x \neq 14 \\ x+2 \neq 16 \end{cases}$
 б) $f(x) = \frac{1}{\sqrt{5-\sqrt{x}}};$ ОДЗ: $\begin{cases} x \geq 0 \\ 5-\sqrt{x} > 0 \end{cases}; \quad x \in [0; 25).$
2. $f(x) = x^4 - 2x; \quad g(x) = \cos x + 1;$
 $f(g(x)) = (\cos x + 1)^4 - 2\cos x - 2; \quad g(f(x)) = \cos(x^4 - 2x) + 1.$
3. а) $f(x) = (7x^3 - 3x^7)^{173}; \quad f'(x) = 173(21x^2 - 21x^6)(7x^3 - 3x^7)^{172};$
 б) $g(x) = \sqrt{x^3 - 3x}; \quad g'(x) = \frac{3x^2 - 3}{2\sqrt{x^3 - 3x}}.$

C-28

$$\text{а) } f(x) = \sin\left(\frac{3x}{7} + 1\right); \quad f'(x) = \frac{3}{7} \cos\left(\frac{3x}{7} + 1\right);$$

$$6) f(x) = \cos x \cos 3x + \sin x \sin 3x = \cos 2x; \quad f'(x) = -2\sin 2x;$$

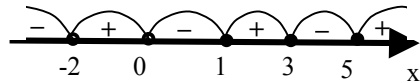
$$в) f(x) = \operatorname{ctg} \left(\frac{\pi}{2} - x \right) + \sin x \sin 2x = \operatorname{tg} x + \sin x \sin 2x;$$

$$f'(x) = \frac{1}{\cos^2 x} + \cos x \sin 2x + 2\cos 2x \sin x.$$

C-29

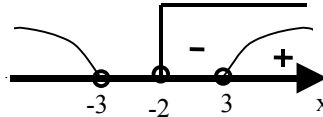
$$1. f(x) = \frac{2x-3}{x^3-5x^2+6x}; \quad \text{ОДЗ: } \begin{matrix} x(x^2-5x+6) \neq 0 \\ x \neq 0 \quad x \neq 2 \quad x \neq 3 \end{matrix}, \text{ значит, } f(x) \text{ непрерывна} \\ \text{при } x \in (-\infty; 0) \cup (0; 2] \cup (2; 3) \cup (3; \infty).$$

$$2. а) \frac{(x-1)(x-3)(x-5)}{x^2+2x} \leq 0;$$



$$x \in (-\infty; -2) \cup (0; 1] \cup [3; 5];$$

$$6) (x^2-9)\sqrt{x+2} < 0; \quad (x-3)(x+3)\sqrt{x+2} < 0; \\ x \in (-2; 3).$$



C-30

$$1. \quad f(x) = \cos \frac{x}{2}; \quad f\left(\frac{\pi}{2}\right) = \frac{\sqrt{2}}{2}; \quad f'(x) = -\frac{1}{2} \sin \frac{x}{2}; \quad f'\left(\frac{\pi}{2}\right) = -\frac{\sqrt{2}}{4};$$

$$y_{\text{кас}} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{4} \left(x - \frac{\pi}{2} \right) = -\frac{\sqrt{2}}{4} \left(x - \frac{\pi}{2} - 2 \right) - \text{уравнение касательной.}$$

$$2. \quad y = 0,5x^2 - 2x + 2; \quad x_0 = 0; \quad y(0) = 2; \quad y' = x - 2; \quad y'(0) = -2; \\ y_{\text{кас}} = 2 - 2x - \text{уравнение касательной}$$

C-31

- $\sqrt{81,12} = 9\sqrt{1+\frac{12}{8100}} = 9\left(1+\frac{12}{8100 \cdot 2}\right) = 9\frac{1}{150}.$
- $1,000007^{100} - 0,999999^{700} \approx 1 + 0,000007 \cdot 100 + 1 - 0,000001 \cdot 700 = 1,0007 + 0,9993 = 2.$

C-32

- $s(t) = 3t - \frac{1}{t+2};$
 $s'(t) = v(t) = 3 + \frac{1}{(t+2)^2};$
 $a(t) = -\frac{2}{(t+2)^3}; F = ma,$
 $F(1) = \frac{-2 \cdot 4}{(1+2)^3} = -\frac{8}{27} \text{ (H)}.$
- a)** $\varphi = 2t - 0,04t^2; \quad \omega = 2 - 0,08t; \quad \omega(2) = 1,04;$
б) $2 - 0,08t = 0; \quad t = 25 \text{ (с)}.$

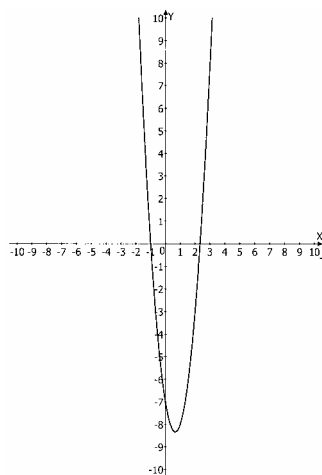
C-33

- $f(x) = x^3 + 3x - 8; f'(x) = 3x^2 + 3 > 0$ всегда, значит, $f'(x)$ возникает на R .
- $f(x) = \frac{x^2}{4} + \frac{9}{x^2}; f'(x) = \frac{x}{2} - \frac{18}{x^3};$
 $f'(x) = 0$ при
 $x^4 = 36; \quad x = \pm\sqrt{6}; x_{\min} = \pm\sqrt{6} - \text{точки минимума}.$

C-34

$f(x) = \frac{1}{(x-3)^2}; f'(x) = -\frac{2}{(x-3)^3} > 0$ при $x \in (-\infty; 3)$, значит, $f(x)$ возрастает при $x \in (-\infty; 3)$;
убывает при $x \in (3; \infty)$ экстремумов нет.

C-35



1. $f(x) = 3x^2 - 4x - 7;$

$$x_{\text{в}} = x_{\text{min}} = \frac{2}{3};$$

$$f(x)_{\text{в}} = 3 \cdot \frac{4}{9} - 4 \cdot \frac{2}{3} - 7 = -8\frac{1}{3};$$

$$x \in R, f(x) \in \left[-8\frac{1}{3}; \infty\right);$$

$$f(x) \text{ возрастает при } x \in \left(\frac{2}{3}; \infty\right); \text{ убывает при } x \in \left(-\infty; \frac{2}{3}\right);$$

$$3x^2 - 4x - 7 = 0; \quad \left. \begin{array}{l} x_1 = -1 \\ x_2 = \frac{7}{3} \end{array} \right\} - \text{нули функции.}$$

2. а) $x^2 - 9x - 22 \leq 0 \quad D = 81 + 88 = 169 \quad x \in [-2; 11]$
 б) $x^2 + 8x + 16 > 0; \quad D = 64 - 64 = 0; x = -4; x \in (-\infty; -4) \cup (4; +\infty).$

C-36

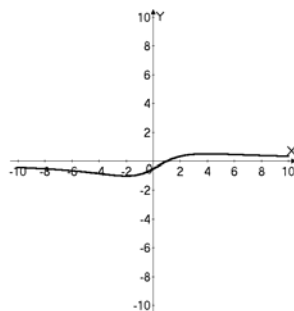
$$f(x) = \frac{4(x-1)}{x^2+8}; f'(x) = \frac{4x^2+32-8x^2+8x}{(x^2+8)^2} = \frac{-4(x^2-2x-8)}{(x^2+8)^2} = 0;$$

$$f'(x) = 0 \text{ при } x_{\text{max}} = 4; x_{\text{min}} = -2$$

$$f(4) = \frac{12}{24} = \frac{1}{2}; \quad f(-2) = -\frac{12}{12} = -1;$$

$f(x)$ возрастает при $x \in (-2; 4)$; убывает при $x \in (-\infty; -2) \cup (4; \infty)$;

$x = 1$ – нуль функции.



C-37

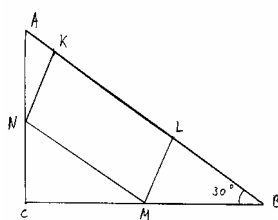
1.

$$f(x) = x^3 - 2x^2 + 8x - 2; \quad x \in [1; 4];$$

$$f'(x) = 3x^2 - 4x + 8; \quad D = 16 - 96 < 0 \Rightarrow \text{экстремумов нет};$$

$f(4) = 62$ – наибольшее значение функции; $f(1) = -5$ – наименьшее.

2.



$$AB = 24; \quad CB = 12\sqrt{3};$$

$$\text{Пусть } KL = x, \text{ значит, } NM = x; \quad CN = \frac{x}{2};$$

$$AN = 12 - \frac{x}{2}; \quad \cos 30^\circ = \frac{KN}{AN};$$

$$KN = \sqrt{36} - \frac{x}{4} \sqrt{3};$$

$$S = 6\sqrt{3}x - \frac{x^2\sqrt{3}}{4}; \quad x_B = \frac{6\sqrt{3} - 2}{\sqrt{3}} = 12; \quad KN = 3\sqrt{3} \text{ см}; \quad KL = 12 \text{ см}.$$

C-38

$$1. \quad \sin \alpha = \frac{1}{3}; \quad \sin \beta = \frac{2}{3};$$

$$0 < \alpha < \frac{\pi}{2}; \quad \frac{3\pi}{2} < \beta < 2\pi;$$

$$\cos \alpha = \frac{2\sqrt{2}}{3}; \quad \sin \beta = -\frac{\sqrt{5}}{3};$$

$$\sin(\alpha - \beta) = \frac{1}{3} \cdot \frac{2}{3} + \frac{\sqrt{5}}{3} \cdot \frac{\sqrt{8}}{3} = \frac{2}{9} + \frac{2\sqrt{10}}{9} = \frac{2+2\sqrt{10}}{9}.$$

2.

$$\sin^2(\pi - \alpha) \cos^2(\pi + \alpha) - \frac{1}{4} \sin^2\left(2\alpha + \frac{3\pi}{2}\right) = \sin^2 \alpha \cos^2 \alpha - \frac{1}{4} \cos^2 2\alpha =$$

$$= \frac{1}{4} \sin^2 2\alpha - \frac{1}{4} \cos^2 2\alpha = -\frac{\cos 4\alpha}{4}.$$

3.

$$\cos \alpha = -\frac{2}{5}; \quad \pi < \alpha < 2\pi;$$

$$\cos \frac{\alpha}{2} = \sqrt{\frac{1 + \cos \alpha}{2}} = \sqrt{\frac{1 - \frac{2}{5}}{2}} = \sqrt{\frac{3}{10}};$$

$$\operatorname{tg} \frac{\alpha}{2} = \sqrt{\frac{1}{\cos^2 \frac{\alpha}{2}} - 1} = \sqrt{\frac{10}{3} - 1} = \sqrt{\frac{7}{3}}.$$

C-39

a) $f(x) = \sin\left(\frac{x}{2} + \frac{\pi}{3}\right);$

$x \in \mathbb{R}; f(x) \in [-1; 1];$

$\sin\left(\frac{x}{2} + \frac{\pi}{3}\right) = 0$ при

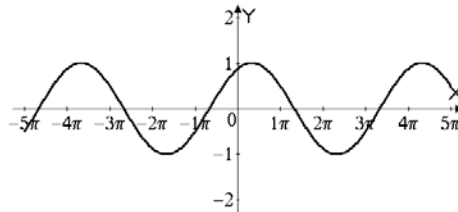
$x = -\frac{2\pi}{3} + 2\pi n$ — нули

функции;

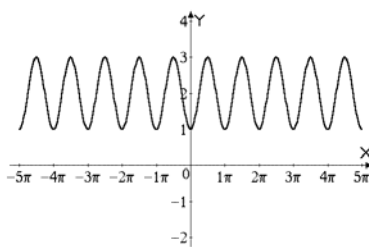
$x_{\max} = \frac{\pi}{3} + 4\pi n; x_{\min} = -\frac{5\pi}{3} + 4\pi n; f\left(\frac{\pi}{3} + 4\pi n\right) = 1; f\left(-\frac{5\pi}{3} + 4\pi n\right) = -1;$

$f(x)$ возрастает при $x \in \left(-\frac{\pi}{4} + 4\pi n; \frac{\pi}{3} + 4\pi n\right);$

убывает при $x \in \left(\frac{\pi}{3} + 4\pi n; \frac{7\pi}{3} + 4\pi n\right).$



б)



$$f(x) = 2 - \cos 2x; \quad x \in R; \quad y \in [1; 3]; \text{ нулей нет;}$$

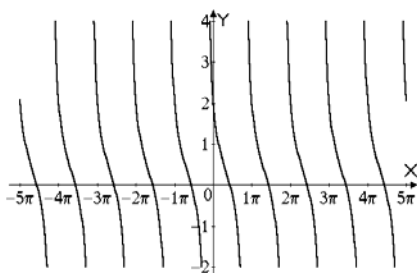
$$f(x) \text{ возрастает при } (\pi n; \frac{\pi}{2} + \pi n);$$

$$\text{убывает при } (\frac{\pi}{2} + \pi n; \pi + \pi n);$$

$$x_{\max} = \frac{\pi}{2} \pi + \pi n; f(\frac{\pi}{2} \pi + \pi n) = 3;$$

$$x_{\min} = \pi n; f(\pi n) = 1.$$

$$\text{в) } f(x) = \frac{1}{3} + \operatorname{tg}\left(\frac{\pi}{3} - x\right)$$



$$y \in R; \quad x \neq \frac{5\pi}{6} + \pi n; \text{ убывает на всей области определения;}$$

экстремумов нет;

$$\text{нули: } \operatorname{tg}\left(\frac{\pi}{3} - x\right) = -\frac{1}{3}; \quad x = \frac{\pi}{3} + \operatorname{arctg} \frac{1}{3} + \pi n.$$

C-40

$$1. \quad \text{а) } \arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}; \quad \text{б) } \arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}; \quad \text{в) } \operatorname{arctg} \sqrt{3} = \frac{\pi}{3}.$$

$$2. \text{ а) } \cos^2\left(3x - \frac{\pi}{3}\right) = \frac{3}{4}; \quad \cos\left(3x - \frac{\pi}{3}\right) = \pm \frac{\sqrt{3}}{2};$$

$$x = \frac{\pi}{9} \pm \frac{\pi}{18} + \pi n \text{ и } x = \frac{\pi}{9} \pm \frac{5\pi}{18} + \pi n;$$

$$6) 4\sin^2 x + 4\cos x = 5; 4\cos^2 x - 4\cos x + 1 = 0; \cos x = \frac{1}{2}; \quad x = \pm \frac{\pi}{3} + 2\pi n.$$

$$3. \quad \text{а) } \operatorname{tg} \frac{x}{2} \leq -\sqrt{3}; \quad x \in \left(-\pi + 2\pi n; -\frac{2\pi}{3} + 2\pi n\right];$$

$$6) \sin\left(2x - \frac{\pi}{6}\right) > \frac{\sqrt{2}}{2}; \quad 2x \in \left(\frac{5\pi}{12} + 2\pi n; \frac{11\pi}{12} + 2\pi n\right);$$

$$x \in \left(\frac{5\pi}{24} + \pi n; \frac{11\pi}{24} + \pi n\right).$$

C-41

$$\begin{cases} \operatorname{tg} \operatorname{tg} 2y = 1 \\ \sqrt{3} \sin 2x - 3 \cos 2y = 0 \end{cases}; \begin{cases} \cos(x+2y) - \cos(x-2y) = \cos(x+y) + \cos(x-2y) \\ \sqrt{3} \sin^2 2x - 3 \cos 2y = 0 \end{cases}$$

$$\begin{cases} x - 2y = \frac{\pi}{2} + \pi n & \sqrt{3} \sin 4y + 3 \cos 2y = 0 \\ \sqrt{3} \sin(4y + \pi + 2\pi n) - 3 \cos 2y = 0 & \cos 2y(2\sqrt{3} \sin 2y + 3) = 0 \end{cases};$$

$$\begin{cases} \cos 2y = 0 \\ x = \frac{\pi}{2} + \pi n + 2y \end{cases}; \quad \begin{cases} y = \frac{\pi}{4} + \frac{\pi k}{2}; \\ x = \pi n + \pi k \end{cases};$$

$$\begin{cases} \sin 2y = -\frac{\sqrt{3}}{2} \\ x = 2y + \frac{\pi}{2} + \pi n \end{cases}; \quad \begin{cases} y = (-1)^{k+1} \frac{\pi}{6} + \frac{\pi k}{2} \\ x = (-1)^{k+1} \frac{\pi}{3} + \pi k + \frac{\pi}{2} + \pi n \end{cases}.$$

C-42

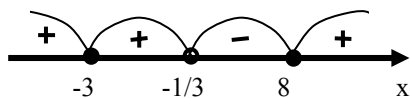
$$1. \text{ а) } x^2 - 5x - 7 < 0; \left(x - \frac{5 - \sqrt{53}}{2}\right) \left(x - \frac{5 + \sqrt{53}}{2}\right) < 0;$$

$$x \in \left(\frac{5 - \sqrt{53}}{2}; \frac{5 + \sqrt{53}}{2}\right); \quad 6) x^2 + 6x + 9 \geq 0 \quad (x+3)^2 \geq 0; \quad x \in R.$$

2.

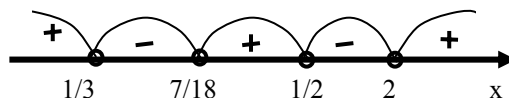
а) $\frac{(x-8)^3(x+3)^8}{3x+1} \geq 0;$

$x \in (-\infty; -\frac{1}{3}) \cup [8; +\infty);$



б) $\frac{5}{3x-1} + \frac{6}{2x-1} < 3;$ $\frac{10x-5+18x-6-18x^2+15x-3}{(3x-1)(2x-1)} < 0;$

$\frac{18x^2-43x+14}{(3x-1)(2x-1)} > 0;$ $\frac{(x-2)\left(x-\frac{7}{18}\right)}{\left(x-\frac{1}{3}\right)\left(x-\frac{1}{2}\right)} > 0;$



$x \in \left(-\infty; \frac{1}{3}\right) \cup \left(\frac{7}{18}; \frac{1}{2}\right) \cup (2; +\infty).$

C-43

а) $y = 3x - 7x^3 + \frac{1}{4}x^8 + x^9;$ $y' = 3 - 21x^2 + 2x^7 + 9x^8;$

б) $y = x\sqrt{x+5};$ $y' = \sqrt{x+5} + \frac{x}{2\sqrt{x+5}};$

в) $y = \cos 0,3x;$ $y' = -0,3\sin 0,3x;$

г) $y = \operatorname{ctg}\left(\frac{\pi}{7} - 3x\right);$ $y' = \frac{3}{\sin^2\left(\frac{\pi}{7} - 3x\right)};$

д) $y = (5x^2 - 1)^8;$ $y' = 8(10x)(5x^2 - 1)^7 = 80x(5x^2 - 1)^7.$

C-44

1. $f(x) = x^2 - 3x - 3;$ $f(0) = -3;$ $f'(x) = 2x - 3;$ $f'(0) = -3;$
 $y_{\text{кас}} = -3 - 3x$ — уравнение касательной.

2. а) $\sqrt{\sqrt{0,999996}} \approx 1 - 0,000004 \cdot \frac{1}{4} = 0,999999;$

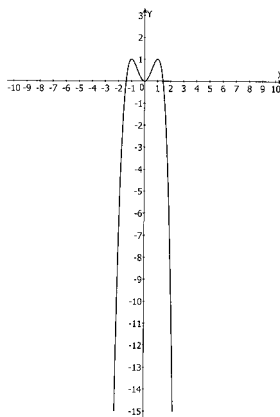
б) $0,99997^{350} \approx 1 - 0,00003 \cdot 350 = 0,9895.$

$$3. \quad x(t) = \frac{2+t}{4+t} = 1 - \frac{2}{t+4}; \quad v(t) = \frac{2}{(t+4)^2};$$

$$a(t) = \frac{-4}{(t+4)^3}; \quad v(1) = \frac{2}{25}; \quad a(1) = -\frac{4}{125}.$$

C-45

1. $f(x) = 2x^2 - x^4 = 0; x = 0; x = \pm 1$ — нули; $f'(x) = 4x(1 - x^2) = 0 \quad x = 0;$
 $x = \pm 1 \quad \max(\pm 1; 1) \quad \min(0; 0)$
 возрастает: $x \in (-\infty; -1] \cup [0; 1];$ убывает: $[-1; 0] \cup [1; +\infty)$
 $x \in R \quad y \leq 1$



$$2. \quad f(x) = \cos^2 \frac{\pi x}{4} \sin \frac{\pi x}{4}; \quad x \in [-2; 2];$$

$$f'(x) = \left(-2 \cos \frac{\pi x}{4} \sin^2 \frac{\pi x}{4} + \cos^3 \frac{\pi x}{4} \right) \frac{\pi}{4}; \quad f'(x) = 0 \text{ при}$$

$$\cos \frac{\pi x}{4} = 0 \quad x = 2 + 4\pi \quad \cos^2 \frac{\pi x}{4} - 2 \sin^2 \frac{\pi x}{4} = 0$$

$$\sin^2 \frac{\pi x}{4} = \frac{1}{3}; \quad \sin \frac{\pi x}{4} = \pm \frac{1}{\sqrt{3}}; \quad x = 4(-1)^k \arcsin \left(\pm \frac{1}{\sqrt{3}} \right) \frac{1}{\pi} + 4\pi n$$

$$\text{наибольшее значение } f \left(\frac{4}{\pi} (-1)^k \arcsin \frac{1}{\sqrt{3}} \right) = \frac{2}{3\sqrt{3}};$$

$$\text{наименьшее значение } f \left(\frac{4}{\pi} (-1)^k \arcsin \left(-\frac{1}{\sqrt{3}} \right) \right) = -\frac{2}{3\sqrt{3}}.$$

ВАРИАНТ 9

С-1

1. $180^\circ = 18^\circ + 2\alpha$; $\alpha = 81^\circ = \frac{9\pi}{20}$; $18^\circ = \frac{\pi}{10}$.
2. а) при повороте на $360^\circ = 60$ мин. при $x = -72^\circ$ $x = 12$ мин. вперед;
б) $360^\circ - 72^\circ = 228^\circ$; $228^\circ + 360^\circ \cdot 11 = 4248^\circ$.
3. $3x + 7x + 17x + 21x = 360^\circ$; $x = 7,5^\circ$;
 $3x = 22^\circ 30'$; $7x = 52^\circ 30'$; $17x = 127^\circ 30'$; $21x = 157^\circ 30'$.
 $22^\circ 30' \approx 0,3927$.
4.
$$\begin{cases} \alpha + \beta = 1 \\ \alpha = \beta^2 \end{cases}; \quad \beta^2 + \beta - 1 = 0; \quad \beta = \frac{-1 + \sqrt{5}}{2};$$
$$\alpha = \frac{6 - 2\sqrt{5}}{2} = 3 - \sqrt{5};$$
$$\alpha = 35^\circ 25'; \quad \beta = 21^\circ 53'.$$

С-2

1.
$$\sqrt{\frac{1 + \sin \alpha}{1 - \sin \alpha}} - \sqrt{\frac{1 - \sin \alpha}{1 + \sin \alpha}} = 2 \operatorname{tg} \alpha;$$
$$\frac{\sqrt{1 + \sin^2 \alpha + 2 \sin \alpha} - \sqrt{1 + \sin^2 \alpha - 2 \sin \alpha}}{\cos \alpha} = \frac{1 + \sin \alpha - 1 + \sin \alpha}{\cos \alpha} = 2 \operatorname{tg} \alpha.$$
2. а) $\frac{\cos 1700^\circ \operatorname{tg} 3400^\circ}{\sin 5000^\circ} < 0$; б) $\sin 7 \cos 9 \operatorname{tg} 11 > 0$.
3.
$$\frac{(\sin \alpha + \cos \alpha)^2 - 1}{\operatorname{tg} \alpha - \sin \alpha \cos \alpha} \cdot \operatorname{tg} \alpha = \frac{2 \sin^2 \alpha}{\operatorname{tg} \alpha - \sin \alpha \cos \alpha} = \frac{2 \sin^2 \alpha \cos \alpha}{\sin \alpha - \sin \alpha \cos^2 \alpha} =$$
$$= \frac{2 \sin \alpha \cos \alpha}{1 - \cos^2 \alpha} = 2 \operatorname{ctg} \alpha;$$
$$\sin \alpha = \frac{2}{\sqrt{5}}; \cos \alpha < 0, \text{ значит, } \alpha \in \text{II четверти}; \cos \alpha = -\frac{1}{\sqrt{5}}; 2 \operatorname{ctg} \alpha = -1$$

C-3

$$1. \quad \operatorname{tg} 31^\circ \operatorname{tg} 33^\circ \operatorname{tg} 35^\circ \dots \operatorname{tg} 59^\circ = \operatorname{tg} 45^\circ; \quad \operatorname{tg} 31^\circ \operatorname{ctg} 31^\circ \dots \operatorname{tg} 43^\circ \operatorname{ctg} 43^\circ = 1.$$

$$2. \quad \frac{\sin\left(\alpha - \frac{3\pi}{2}\right) \operatorname{tg}\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)}{1 + \cos\left(\alpha - \frac{5\pi}{2}\right)} = \frac{-\cos \alpha \operatorname{tg}\left(\frac{\pi}{4} + \frac{\alpha}{2}\right)}{1 + \sin \alpha} =$$

$$= -\frac{\cos \alpha}{1 + \sin \alpha} \cdot \frac{1 + \operatorname{tg} \frac{\alpha}{2}}{1 - \operatorname{tg} \frac{\alpha}{2}} = -\frac{\cos \alpha}{1 + \sin \alpha} \cdot \frac{\cos \frac{\alpha}{2} + \sin \frac{\alpha}{2}}{\cos \frac{\alpha}{2} - \sin \frac{\alpha}{2}} = -\frac{1 + 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2}}{1 + \sin \alpha} =$$

$$= -1.$$

$$3. \quad \sin\left(2\varphi - \frac{\pi}{2}\right) \cos(3\varphi + \pi) = \sin(2\varphi - \pi) \sin(\pi - 3\varphi) - \sin\left(\frac{3\pi}{2} + \varphi\right)$$

$$\cos 2\varphi \cos 3\varphi = -\sin 2\varphi \sin 3\varphi + \cos \varphi; \quad \cos(2\varphi - 3\varphi) = \cos \varphi.$$

C-4

$$1. \quad \cos \frac{\pi}{9} \cos \frac{2\pi}{9} \cos \frac{4\pi}{9} = \cos 20^\circ \cos 40^\circ \cos 80^\circ =$$

$$= \frac{1}{2} (\cos 60^\circ + \cos 20^\circ) \cos 80^\circ = \frac{1}{2} \left(\frac{1}{2} \cos 80^\circ + \frac{1}{2} (\cos 60^\circ + \cos 100^\circ) \right) =$$

$$= \frac{1}{2} \left(\frac{1}{2} \cos 80^\circ + \frac{1}{4} - \frac{1}{2} \cos 80^\circ \right) = \frac{1}{8}.$$

$$2. \quad \frac{2 \sin 2\alpha - 3 \cos 2\alpha}{4 \sin 2\alpha + 5 \cos 2\alpha} = \left(\operatorname{tg} \alpha = 3 \quad \operatorname{tg} 2\alpha = \frac{6}{1-9} = -\frac{3}{4} \right) = \frac{2 \operatorname{tg} 2\alpha - 3}{4 \operatorname{tg} 2\alpha + 5} =$$

$$= \frac{-\frac{3}{2} - 3}{-3 + 5} = \frac{-9}{4} = \cos 8\alpha.$$

$$3. \quad \cos^4 2\alpha - 6 \cos^2 2\alpha \sin^2 2\alpha + \sin^4 2\alpha = 1 - 8 \cos^2 2\alpha \sin^2 2\alpha =$$

$$= 1 - 2 \sin^2 4\alpha.$$

C-5

1.

$$(\cos t - \sin t)(1 + \cos t + \sin t) = 0$$

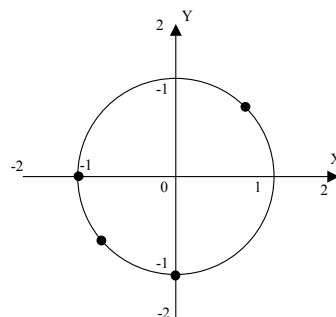
$$\cos t - \sin t = 0 \quad t = \frac{\pi}{4} + \pi n$$

$$1 + \cos t + \sin t = 0$$

$$\sin\left(t + \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$$

$$t = (-1)^{k+1} \frac{\pi}{4} - \frac{\pi}{4} + \pi k$$

$$t \in [0; 2\pi]: t = \frac{\pi}{4}; \frac{5\pi}{4}; \pi; \frac{3\pi}{2}.$$

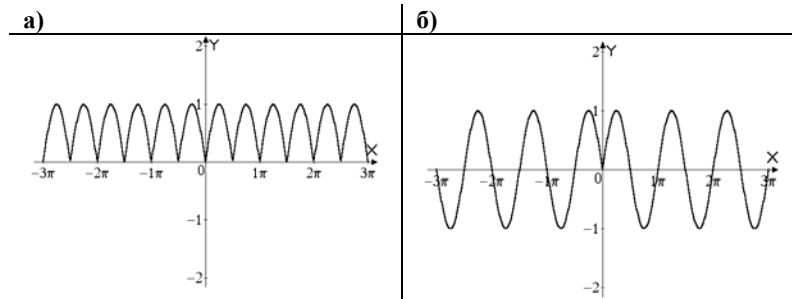


2.

$$\cos\left(\sin \frac{\pi}{7}\right) > \sin\left(\cos \frac{\pi}{7}\right), \text{ т.к. } \cos \frac{\pi}{7} > 0 \Rightarrow \sin\left(\cos \frac{\pi}{7}\right) < \cos \frac{\pi}{7}, \text{ а}$$

$$\cos\left(\sin \frac{\pi}{7}\right) > \cos \frac{\pi}{7} = \cos \frac{\pi}{7} > \sin\left(\cos \frac{\pi}{7}\right) \quad \text{ч.т.д.}$$

3.



C-6

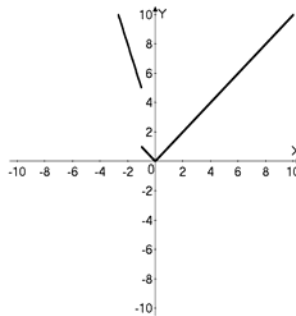
$$1. \text{ а) } f(x) = \frac{3x^2 + 4x - 5x^3}{x(2 - \sqrt{x+1})}; \text{ ОДЗ: } \begin{cases} x \neq 0 \\ 2 - \sqrt{x+1} \neq 0 \\ x \geq -1 \end{cases} \quad x \geq -1, x \neq 0, x \neq 3,$$

значит, $x \in [-1; 0) \cup (0; 3) \cup (3; \infty)$;

$$6) f(x) = \sqrt{9 - 2\sqrt{x}}; \quad \text{ОДЗ: } \begin{cases} x \geq 0 \\ 9 - 2\sqrt{x} \geq 0 \end{cases}; \quad x \in \left[0; \frac{81}{4}\right].$$

$$2. \quad f(x) = \begin{cases} |x| & x > -1 \\ 2 - 3x & x \leq -1 \end{cases}$$

$$a) f(-2) = 8; \quad f(-1) = 5; \quad f(3) = 3; \quad f(x^2) = \begin{cases} x^2, & x > 1 \\ 2 - 3x^2, & x \leq -1 \end{cases};$$



б)

С-7

1. а) да; б) да.

$$2. \quad f_1(x) = \frac{f(-x) + f(x)}{2}; \quad f_2(x) = \frac{f(x) - f(-x)}{2} \quad \text{пусть существуют}$$

2 представления

$$f(x) = f_1(x) + f_2(x) = g_1(x) + g_2(x), \quad \text{где } f_1(x) \text{ и } g_1(x) - \text{четные,}$$

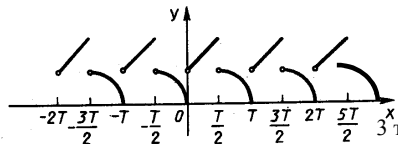
$$g_2(x) \text{ и } f_2(x) - \text{нечетные;}$$

$$f_1(x) - g_1(x) = g_2(x) - f_2(x) \Rightarrow \text{слева четная функция;}$$

$$\text{справа нечетная} \Rightarrow f_1(x) = g_1(x); \quad g_2(x) = f_2(x).$$

С-8.

1.



2. а) $f(x) = |\sin x| + \operatorname{tg} 2x$; $f_1(x) = |\sin x|$ $T_1 = \pi$;
 $f_2(x) = \operatorname{tg} 2x$; $T_2 = \frac{\pi}{2} \Rightarrow T = \pi$;
 б) $f(x) = \cos\left(\sqrt{2}x - \frac{\pi}{3}\right)$; $T = \frac{2\pi}{\sqrt{2}}$.
3. а) $f(x) = \sin x^2$, пусть T – период $\Rightarrow \sin x^2 = \sin(x+T)^2$,
 что неверно;
 б) $f(x) = \cos x \cos \sqrt{2}x = \frac{1}{2}(\cos x(1+\sqrt{2}) + \cos x(1-\sqrt{2}))$;

$$\begin{aligned} f_1(x) &= \cos x(1+\sqrt{2}) & T_1 &= \frac{2\pi}{1+\sqrt{2}} \\ f_2(x) &= \cos x(1-\sqrt{2}) & T_2 &= \frac{2\pi}{1-\sqrt{2}} \end{aligned} \quad T_1 = nT_2, \text{ значит, } f(x) \text{ не является}$$

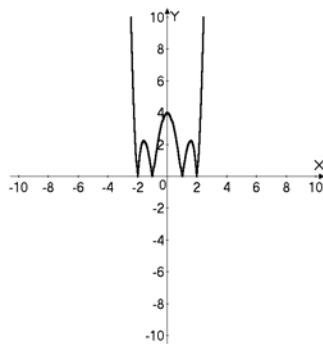
периодической.

С-9

1. а) $f(x) = \begin{cases} x^2 + 2x, & x \leq 0; \\ x^2 - 4x, & x > 0; \end{cases} \quad \begin{cases} x_B = -1 \\ x_B = 2 \end{cases}$
 $f(x)$ убывает при $x \in (-\infty; -1) \cup (0; 2)$; возрастает при $x \in (-1; 0) \cup (2; +\infty)$.
 б) $f(x) = x + \frac{1}{x}$ $f'(x) = 1 - \frac{1}{x^2} > 0$ при $x^2 > 1$, значит,
 $f(x)$ возрастает при $x \in (-\infty; -1) \cup (1; +\infty)$; убывает при $x \in (-1; 0) \cup (0; 1)$.
 2. а) возрастает; б) нет; в) возрастает; г) убывает;
 д) возрастает; е) нет.
 3. $\sin 1, \cos 1, \operatorname{tg} 1, \operatorname{ctg} 2$.
 Ответ: $\operatorname{ctg} 2, \cos 1, \sin 1, \operatorname{tg} 1$.

С-10

1. $f(x) = |x^4 - 5x^2 + 4|$; $y = x^4 - 5x^2 + 4$;
 $y' = 2x(2x^2 - 5) = 0$ при $x_{\max} = 0$ и $x_{\min} = \pm \sqrt{\frac{5}{2}}$;
 $y = 0$ при $x_{\min} = \pm 1$; $x_{\max} = \pm 2$.



2. а) $f(x) = \sin |x + 2|$; $|x + 2| = \frac{\pi}{2} + 2\pi n$, $n \in \mathbb{N} \cup \{0\}$;

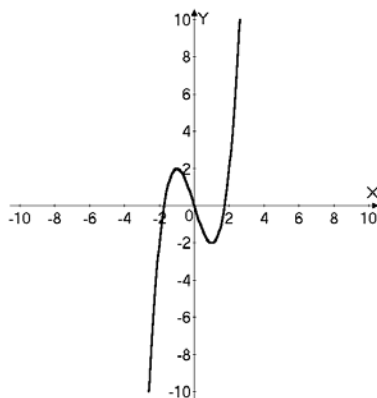
$$x_{\max} = -2 \pm \left(-\frac{\pi}{2} + 2\pi n\right); x_{\min} = -2 \pm \left(-\frac{\pi}{2} + 2\pi k\right) \quad k \in \mathbb{N};$$

б) $f(x) = \cos 4x + \cos 2x - \frac{1 - \operatorname{tg}^2 x}{1 + \operatorname{tg}^2 x} =$

$$= \cos 4x + \cos 2x - \cos 2x = \cos 4x;$$

$$x_{\max} = \frac{\pi n}{2}; \quad x_{\min} = -\frac{\pi}{4} + \frac{\pi n}{2}.$$

C-11



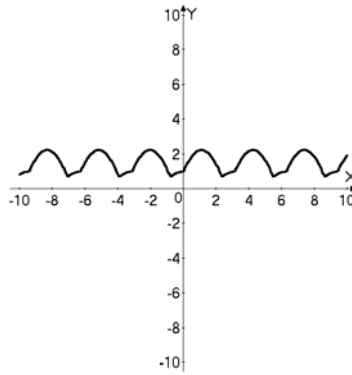
$y = x^3 - 3x$; $x = 0$, $x = \pm\sqrt{3}$ — нули функции;

$y' = 3(x^2 - 1) = 0$ при $x = \pm 1$; y возрастает при $(-\infty; -1) \cup (1; +\infty)$;
убывает при $[-1; 1]$; $x_{\max} = -1$; $x_{\min} = 1$.

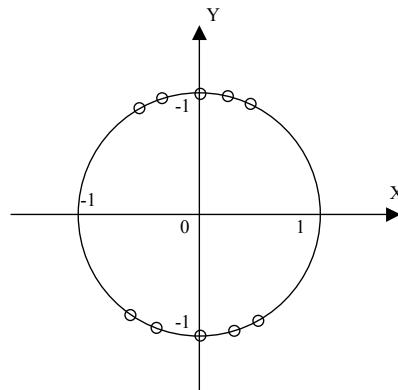
C-12

1. $f(x) = \sin^2 x - 2\sin x + 3$;
 min: $\sin x = 1 \quad x = \frac{\pi}{2} + 2\pi n \quad f(x) = 2$
 max: $\sin x = -1 \quad x = -\frac{\pi}{2} + 2\pi n \quad f(x) = 6$, значит, $f(x) \in [2; 6]$.

2.



3.



$$\frac{\operatorname{tg}^2 t - 5}{\operatorname{tg}^2 t - 3} \geq 2; \quad \frac{\operatorname{tg}^2 t - 2\operatorname{tg}^2 t + 1}{\operatorname{tg}^2 t - 3} \geq 0; \quad \frac{\operatorname{tg}^2 t - 1}{\operatorname{tg}^2 t - 3} \leq 0;$$

$$t \in \left(-\frac{\pi}{3} + \pi n; -\frac{\pi}{4} + \pi n \right] \cup \left[\frac{\pi}{4} + \pi n; \frac{\pi}{3} + \pi n \right).$$

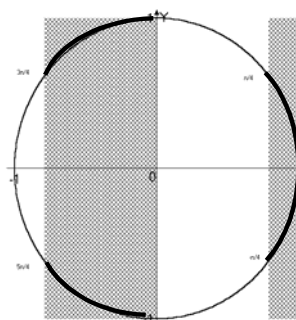
C-13

1. **a)** $\sin(\arccos 0,28) = \sqrt{1 - 0,0784} = 0,96$;
 б) $\arcsin \sin 10 = -\arcsin \sin(10 - 3\pi) = 3\pi - 10$.
2. $\arcsin x + \arccos x = \frac{\pi}{2}$; $\arcsin x = \frac{\pi}{2} - \arccos x$;
 $x = \sin\left(\frac{\pi}{2} - \arccos x\right)$.
3. **a)** $\cos(5\arccos 0,7321) \approx -0,8223$;
 б) $\sin(4\arcsin(0,0237) + \arccos 0,67) \approx 0,8025$.

C-14

- a)** $4\sin x \cos x = -1$; $\sin 2x = -\frac{1}{2}$; $x = (-1)^{k+1} \frac{\pi}{12} + \frac{\pi k}{2}$;
- б)** $\frac{\operatorname{tg} x + \operatorname{tg} 2x}{1 - \operatorname{tg} x \operatorname{tg} 2x} = \frac{1}{\sqrt{3}}$; $\operatorname{tg} 3x = \frac{1}{\sqrt{3}}$; $x = \frac{\pi}{18} + \frac{\pi n}{3}$;
- в)** $\left| \cos\left(3x - \frac{\pi}{14}\right) \right| = \frac{1}{2}$; $\cos\left(3x - \frac{\pi}{14}\right) = \pm \frac{1}{2}$;
 $x = \pm \frac{\pi}{9} + \frac{\pi}{42} + 2\pi n$; $x = \pm \frac{2\pi}{9} + \frac{\pi}{42} + 2\pi n$.

C-15



$$\cos t \operatorname{tg} 2t \leq 0;$$

$$x \in \left(-\frac{\pi}{4} + 2\pi n; \frac{\pi}{4} + 2\pi n\right) \cup \left[\frac{\pi}{2} + 2\pi n; \frac{3\pi}{4} + 2\pi n\right) \cup \left(\frac{5\pi}{4} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right].$$

C-16

$$\text{a) } \frac{\operatorname{tg}\left(x + \frac{\pi}{7}\right) + \operatorname{tg} 2x}{1 - \operatorname{tg} 2x \operatorname{tg}\left(x + \frac{\pi}{7}\right)} \leq \frac{1}{\sqrt{3}}; \quad \operatorname{tg}\left(3x + \frac{\pi}{7}\right) \leq \frac{1}{\sqrt{3}};$$

$$x \in \left(-\frac{3\pi}{7} + \frac{\pi n}{3}; \frac{\pi}{26} + \frac{\pi n}{3}\right), \text{ но по ОДЗ } x \neq \frac{5\pi}{14} + \pi k, \text{ значит,}$$

$$x \in \left(-\frac{9\pi}{42} + \pi k; \frac{\pi}{126} + \pi k\right) \cup \left(\frac{5\pi}{42} + \pi k; \frac{\pi}{4} + \pi k\right) \cup$$

$$\cup \left(\frac{\pi}{4} + \pi k; \frac{43}{126}\pi + \pi k\right) \cup \left(\frac{19\pi}{42} + \pi k; \frac{85\pi}{126} + \pi k\right)$$

$$\text{б) } \sin^2 x \geq \frac{1}{2} \quad \begin{cases} \sin x \geq \frac{\sqrt{2}}{2} \\ \sin x \leq -\frac{\sqrt{2}}{2} \end{cases}; \quad x \in \left[\frac{\pi}{4} + \pi n; \frac{3\pi}{4} + \pi n\right].$$

C-17

$$\text{a) } \sqrt{2} \sin \frac{x}{2} + 1 = \cos x; \quad \sin \frac{x}{2} \left(2 \sin \frac{x}{2} + \sqrt{2}\right) = 0;$$

$$x = 2\pi n; \quad x = (-1)^{k+1} \frac{\pi}{2} + 2\pi n;$$

$$\text{б) } \sin x \sin 3x = \frac{1}{2}; \quad \cos(2x) - \cos 4x = 1;$$

$$\cos 2x(1 - 2\cos 2x) = 0; \quad x = \frac{\pi}{4} + \frac{\pi n}{2}; \quad x = \pm \frac{\pi}{6} + \pi n.$$

C-18

$$\text{a) } 4\cos^2 x + \sin x \cos x + 3\sin^2 x = 3; \quad \cos x(\cos x + \sin x) = 0;$$

$$x = \frac{\pi}{2} + \pi n \quad x = -\frac{\pi}{4} + \pi n;$$

$$\text{б) } \sin^5 x - \sin^4 x \cos x = 2\sin^3 x \cos^2 x;$$

$$\sin^3 x(\sin^2 x - \sin x \cos x - 2\cos^2 x) = 0; \quad x = \pi n; \quad \cos x \neq 0;$$

$$\operatorname{tg}^2 x - \operatorname{tg} x - 2 = 0; \quad \operatorname{tg} x = 2; \quad x = \operatorname{arctg} 2 + \pi k;$$

$$\operatorname{tg} x = -1; \quad x = -\frac{\pi}{4} + \pi k.$$

C-19.

$$\begin{cases} \operatorname{tg} 2y = 1 \\ \sin 2x = \sqrt{3} \cos 2y \end{cases}; \begin{cases} \cos(x - 2y) = 0 \\ \sin 2x = \sqrt{3} \cos 2y \end{cases};$$

$$\begin{cases} x = 2y + \frac{\pi}{2} + \pi n \\ \sin(4y + \pi) = \sqrt{3} \cos 2y \end{cases}; \begin{cases} \sqrt{3} \cos 2y + \sin 4y = 0 \\ \cos 2y(\sqrt{3} + 2 \sin 2y) = 0 \end{cases};$$

$$\begin{cases} y = -\frac{\pi}{4} + \frac{\pi k}{2} \\ x = \pi n + \pi k \end{cases}; \begin{cases} y = (-1)^{k+1} \frac{\pi}{6} + \frac{\pi k}{2} \\ x = (-1)^{k+1} \frac{\pi}{3} + \pi k + \frac{\pi}{2} + \pi n \end{cases}.$$

C-20

a) $\sin 3x \sin^3 x + \cos 3x \cos^3 x = \frac{1}{2\sqrt{2}};$
 $\sin 3x \sin x + \cos 3x \cos x - \sin 3x \cos^2 x \sin x -$
 $-\cos 3x \cos x \sin^2 x = \frac{1}{2\sqrt{2}};$
 $\cos 2x - \frac{1}{2} (\cos 2x + \sin 3x \cos 2x \sin x - \cos 3x \cos x \cos 2x) = \frac{1}{2\sqrt{2}};$
 $\frac{1}{2} \cos 2x + \frac{1}{2} (\cos 2x \cos 4x) = \frac{1}{2\sqrt{2}};$
 $2\cos^3 2x = \frac{1}{\sqrt{2}}; \quad \cos 2x = \frac{1}{\sqrt{2}}; \quad x = \pm \frac{\pi}{8} + \pi n;$
б) $\sin x + \sin 2x + \sin 3x = 1 + \cos x + \cos 2x;$
 $2\sin 2x \cos x + \sin 2x - \cos x - 2\cos^2 x = 0;$
 $2\cos x(\sin 2x - \cos x) + \sin 2x - \cos x = 0;$
 $(\sin 2x - \cos x)(2\cos x + 1) = 0;$
 $\cos x(2\sin x - 1)(2\cos 3x + 1) = 0;$
 $x = \frac{\pi}{2} + \pi n;$
 $x = (-1)^k \frac{\pi}{6} + \pi k;$
 $x = \pm \frac{2\pi}{3} + 2\pi z.$

C-21

- $f(x) = -\frac{16}{x}; \quad g(x) = x^2 - 1; \quad x_0 = 2; \quad \Delta x = 0,1;$
 $\Delta f(x_0) = -\frac{16}{x_0 + \Delta x} + \frac{16}{x_0} = -\frac{16}{2,1} + 8 \approx -0,38;$
 $\Delta g(x_0) = -2\Delta x x_0 + \Delta x^2 = 0,01 - 0,4 = -0,39$, значит, $\Delta g(2) < \Delta f(2);$
 $x_0 = 2 \quad \Delta x = 0,2 \quad \Delta f = -\frac{16}{2,2} + 8 \approx 0,73;$
 $\Delta g(x_0) = 0,04 - 0,8 = -0,76$, значит, $\Delta f(2) > \Delta g(2).$
- $f(x) = x^3 - 2x^2 + 4x - 3$
 $\Delta f(x_0) = x_0^3 + (\Delta x)^3 + 3x_0(\Delta x)^2 + 3x_0^2\Delta x - 2x_0^2 - 2(\Delta x)^2 - 4x_0\Delta x +$
 $+ 4x_0 + 4\Delta x - 3 - x_0^3 + 2x_0^2 - 4x_0 + 3;$
 $\frac{\Delta f(x_0)}{\Delta x} = (\Delta x)^2 + 3x_0\Delta x + 3x_0^2 - 2\Delta x - 4x_0 + 4;$
 $\lim_{\Delta x \rightarrow 0} \frac{\Delta f(x_0)}{\Delta x} = 3x_0^2 - 4x_0 + 4.$

C-22

- $m = 3 - 2t; \quad x(t) = t^2 + 3t + 1; \quad m(1) = 1;$
 $v(t) = 2t + 3; \quad a(t) = 2; \quad F = 2 \text{ Н}.$
- $\text{а) } f(x) = 4\sqrt{x} - x^3; \quad f'(x) = \frac{2}{\sqrt{x}} - 3x^2;$
 $\text{б) } f(x) = \frac{x-2}{x-1} = 1 - \frac{1}{x-1}; \quad f'(x) = \frac{1}{(x-1)^2}.$

C-23

- $\text{а) } (-2; 1) (3; 3)1); \quad \text{б) } \lim_{x \rightarrow -2} f(x) \text{ не существует; } \lim_{x \rightarrow 3} f(x) = 1;$
 $\text{в) } y \in (-2; 2] \cup \{3\}.$
- $f(x) = \frac{x+1-4}{\sqrt{x+1}-2} = \sqrt{x+1} + 2, \quad x \neq 3;$
 $\left| \sqrt{x+1} - 2 \right| < 0,1; \quad x \in (2,61; 3,41), \text{ значит, } \delta = 0,39.$

C-24

1. a) $\lim_{x \rightarrow 1} y = \frac{\lim_{x \rightarrow 1} f^2(x) - \lim_{x \rightarrow 1} g(x)}{3 \lim_{x \rightarrow 1} f(x) + 4 \lim_{x \rightarrow 1} g(x)} = \frac{4+1}{6-4} = \frac{5}{2};$
 б) $\lim_{x \rightarrow 1} y = \left(\sqrt{\lim_{x \rightarrow 1} f(x)} + \lim_{x \rightarrow 1} g(x) \right)^2 + \left(\sqrt{\lim_{x \rightarrow 1} f(x)} - \lim_{x \rightarrow 1} g(x) \right)^2 =$
 $= 2 \lim_{x \rightarrow 1} f(x) + 2 \lim_{x \rightarrow 1} g^2(x) = 2(2+1) = 6.$
2. a) $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2} = \lim_{x \rightarrow 2} x^2 + 2x + 4 = 12;$
 б) $\lim_{x \rightarrow 2} \frac{\sqrt{3x-2} - 2}{x - 2} = \lim_{x \rightarrow 2} \frac{3x - 6}{(x - 2)(\sqrt{3x-2} + 2)} = \frac{3}{4}.$

C-25

1. a) $f(x) = \sqrt{x^3} - \sqrt{x} - 3x^{18}; \quad f'(x) = \frac{3}{2}\sqrt{x} - \frac{1}{2\sqrt{x}} - 54x^{17};$
 б) $g(x) = (x^2 + 3x)\sqrt{x}; \quad g'(x) = (2x + 3)\sqrt{x} + \frac{x^2 + 3x}{2\sqrt{x}}.$
2. $f(x) = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases} = x|x|; \quad \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = \lim_{x \rightarrow 0} \frac{x|x|}{x} = \lim_{x \rightarrow 0} |x| = 0.$

C-26

- a) $f(x) = (x - 2)^2(x + 4); \quad f'(x) = 2(x - 2)(x + 4) = 0;$
 $x = 2 \quad x = -4; \quad f'(x) > 0; \quad x \in (-\infty; -4) \cup (2; +\infty);$
 $f'(x) < 0 \quad x \in (-4; 2);$
 б) $f(x) = \frac{2x - 1}{(x - 1)^2};$
 $f'(x) = \frac{2(x - 1)^2 - 2(x - 1)(2x - 1)}{(x - 1)^4} = \frac{2x - 2 - 4x + 2}{(x - 1)^4} = \frac{-2x}{(x - 1)^4} = 0 \text{ при } x = 0;$
 $f'(x) > 0 \quad x < 0;$
 $f'(x) < 0 \quad x > 0, \quad x \neq 1.$

C-27

1. а) $f(x) = \frac{1}{\sqrt{2-\sqrt{x^2-7}}}$; ОДЗ: $\begin{cases} x^2-7 \geq 0 \\ 2-\sqrt{x^2-7} > 0 \end{cases}$; $\begin{cases} x^2 \geq 7 \\ x^2 < 11 \end{cases}$;
 $x \in (-\sqrt{11}; -\sqrt{7}] \cup [\sqrt{7}; \sqrt{11})$;
 б) $f(x) = \sqrt{x - \sqrt{x - 2\sqrt{x}}}$;
 ОДЗ $\begin{cases} x \geq 0 \\ x - 2\sqrt{x} \geq 0 \\ x - \sqrt{x - 2\sqrt{x}} \geq 0 \end{cases}$; $\begin{cases} x \geq 0 \\ x^2 \geq 4x \\ x^2 \geq x - 2\sqrt{x} \geq 0 \end{cases}$; $\begin{cases} x \geq 0 \\ x(x-4) \geq 0 \end{cases}$;
 $x \in \{0\} \cup [4; +\infty)$.
2. $f(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$; $f(f(x)) = 1 - \frac{1}{1 - \frac{1}{x}} = 1 - \frac{x}{x-1} = -\frac{1}{x-1}$;
 $f(f(f(x))) = -\frac{1}{1 - \frac{1}{x} - 1} = x$; $f(f(f(f(x)))) = 1 - \frac{1}{x}$;
 $f_n(x) = 1 - \frac{1}{x}$, $n=3k-2$; $f_n(x) = x$; $n=3k$; $f_n(x) = -\frac{1}{x-1}$; $n=3k-1$
 ОДЗ: для $f: x \in (-\infty; 0) \cup (0; \infty)$; для $f_a: x \in (-\infty; 0) \cup (0; 1) \cup (1; \infty)$.

3.

а) $f(x) = \sqrt{3x^3 + 2x^2 - 12} = \frac{9x^2 + 4x}{2\sqrt{3x^3 + 2x^2 - 12}}$;
 б) $f(x) = (x^3 - x\sqrt{x})^9$; $f'(x) = 9(3x^2 - \frac{3}{2}\sqrt{x})(x^3 - x\sqrt{x})^8$.

C-28

а) $f(x) = \sin 2x \cos 3x + \cos 2x \sin 3x = \sin 5x$; $f'(x) = 5\cos 5x$;
 б) $f(x) = \frac{\operatorname{tg} x - \operatorname{tg}(x-1)}{1 + \operatorname{tg} x \operatorname{tg}(x-1)} = \operatorname{tg} 1$; $f'(x) = 0$;
 в) $f(x) = \sin^3 2x + \cos^3 2x$; $f'(x) = 6(\sin^2 2x \cos 2x - \sin 2x \cos^2 2x)$.

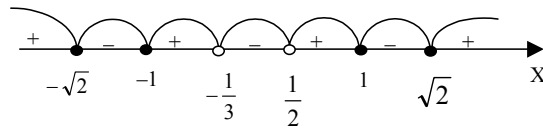
C-29

$$1. \quad \lim_{x \rightarrow a} \frac{x^2 - a^2}{\sqrt{x} - \sqrt{a}} = \lim_{x \rightarrow a} (\sqrt{x} + \sqrt{a})(x + a) = 4\sqrt{a}a \leq 32;$$

$$\frac{3}{a^2} \leq 8; \quad a \leq 4, \text{ значит, } 0 < a \leq 4.$$

$$2. \text{ а) } \frac{x^4 - 3x^2 + 2}{6x^2 - x - 1} \leq 0; \quad \frac{(x^2 - 2)(x^2 - 1)}{\left(x - \frac{1}{2}\right)\left(x + \frac{1}{3}\right)} \leq 0;$$

$$\frac{(x - \sqrt{2})(x + \sqrt{2})(x - 1)(x + 1)}{\left(x - \frac{1}{2}\right)\left(x + \frac{1}{3}\right)} \leq 0; \quad x \in \left(-\frac{1}{3}; \frac{1}{2}\right).$$

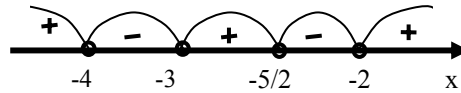


$$\text{б) } \frac{1}{x+2} + \frac{2}{x+3} < \frac{3}{x+4};$$

$$\frac{x^2 + 7x + 12 + 2x^2 + 12x + 16 - 3x^2 - 15x - 18}{(x+2)(x+3)(x+4)} < 0;$$

$$\frac{4x+10}{(x+2)(x+3)(x+4)} < 0;$$

$$x \in (-4; -3) \cup \left(-\frac{5}{2}; -2\right).$$



C-30

$$1. \quad f(x) = \sqrt{x}; \quad f'(x) = \frac{1}{2\sqrt{x}} = \operatorname{tg} 60^\circ = \sqrt{3}; \quad x = \frac{1}{12}; \quad f\left(\frac{1}{12}\right) = \frac{1}{2\sqrt{3}},$$

$$\text{значит искомая точка } \left(\frac{1}{12}; \frac{1}{2\sqrt{3}}\right).$$

$$2. \quad f(x) = \cos\left(\frac{x}{3} - \frac{\pi}{12}\right); \quad f(\pi) = \frac{\sqrt{2}}{2};$$

$$f'(x) = -\frac{1}{3} \sin\left(\frac{x}{3} - \frac{\pi}{12}\right); \quad f'(\pi) = -\frac{\sqrt{2}}{6}; \quad y_{\text{кас}} = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{6}(x - \pi) - \text{ур. кас.}$$

C-31

- $\left(\sqrt{4,0008} - \sqrt{0,9998}\right)^{40} \approx (2(1 + 0,0001) - 1 + 0,0001)^{40} \approx$
 $\approx 1 + 0,0001 \cdot 40 \cdot 3 \approx 1,012.$
- $\sin 64^\circ = \sin 60^\circ \cos 4^\circ + \cos 60^\circ \sin 4^\circ \approx 0,866 + \frac{1}{2} \cdot 0,0698 = 0,9009.$

C-32

- $s(t) = -\frac{1}{3}t^3 + 4t^2 + 5t; \quad v(t) = -t^2 + 8t + 5; \quad a(t) = -2t + 8;$
a) $t = 4; \quad \text{б)}$ $v(4) = 21 \text{ м/с}.$
- $s(t) = \frac{1}{(t-2)^2}; \quad v(t) = \frac{-2}{(t-2)^3}; \quad a(t) = \frac{6}{(t-2)^4}; \quad F = \frac{6m}{(t-2)^4}.$

C-33.

- $f(x) = 3x^3 - 2x^2 + 3x - 2; \quad f'(x) = 9x^2 - 4x + 3;$
 $\frac{D}{4} = 4 - 27 < 0 \Rightarrow f(x) \text{ всегда возрастает.}$
- $f(x) = \operatorname{tg}^3 x - \operatorname{tg} x - 3; \quad \text{ОДЗ: } x \neq \frac{\pi}{2} + \pi n$
 $f'(x) = 3\operatorname{tg}^2 x \frac{1}{\cos^2 x} - \frac{1}{\cos^2 x} = 0; \quad f'(x) = 0 \text{ при } \operatorname{tg} x = \frac{\pm 1}{\sqrt{3}};$
 $x = \pm \frac{\pi}{6} + \pi n; \quad x_{\min} = -\frac{\pi}{6} + \pi n; \quad x_{\max} = \frac{\pi}{6} + \pi n.$

C-34

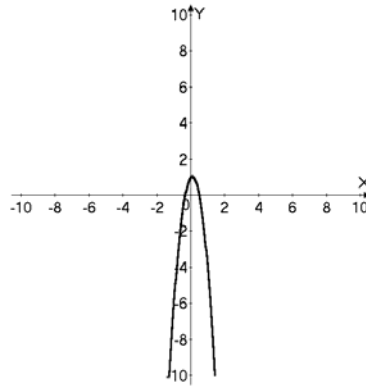
$$f(x) = \frac{(x-2)(6+x)}{(x-1)^2}; \quad f(x) = \frac{(6+x+x-2)(x-1)^2 - 2(x-1)(x-2)(x+6)}{(x-1)^4} =$$
$$= \frac{(x-1)(2x^2 + 2x - 4 - 2x^2 - 8x + 24)}{(x-1)^4} = \frac{20 - 6x}{(x-1)^3} = 0;$$

$$x = \frac{10}{3} - \max \frac{3x-10}{(x-1)^3} > 0;$$

убывает: $(1; \frac{3}{10}]$; возрастает: $(-\infty; 1) \cup [\frac{3}{10}; +\infty)$.

C-35

1.



$$h(x) = -6x^2 + x + 1; x_{\text{в}} = \frac{1}{12} = x_{\text{max}};$$

$$h_{\text{в}} = -\frac{1}{24} + \frac{1}{12} + 1 = 1\frac{1}{24}; \quad x \in R, y \leq 1\frac{1}{24};$$

$$\text{возрастает: } x \leq \frac{1}{12}; \quad \text{убывает: } x \geq \frac{1}{12};$$

$$\text{нули: } x_1 = \frac{-1-5}{-12} = \frac{1}{2} \text{ и } x_2 = -\frac{1}{3}.$$

2.

$$5x^2 + 8x - 4 \geq 0 \quad (x+2)\left(x - \frac{2}{5}\right) \geq 0;$$

$$x \in (-\infty; -2] \cup \left[\frac{2}{5}; +\infty\right).$$

3.

$$x^3 + 3x^2 + 3x + 1 > 0; (x+1)(x^2 - x + 1 + 3x) > 0;$$

$$(x+1)(x+1)^2 > 0; \quad (x+1)^3 > 0.$$

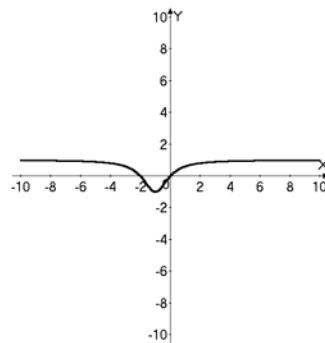
С-36

$$f(x) = \frac{x^2 + 2x}{x^2 + 2x + 2} = 1 - \frac{2}{x^2 + 2x + 2};$$

$$f'(x) = \frac{4x + 4}{(x^2 + 2x + 2)^2} = 0 \text{ при}$$

$x = -1$; $f(x)$ возрастает при $x > -1$;
убывает при $x < -1$;

$$x_{\min} = -1; \quad -f(-1) = \frac{-1}{1} = -1.$$



С-37.

$$1. \quad f(x) = \frac{2x}{x^2 + 1}; \quad f'(x) = \frac{2x^2 + 1 - 4x^2}{(x^2 + 1)^2} = \frac{1 - 2x^2}{(x^2 + 1)^2} = 0 \text{ при}$$

$$x = \pm \frac{1}{\sqrt{2}}; \quad x_{\max} = \frac{1}{\sqrt{2}}; \quad -f\left(\frac{1}{\sqrt{2}}\right) = \frac{2}{3} - \text{наибольшее значение};$$

$$x_{\min} = -\frac{1}{\sqrt{2}}; \quad -f\left(-\frac{1}{\sqrt{2}}\right) = \frac{-2}{3} - \text{наименьшее значение}.$$

2.

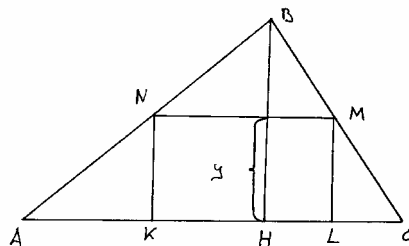
Пусть $LK = x$ $LM = y$;

тогда из подобия:

$$\frac{3-y}{3} = \frac{x}{4} \Rightarrow x = 4 - \frac{4y}{3};$$

$$S = xy = 4y - \frac{4y^2}{3};$$

$$S' = 4 - \frac{8}{3}y = 0 \text{ при } y = \frac{3}{2}$$



$X = 2; S = 3 \text{ M}^2. \text{C-38}$

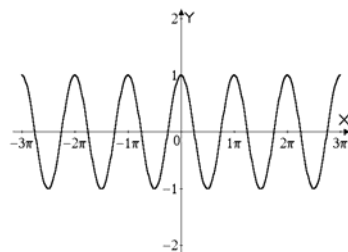
$$\begin{aligned}
 1. \quad & \sin \alpha = \frac{3}{5}, \cos \beta = \frac{4}{5}, \operatorname{tg} \gamma = \frac{3}{4}; \\
 & 0 < \alpha < \frac{\pi}{2}, 0 < \beta < \frac{\pi}{2}, \pi < \gamma < \frac{3\pi}{2}; \\
 & \cos \alpha = \frac{4}{5}, \sin \beta = \frac{3}{5}, \sin^2 \gamma = \frac{9}{16} - \frac{9}{16} \sin^2 \gamma; \sin \gamma = -\frac{3}{5}, \cos \gamma = -\frac{4}{5}; \\
 & \sin(\alpha + \beta + \gamma) = \sin \alpha \cos \beta \cos \gamma + \cos \alpha \sin \beta \cos \gamma + \\
 & + \cos \alpha \cos \beta \sin \gamma - \sin \alpha \sin \beta \sin \gamma = \\
 & = -\frac{3}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} - \frac{4}{5} \cdot \frac{3}{5} \cdot \frac{4}{5} - \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{3}{5} + \frac{3}{5} \cdot \frac{3}{5} \cdot \frac{3}{5} = -\frac{16}{25} \cdot \left(\frac{9}{5}\right) + \frac{27}{125} = -\frac{117}{125}.
 \end{aligned}$$

$$\begin{aligned}
 2. \quad & \frac{\sin^2 2\alpha + 4\sin^4 \alpha - 4\sin^2 \alpha \cos^2 \alpha}{4 - \sin^2 2\alpha - 4\sin^2 \alpha} = \operatorname{tg}^4 \alpha; \\
 & \frac{4\sin^4 \alpha}{4\cos^2 \alpha - \sin^2 2\alpha} = \frac{4\sin^4 \alpha}{4\cos^2 \alpha(1 - \sin^2 \alpha)} = \operatorname{tg}^4 \alpha.
 \end{aligned}$$

$$\begin{aligned}
 3. \quad & \text{a) } \frac{\operatorname{tg} 7^\circ + \operatorname{tg} 68^\circ}{1 - \operatorname{tg} 7^\circ \operatorname{tg} 68^\circ} = \operatorname{tg} 75^\circ = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}; \\
 & \text{б) } \cos 16^\circ \cos 59^\circ - \sin 16^\circ \sin 59^\circ = \cos 75^\circ = \\
 & = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}.
 \end{aligned}$$

C-39

$$\begin{aligned}
 \text{a) } \quad & \text{см.рис;} \\
 & f(x) = \sin\left(2x + \frac{\pi}{2}\right) = 0 \text{ при} \\
 & x = -\frac{\pi}{4} + \frac{\pi n}{2} \text{ -нули;} \\
 & x \in R, f(x) \in [-1; 1];
 \end{aligned}$$



убывает при $x \in \left(\pi n; \frac{\pi}{2} + \pi n \right)$; возрастает при $x \in \left(-\frac{\pi}{2} + \pi n; \pi n \right)$;

$$x_{\max} = \pi n, \quad x_{\min} = -\frac{\pi}{2} + \pi n;$$

б)

$$f(x) = \cos\left(\frac{x}{2} - \frac{\pi}{8}\right);$$

см.рис.

$$x = \frac{5\pi}{4} + 2\pi n \text{ - нули;}$$

$$x \in R, \quad f(x) \in [-1; 1];$$

возрастает при

$$x \in \left(-\frac{7\pi}{4} + 4\pi n; \frac{\pi}{4} + 4\pi n \right);$$

$$\text{убывает при } x \in \left(\frac{\pi}{4} + 4\pi n; \frac{9\pi}{4} + 4\pi n \right)$$

$$x_{\max} = \frac{\pi}{4} + 4\pi n; \quad x_{\min} = -\frac{7\pi}{4} + 4\pi n;$$

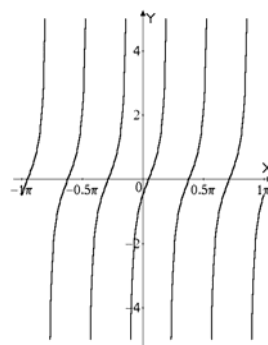
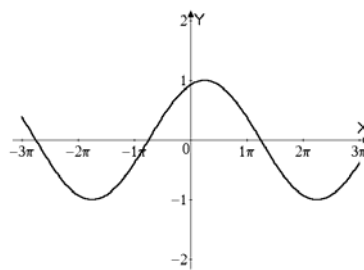
в) см.рис;

$$f(x) = \operatorname{tg}\left(3x - \frac{\pi}{7}\right);$$

$$x = \frac{\pi}{21} + \pi n \text{ - нули;}$$

$$x \neq \frac{3\pi}{14} + \frac{\pi n}{3}, \quad f(x) \in R;$$

возрастает на обл. опр; экстремумов нет.



С-40

$$1. \quad \text{а) } \sin\left(2 \arcsin \frac{3}{5}\right) = 2 \sin\left(\arcsin \frac{3}{5}\right) \cos\left(\arccos \frac{4}{5}\right) = \frac{24}{25};$$

$$\text{б) } \operatorname{arctg} \sqrt{2} + \operatorname{arctg} \frac{1}{\sqrt{2}} = A, \quad \frac{\sqrt{2} + \frac{1}{\sqrt{2}}}{1 - 1} = \operatorname{tg} A \Rightarrow A = \frac{\pi}{2}.$$

2. а) $\cos x \cos 2x \cos 4x = \frac{1}{8}$;
 $\sin x \neq 0$, $x \neq \pi L$;
 $\frac{8 \sin x \cos x \cos 2x \cos 4x}{\sin x} = 1$; $\sin 8x = \sin x$;
 $\sin \frac{7x}{2} \cos \frac{9x}{2} = 0$; $x = \frac{2\pi n}{7}$, $x = \frac{\pi}{9} + \frac{2\pi k}{9}$; $n \neq 7\pi Z$; $k \neq 9p + 4$;
 б) $\cos^2 2x + \cos^2 4x - \sin^2 6x - \sin^2 8x = 0$;
 $\cos 4x + \cos 8x + \cos 12x + \cos 16x = 0$;
 $\cos 10x \cos 6x + \cos 10x \cos 2x = 0$; $\cos 10x \cos 4x \cos 2x = 0$;
 $x = \frac{\pi}{20} + \frac{\pi n}{10}$, $x = \frac{\pi}{8} + \frac{\pi n}{4}$, $x = \frac{\pi}{4} + \frac{\pi n}{2}$.

3. а) $\sin x < \cos x$;
 $\sin \left(x - \frac{\pi}{4} \right) < 0$; $x \in \left(-\frac{3\pi}{4} + 2\pi n ; \frac{\pi}{4} + 2\pi n \right)$;
 б) $\sin x \left(\cos x + \frac{1}{2} \right) \leq 0$; $\sin x \left(\cos x + \frac{1}{2} \right) = 0$ при $x = \pi n$;
 $x = \pm \frac{2\pi}{3} + 2\pi k$, значит $x \in \left[\frac{2\pi}{3} + 2\pi n ; \pi + 2\pi n \right] \cup \left[\frac{-2\pi}{3} + 2\pi n ; 2\pi + 2\pi n \right]$.

C-41

$$\begin{cases} 2 \cos x = 3 \operatorname{tg} y \\ 2 \cos y = 3 \operatorname{tg} z \\ 2 \cos z = 3 \operatorname{tg} x \end{cases} \begin{cases} 4 \cos^2 x = 9 \operatorname{tg}^2 y \\ 4 \cos^2 y = 9 \operatorname{tg}^2 z \\ 4 \cos^2 z = 9 \operatorname{tg}^2 x \end{cases}$$

Пусть $\cos^2 x = a$, $\cos^2 y = b$, $\cos^2 z = c$;

$$\begin{cases} 4a = 9 \cdot \frac{1-b}{b} \\ 4b = 9 \cdot \frac{1-c}{c} \\ 4c = 9 \cdot \frac{1-a}{a} \end{cases} ; a = \frac{9}{4} \cdot \frac{1-b}{b} ; c = \frac{9}{4b+9} ; \frac{4 \cdot 9}{4b+9} = 9 \cdot \frac{1 - \frac{9-9b}{4b}}{\frac{9-9b}{4b}} ;$$

$$36(1-b) = 52b^2 - 36b + 117b - 81 ; 52b^2 + 117b - 117 = 0 ;$$

$$b_1 = \frac{3}{4}; b_2 = 3, \text{ постор. корень, т.к. } \cos^2 \gamma \leq 1;$$

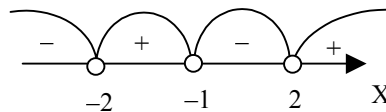
$$a = \frac{9}{4} \cdot \frac{1}{4} \cdot \frac{4}{3} = \frac{3}{4}; c = \frac{9}{4 \cdot \frac{3}{4} + 9} = \frac{9}{12} = \frac{3}{4};$$

$$\begin{cases} \cos x = \pm \frac{\sqrt{3}}{2} \\ \cos y = \pm \frac{\sqrt{3}}{2}; \\ \cos z = \pm \frac{\sqrt{3}}{2} \end{cases}$$

$$\begin{aligned} & \left(\frac{\pi}{6} + 2\pi k; \frac{\pi}{6} + 2\pi k; \frac{\pi}{6} + 2\pi k \right); \left(\frac{5\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n \right); \\ & \left(\frac{\pi}{6} + 2\pi n; \frac{7\pi}{6} + 2\pi n; -\frac{\pi}{6} + 2\pi n \right); \left(-\frac{\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n; \frac{7\pi}{6} + 2\pi n \right); \\ & \left(\frac{7\pi}{6} + 2\pi n; -\frac{\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n \right); \left(\frac{5\pi}{6} + 2\pi n; -\frac{\pi}{6} + 2\pi n; \frac{7\pi}{6} + 2\pi n \right); \\ & \left(\frac{7\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n; -\frac{\pi}{6} + 2\pi n \right); \left(-\frac{\pi}{6} + 2\pi n; \frac{7\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n \right). \end{aligned}$$

C-42

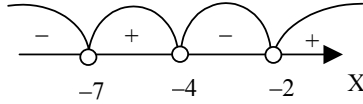
1. а) $x^2 - 12|x| + 32 \geq 0; |x| = y \geq 0; y^2 - 12y + 32 \geq 0;$
 $y \in [0; 4] \cup [8; +\infty); x \in (-\infty; -8] \cup [-4; 4] \cup [8; +\infty);$
 б) $1 + \frac{12}{x^2} - \frac{7}{x} < 0$, ОДЗ: $x \neq 0; (x^2 - 7x + 12) < 0; x \in (3; 4).$
2. а) $\frac{4}{x+2} > 3 - x; \frac{4 + x^2 - x - 6}{x+2} > 0; \frac{(x-2)(x+1)}{(x+2)} > 0;$
 $x \in (-2; -1) \cup (2; +\infty);$



$$6) \frac{(x-2)(x-4)(x-7)}{(x+2)(x+4)(x+7)} > 1;$$

$$\frac{(x-2)(x^2-11x+28)-(x+2)(x^2+11x+28)}{(x+2)(x+4)(x+7)} > 0;$$

$$\frac{-26x^2-112}{(x+2)(x+4)(x+7)} > 0; \quad \frac{13x^2+56}{(x+2)(x+4)(x+7)} < 0;$$



$$x \in (-\infty; -7) \cup (-4; -2).$$

C-43

$$1. \quad a) y = \frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3}; \quad y' = -\frac{1}{x^2} - \frac{4}{x^3} - \frac{9}{x^4}$$

$$6) y = \frac{\sqrt{x^2+1}}{x} = \sqrt{1 + \frac{1}{x^2}}; \quad y' = \frac{-\frac{2}{x^3}}{2\sqrt{1 + \frac{1}{x^2}}} = -\frac{1}{x^3 \sqrt{1 + \frac{1}{x^2}}}.$$

$$b) y = (2 - x^2) \cos x + 2x \sin x;$$

$$y' = (x^2 - 2) \sin x - 2x \cos x + 2 \sin x + 2x \cos x = \\ = (x^2 - 2) \sin x + 2 \sin x = x^2 \sin x;$$

$$r) y = (x^3 - x^2)^{66}; \quad y' = 66(3x^2 - 2x)(x^3 - x^2)^{65}.$$

$$2. \quad f(x) = x^3 + x - \sqrt{2}; \quad f'(x) = 3x^2 + 1; \quad g(x) = 3x^2 + x - \sqrt{3}; \\ g'(x) = 6x + 1; \quad f'(x) - g'(x) = 3x^2 - 6x > 0; \quad x \in (-\infty; 0) \cup (2; +\infty).$$

C-44

$$1. \quad f(x) = -x^2 - 2x; \quad f'(x) = -2x - 2;$$

$$y_{кас} = -x_0^2 - 2x_0 - 2(x_0 + 1)(x - x_0) = -2(x_0 + 1)x + x_0^2;$$

$$1 = -2(x_0 + 1)x + x_0^2; \quad x_0^2 - 2x_0 - 3 = 0; \quad x_0 = 3, \quad x_0 = -1;$$

$$y_1 = -8x + 9, \quad y_2 = 1 - \text{уравнения касательных.}$$

$$2. \quad a) \left(\sqrt{4,000008} - \sqrt{0,999996} \right)^{100} \approx (1,000002 + 0,000002)^{100} \approx \\ \approx 1 + 0,0004 = 1,0004 ;$$

$$6) \sin 32^0 = \sin 30^0 \cos 2^0 + \cos 30^0 \sin 2^0 \approx \frac{1}{2} \cdot 0,9994 + 0,866 \cdot 0,0349 = \\ = 0,0302 + 0,4997 = 0,5299 .$$

$$3. \quad S(t) = \frac{t}{t^2 + 4} ; V(t) = \frac{t^2 + 4 - 2t^2}{(t^2 + 4)^2} = \frac{4 - t^2}{(t^2 + 4)^2} = 0 \text{ при}$$

$$t = \pm 2, \text{ но } t \geq 0 \Rightarrow t = 2;$$

$$a(t) = - \frac{2t(t^2 + 4)^2 + 4t(t^2 + 4)(4 - t^2)}{(t^2 + 4)^4};$$

$$a(2) = - \frac{4 \cdot 64 + 8(8)(0)}{8^4} = - \frac{4}{64} = - \frac{1}{16};$$

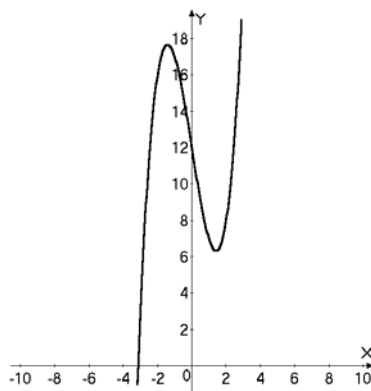
$$F = - \frac{3}{16} \text{ Н.}$$

C-45

1.

$$x^3 - 6x + 12 = 0 ; x^3 = 6x - 12 ;$$

см.рис. 1 корень.



2. Пусть основание a , а сторона – e ;

$$H = \sqrt{b^2 - \frac{a^2}{4}}, S = \frac{1}{2}a\sqrt{b^2 - \frac{a^2}{4}}; a + 2b = P;$$

$$S^2 = \frac{1}{4}a^2b^2 - \frac{a^4}{16}; b = \sqrt{\frac{4S^2}{a^2} + \frac{a^2}{4}}; a + 2\sqrt{\frac{4S^2}{a^2} + \frac{a^2}{4}} = P;$$

$$P' = 1 + \frac{-S^2 8/a^3 + a/2}{\sqrt{4\frac{S^2}{a^2} + \frac{a^2}{4}}} = 0; -\frac{a}{2} + \frac{8S^2}{a^3} = \sqrt{\frac{4S^2}{a^2} + \frac{a^2}{4}};$$

$$\frac{64S^4}{a^6} + \frac{a^2}{4} - \frac{8S^2}{a^2} = \frac{4S^2}{a^2} + \frac{a^2}{4}; \frac{64S^4}{a^6} - \frac{2S^2}{a^2} = 0;$$

$$64S^4 - 2S^2a^4 = 0; a^4 = 32S^2; a = 2\sqrt{2}\sqrt{S};$$

$$e = \sqrt{\frac{4S^2}{8S} + \frac{8S}{4}} = \sqrt{\frac{S}{2} + 2S} = \sqrt{\frac{5S}{2}}.$$

ВАРИАНТ 10

С-1

1. $\alpha = 36^0, 360^0 - 2\alpha = 2\beta, \beta = 144^0$;

$$36^0 = \frac{\pi}{180} \cdot 36 = \frac{\pi}{5}; 144^0 = \frac{\pi}{180} \cdot 144 = \frac{4\pi}{5}.$$

2. $360^\circ - 60 \text{ мин.}; x^\circ - 24 \text{ мин.}$

а) $x = 144^0$; б) $x = 360^0 - 144^0 = 216^0$; $216^0 + 360 \cdot 11 = 4176$.

3. $8x + 13x + 23x + 28x = 360^0$; $x = 5$; $8x = 40^0 = \frac{\pi}{180} \cdot 40 = \frac{2\pi}{9}$.

4. $\begin{cases} x + y = 4 \\ x = y^2 \end{cases}; y^2 + y - 4 = 0; D=17; y_{12} = (-1 \pm \sqrt{17}) \cdot \frac{1}{2}, \text{ но}$

$$y > 0 \Rightarrow y = \frac{-1 + \sqrt{17}}{2}; y = 89^0 28', x = 139^0 43'.$$

C-2

$$1. \sqrt{\frac{1+\sin \alpha}{1-\sin \alpha}} - \sqrt{\frac{1-\sin \alpha}{1+\sin \alpha}} = -2\operatorname{tg} \alpha ; \frac{1+\sin \alpha - 1 + \sin \alpha}{-\sqrt{1-\sin^2 \alpha}} = \frac{2\sin \alpha}{-\cos \alpha} = -2\operatorname{tg} \alpha .$$

$$2. \quad \text{a) } \frac{\cos 1100^0 \sin 2200^0}{\operatorname{tg} 2980^0} < 0 ;$$

$$\text{б) } \sin 6 \operatorname{tg} 8 \cos 10 < 0 .$$

$$3. \quad \frac{\frac{2}{\operatorname{tg} \alpha + \operatorname{ctg} \alpha} + \operatorname{tg} \alpha \operatorname{ctg} \alpha}{(\sin \alpha + \cos \alpha)^2} - \sin \alpha = \frac{2\sin \alpha \cos \alpha + 1}{1 + 2\sin \alpha \cos \alpha} - \sin \alpha =$$

$$= 1 - \sin \alpha$$

$$\operatorname{tg} \alpha = 2, \sin \alpha < 0; \sin^2 \alpha = 4 - 4\sin^2 \alpha; \sin \alpha = -\frac{2}{\sqrt{5}}$$

$$1 - \sin \alpha = \frac{\sqrt{5} + 2}{\sqrt{5}} .$$

C-3

1.

$$\operatorname{ctg} 13^0 \cdot \operatorname{ctg} 17^0 \cdot \operatorname{ctg} 21^0 \dots \operatorname{ctg} 77^0 = \operatorname{tg} 13^0 \cdot \operatorname{ctg} 13^0 \cdot \operatorname{ctg} 17^0 \cdot \operatorname{tg} 17^0 \dots \operatorname{ctg} 45^0 = 1 .$$

$$2. \quad \frac{\cos\left(2\alpha - \frac{\pi}{2}\right) + \sin(3\pi - 4\alpha) - \cos\left(\frac{5\pi}{2} + 6\alpha\right)}{4\sin(5\pi - 3\alpha)\cos(\alpha - 2\pi)} =$$

$$= \frac{\sin 2\alpha + \sin 4\alpha + \sin 6\alpha}{4\sin 3\alpha \cos \alpha} =$$

$$= \frac{2\sin 3\alpha(\cos \alpha + \cos 3\alpha)}{4\sin 3\alpha \cos \alpha} = \frac{2\cos 2\alpha \cos \alpha}{2\cos \alpha} = \cos 2\alpha .$$

$$3. \quad \cos\left(4t + \frac{\pi}{2}\right)\cos(t - \pi) - \cos\left(\frac{3\pi}{2} + 3t\right) = \sin\left(\frac{3\pi}{2} - 4t\right)\sin(t + \pi);$$

$$\sin 4t \cos t - \sin 3t = \cos 4t \sin t ;$$

$$\frac{1}{2}(\sin 5t + \sin 3t) - \sin 3t = \frac{1}{2}(\sin 5t - \sin 3t);$$

C-4

1.
$$\cos \frac{\pi}{7} \cos \frac{2\pi}{7} \cos \frac{4\pi}{7} = \frac{\sin \frac{8\pi}{7}}{8 \sin \frac{\pi}{7}} = -\frac{\sin \frac{\pi}{7}}{8 \sin \frac{\pi}{7}} = -\frac{1}{8}.$$
2.
$$\operatorname{tg} \alpha = 3, \sin^2 \alpha = 9 - 9 \sin^2 \alpha; \sin \alpha = \pm \frac{3}{\sqrt{10}}, \cos \alpha = \pm \frac{1}{\sqrt{10}};$$

$$\sin 2\alpha = \frac{6}{10}, \cos 2\alpha = -\frac{8}{10} = -\frac{4}{5};$$

$$\frac{3 \sin 2\alpha - 4 \cos 2\alpha}{5 \cos 2\alpha - \sin 2\alpha} = \left(\frac{9}{5} + \frac{4}{5} \right) : \left(-4 - \frac{3}{5} \right) = -\frac{13}{5} \cdot \frac{5}{23} = -\frac{13}{23}.$$
3.
$$\frac{(1 + \operatorname{tg} 2\alpha)^2 - 2 \operatorname{tg}^2 2\alpha}{1 + \operatorname{tg}^2 2\alpha} - \sin 4\alpha - 1 =$$

$$= \frac{(1 - \operatorname{tg}^2 2\alpha + 2 \operatorname{tg} 2\alpha) \cos^2 2\alpha}{1 + \operatorname{tg}^2 2\alpha} - \sin 4\alpha - 1 =$$

$$= \cos^2 2\alpha - \sin^2 2\alpha + \sin 4\alpha - \sin 4\alpha - 1 = \cos 4\alpha - 1.$$

C-5

1. см. рис.
 $(\cos t + \sin t)(1 + \cos t - \sin t) = 0;$

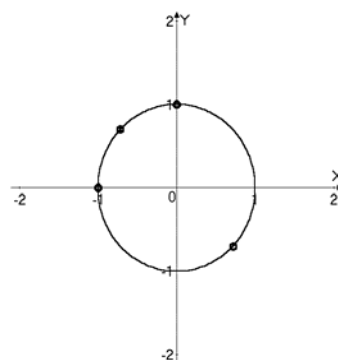
$$t = -\frac{\pi}{4} + \pi n$$

$$\sin \left(t - \frac{\pi}{4} \right) = \frac{\sqrt{2}}{2},$$

$$t = -\frac{\pi}{4} + \pi n + (-1)^k \frac{\pi}{4}$$

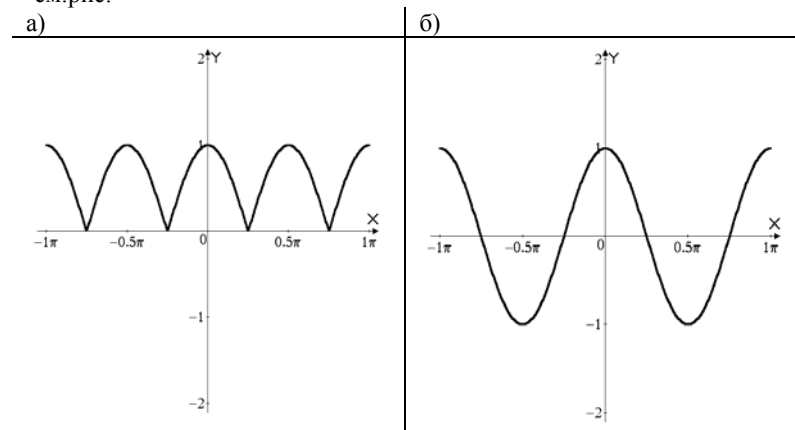
$$t \in [0; 2\pi], t = \frac{3\pi}{4}, t = \frac{\pi}{2}, t = \pi,$$

$$t = \frac{7\pi}{4}.$$



2. $\cos(\sin 1) > \sin(\cos 1), x \in \left[0; \frac{\pi}{2}\right]; \sin(\cos 1) < \cos 1;$
 $\sin 1 < 1 \Rightarrow \cos(\sin 1) > \cos 1 > \sin(\cos 1).$

3.
см.рис.



C-6

1. а) $f(x) = \frac{\frac{1}{2}x^2 - \frac{1}{3}x + 2x^3}{x(4 - \sqrt{x-1})};$

ОДЗ: $\begin{cases} x \neq 0 \\ 4 \neq \sqrt{x-1}; x \geq 1, x \neq 17, \text{ значит, } x \in [1; 17) \cup (17; \infty); \\ x \geq 1 \end{cases}$

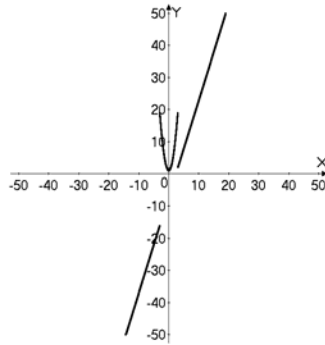
б) $f(x) = \sqrt{3 - 4\sqrt{x}}$

ОДЗ: $\begin{cases} x \geq 0 \\ 3 \geq 4\sqrt{x}; x \in \left[0; \frac{9}{16}\right]. \end{cases}$

2. $f(x) = \begin{cases} 2x^2 + 1, & |x| < 3 \\ 3x - 7, & |x| \geq 3 \end{cases};$

а) $f(-3) = -16; f(2) = 9, f(5) = 8; f(x^2 + 4) = 3x^2 + 5;$

б) см.рис.



C-7

1. а) да; б) нет.

$$2. \quad \frac{f(x) + f(-x)}{2} - \text{четная}; \quad \frac{f(x) - f(-x)}{2} - \text{нечетная};$$

$$\Rightarrow \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2} = f(x);$$

единственность: пусть $f(x) = g_1(x) + \varphi_1(x) = g_2(x) + \varphi_2(x)$;

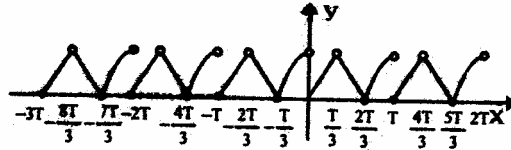
где $g_i(x)$ - четная, $\varphi_i(x)$ - нечетная $i = 1, 2 \Rightarrow$;

$g_1(x) - g_2(x) = \varphi_2(x) - \varphi_1(x)$, а это возможно только при

$g_1(x) = g_2(x)$ и $\varphi_2(x) = \varphi_1(x)$.

C-8

1.



$$2. \quad \text{а) } f(x) = |\cos x| + \operatorname{ctg} \frac{x}{3}; \quad f_1(x) = |\cos x|, \quad T_1 = \pi;$$

$$f_2(x) = \operatorname{ctg} \frac{x}{3}, \quad T_2 = 3\pi, \quad \text{значит период } f(x): T = 3\pi.$$

$$\text{б) } f(x) \sin\left(\sqrt{3}x - \frac{\pi}{9}\right); T = \frac{2\pi\sqrt{3}}{3}.$$

$$3. \quad \text{а) } f(x) = \sin \sqrt{|x|}; \text{ Пусть } T - \text{ период; } \Rightarrow f(x) = f(x + T)$$

$$\sin \sqrt{|x|} = \sin \sqrt{|x + T|}, \text{ чего очевидно не может быть}$$

(легко видеть при $-T < x < 0$), значит, $f(x)$ не периодична;

$$\text{б) } f(x) = \cos x + \cos \sqrt{2}x; f_1(x) = \cos x, T_1 = 2\pi$$

$f_2(x) = \cos \sqrt{2}x, T_2 = 2\sqrt{2}\pi$; не существует $n \in \mathbb{N}$ $T_1 = 2\sqrt{2}\pi n$,
значит, $f(x)$ не периодична.

C-9

$$1. \quad \text{а) } f(x) = \begin{cases} x^2 + 4x, & x \leq 0 \\ x^2 - 2x, & x > 0 \end{cases}$$

$f(x)$ возрастает при $x \in (-2; 0) \cup (1; +\infty)$; убывает при $x \in (-\infty; -2] \cup [0; 1]$;

$$\text{б) } f(x) = \frac{2x}{1+x^2}; f'(x) = \frac{-2x^2+2}{(x^2+1)^2}$$

$f(x)$ возрастает при $x \in [-1; 1]$; убывает при $x < -1, x > 1$.

$$2. \quad \text{а) } f(x) = 3x, g(x) = 2x; \text{ б) } f(x) = 2x, g(x) = 3x;$$

$$\text{в) } f(x) = 4x + |x|; g(x) = 4x; \text{ г) } f(x) = 4x + \sin x; g(x) = 4x.$$

$$3. \quad \sin 2, \cos 2, \operatorname{tg} 2, \operatorname{ctg} 3; \quad \text{Ответ: } \sin 2, \cos 2, \operatorname{tg} 2, \operatorname{ctg} 3.$$

C-10

1.

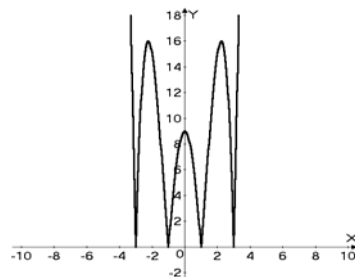
см. рис.

$$f(x) = |x^4 - 10x^2 + 9|;$$

$$x_{\min} = \pm 1$$

$$x_{\max} = \pm \sqrt{5};$$

$$x_{\max} = 0; x_{\min} = \pm 3.$$



2. а) $f(x) = 2 \cos|x-1|$; $f(x) = 2$ при $|x-1| = 2\pi n + 2\pi$; $x = 1 \pm 2\pi n$,
 $n \in \mathbb{N}$; $x_{\max} = 1 \pm 2\pi n$; $x_{\min} = 1 + \pi + 2\pi n$; $x_{\min} = 1 - \pi + 2\pi n$.

$$\text{б) } f(x) = \sin 3x + \sin 2x - \frac{2 \operatorname{tg} x}{1 + \operatorname{tg}^2 x} = \sin 3x ;$$

$$x_{\max} = \frac{\pi}{6} + \frac{2\pi n}{3}; \quad x_{\min} = -\frac{\pi}{6} + \frac{2\pi n}{3}.$$

С-11

см.рис.

$$y = x^4 - 2x^2 ;$$

$$\text{нули: } x = 0, \quad x = \pm\sqrt{2} ;$$

$$y' = 4x(x^2 - 1) = 0 \text{ при}$$

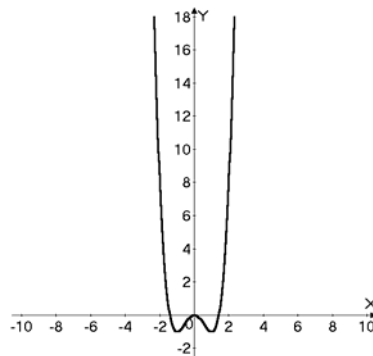
$$x_{\max} = 0, \quad x_{\min} = \pm 1$$

$$y(\pm 1) = -1 ,$$

$$y(0) = 0$$

y убывает при $x < -1$, $x \in [0; 1]$;

возрастает при $x \in [-1; 0] \cup (1; \infty)$



С-12

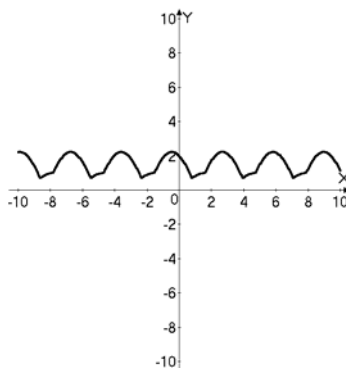
1.

$$f(x) = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x} + \frac{1 - \operatorname{tg}^2 x}{2 \operatorname{tg} x} = \operatorname{tg} x + \operatorname{ctg} x = \operatorname{tg} x + \frac{1}{\operatorname{tg} x} ;$$

$$\text{ОДЗ: } \begin{cases} \operatorname{tg} x \neq \pm 1 \\ \operatorname{tg} x \neq 0 \\ \cos x \neq 0 \end{cases} ; \begin{cases} x \neq \pm \frac{\pi}{4} + \pi n \\ x \neq \pi n \\ x \neq \frac{\pi}{2} + \pi n \end{cases} , \text{ значит, } x \in \mathbb{R} , \text{ кроме } \frac{\pi}{4} ;$$

$$f(x) \in (-\infty; 2] \cup [2; \infty) ,$$

2. см.рис



3.

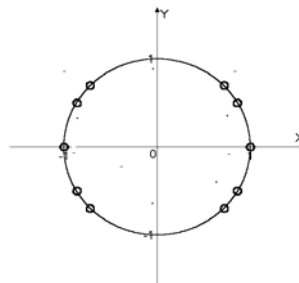
см.рис.

$$\frac{\operatorname{ctg}^2 t - 5}{\operatorname{ctg}^2 t - 3} < 2$$

$$\text{ОДЗ: } \begin{cases} \operatorname{ctg} t \neq \pm \sqrt{3} \\ \sin t \neq 0 \end{cases}, \begin{cases} t \neq \pm \frac{\pi}{6} + \pi n \\ t \neq \pi n \end{cases}$$

$$\frac{-\operatorname{ctg}^2 t + 1}{\operatorname{ctg}^2 t - 3} < 0; \quad \frac{\operatorname{ctg}^2 t - 1}{\operatorname{ctg}^2 t - 3} > 0;$$

$$t \in \left(-\frac{3\pi}{4} + \pi n; -\frac{\pi}{4} + \pi n\right) \cup \left(-\frac{\pi}{6} + \pi n; \pi n\right) \cup \left(\pi n; \frac{\pi}{6} + \pi n\right).$$



C-13

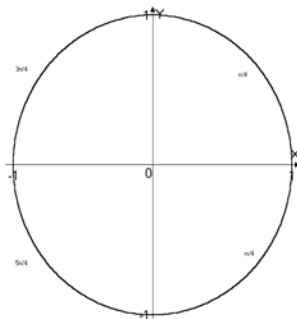
- 1)
 - а) $\cos(\arcsin(-0,96)) = \sqrt{1 - 0,96^2} = 0,28$;
 - б) $\arccos(\cos 10) = 4\pi - 10$.
- 2) $\operatorname{arctg} x + \operatorname{arcctg} x = \frac{\pi}{2}$; $x = \operatorname{ctg}(\operatorname{arcctg} x)$.
- 3)
 - а) $\sin(7 \arcsin(0,1235)) \approx 0,7622$;
 - б) $\cos(4 \arccos 0,12 + \arcsin 0,3375) \approx 0,9906$.

C-14

- а) $4 \sin x \cos x = -\sqrt{3}$; $\sin 2x = -\frac{\sqrt{3}}{2}$; $x = (-1)^{k+1} \frac{\pi}{6} + \frac{\pi k}{2}$;
 б) $\frac{\operatorname{tg} 5x - \operatorname{tg} 2x}{1 + \operatorname{tg} 2x \operatorname{tg} 5x} = -1$; $\operatorname{tg} 3x = -1$; $x = -\frac{\pi}{12} + \frac{\pi k}{3}$;
 в) $\left| \sin \left(9x + \frac{\pi}{7} \right) \right| = \frac{1}{\sqrt{2}}$; $9x = \frac{3\pi}{28} + \frac{\pi n}{2}$; $x = \frac{3\pi}{252} + \frac{\pi n}{18}$; $x = \frac{\pi}{84} + \frac{\pi n}{18}$.

C-15

см.рис



$$\operatorname{tg} t \cos 2t \geq 0;$$

$$t \in \left(-\frac{\pi}{2} + 2\pi n; -\frac{\pi}{4} + 2\pi n \right) \cup \left[\frac{\pi}{4} + 2\pi n; \frac{\pi}{2} + 2\pi n \right) \cup \left[\frac{3\pi}{4} + 2\pi n; \frac{5\pi}{4} + 2\pi n \right).$$

C-16

- а) $\frac{\operatorname{tg} 3x - \operatorname{tg} \left(x - \frac{2\pi}{7} \right)}{1 + \operatorname{tg} 3x \operatorname{tg} \left(x - \frac{2\pi}{7} \right)} > \sqrt{3}$; $\operatorname{tg} \left(2x + \frac{2\pi}{7} \right) > \sqrt{3}$;
 $2x \in \left(\frac{\pi}{21} + \pi n; \frac{3\pi}{14} + \pi n \right)$; $x \in \left(\frac{\pi}{42} + \frac{\pi n}{2}; \frac{3\pi}{28} + \frac{\pi n}{2} \right)$;
 б) $\cos^2 x \leq \frac{1}{2}$; $\cos x \in \left[-\frac{\sqrt{2}}{2}; \frac{\sqrt{2}}{2} \right]$; $x \in \left[\frac{\pi}{4} + \pi n; \frac{3\pi}{4} + \pi n \right]$.

C-17

$$\text{a)} \quad 2 \cos^2 \left(x + \frac{\pi}{6} \right) + 3 \sin \left(\frac{\pi}{3} - x \right) + 1 = 0 ;$$

$$2 \cos^2 \left(x + \frac{\pi}{6} \right) + 3 \cos \left(\frac{\pi}{6} + x \right) + 1 = 0 ;$$

$$\cos \left(x + \frac{\pi}{6} \right) = -1 \quad \cos \left(x + \frac{\pi}{6} \right) = -\frac{1}{2} ;$$

$$x = \frac{5\pi}{6} + 2\pi n ; \quad x = \pm \frac{2\pi}{3} + 2\pi n - \frac{\pi}{6} ;$$

$$\text{б)} \quad \sin 2x - \sin 3x = 0 ;$$

$$\sin \frac{x}{2} \cos \frac{5x}{2} = 0 ; \quad x = 2\pi n ; \quad x = \frac{\pi}{5} + \frac{2\pi n}{5} .$$

C-18

$$\text{a)} \quad 3 \sin \left(x - \frac{\pi}{4} \right) = 2 \cos \left(x + \frac{\pi}{3} \right) ;$$

$$\frac{3\sqrt{2}}{2} \sin x - \frac{3\sqrt{2}}{2} \cos x = \cos x - \sqrt{3} \sin x ;$$

$$\frac{3\sqrt{2} - 2\sqrt{3}}{2} \sin x = \frac{3\sqrt{2} + 2}{2} \cos x ; \quad \operatorname{tg} x = \frac{3\sqrt{2} + 2}{3\sqrt{2} - 2\sqrt{3}} ;$$

$$x = \operatorname{arctg} \frac{3\sqrt{2} + 2}{3\sqrt{2} - 2\sqrt{3}} + \pi n ;$$

$$\text{б)} \quad \cos^2 \left(x + \frac{\pi}{4} \right) - 2 \sin \left(x + \frac{\pi}{4} \right) \cos \left(x + \frac{\pi}{4} \right) - 3 \cos^2 \left(\frac{\pi}{4} - x \right) = 0$$

$$\operatorname{tg}^2 \left(x + \frac{\pi}{4} \right) - 2 \operatorname{tg} \left(x + \frac{\pi}{4} \right) - 3 = 0 \quad \sin^2 \left(x + \frac{\pi}{4} \right) \neq 0$$

$$\operatorname{tg} \left(x + \frac{\pi}{4} \right) = 3 ; \quad \operatorname{tg} \left(x + \frac{\pi}{4} \right) = -1 ; \quad x = \operatorname{arctg} 3 - \frac{\pi}{4} + \pi n ; \quad x = -\frac{\pi}{2} + \pi n .$$

C-19

$$\begin{cases} \cos x \sin y = \frac{1}{2} \\ \sin 2x = -\sin 2y \end{cases}; \begin{cases} \sin(x+y)\cos(x-y) = 0 \\ \cos x \sin y = \frac{1}{2} \end{cases};$$

$$1. \begin{cases} \sin(x+y) + \sin(x-y) = 1 \\ x+y = \pi n \end{cases}; \begin{cases} x = \frac{\pi}{2} + y + \pi k \\ x = \pi n - y \end{cases}; \begin{cases} x = \frac{\pi}{4} + \frac{\pi k}{2} + \frac{\pi n}{2} \\ y = \frac{\pi n}{2} - \frac{\pi}{4} - \frac{\pi k}{2} \end{cases};$$

$$2. \begin{cases} (x-y) = \frac{\pi}{2} + \pi k \\ x+y = \pi n \end{cases} \text{ тоже самое.}$$

C-20

a) $\cos x \cos 2x \cos 4x = \frac{1}{8}$;

$\sin x \neq 0 \quad x \neq \pi k$; $\sin 8x = \sin x$; $x = \frac{2\pi n}{7}$; $n \neq 7L$;

$\sin \frac{7x}{2} \cos \frac{9x}{2}$; $x = \frac{\pi}{9} + \frac{2\pi p}{9}$, но $x \neq \pi k$; $p \neq 9z + 4$;

б) $\sin x + \sin 2x + \sin 3x + \sin 4x + \sin 5x = 0$;

$2 \sin 3x \cos 2x + 2 \sin 3x \cos x + \sin 3x = 0$;

$\sin 3x(2 \cos 2x + 2 \cos x + 1) = 0$;

$x = \frac{\pi n}{3}$; $4 \cos^2 x + 2 \cos x - 1 = 0$; $x = \pm \arccos\left(\frac{-1 \pm \sqrt{5}}{4}\right) + 2\pi n$.

C-21

1) $f(x) = \frac{2}{x}$; $g(x) = \frac{2-x^2}{8}$; $\Delta f(x_0) = \frac{2}{x_0 + \Delta x} - \frac{2}{x_0} = -\frac{\Delta x}{2 + \Delta x}$;

$\Delta g(x_0) = \frac{2 - x_0^2 + 2x_0\Delta x - \Delta x^2}{8} = \frac{-2 + 4\Delta x - \Delta x^2}{8}$

$\Delta g(x_0) = \frac{2 - (x_0 + \Delta x)^2}{8} - \frac{2 - x_0^2}{8} = -\frac{4\Delta x + \Delta x^2}{2}$;

$$\Delta x = 0,1 \quad \Delta f(x_0) \approx -0,048; \quad \Delta g(x_0) \approx -0,051; \quad \Delta f(x_0) > \Delta g(x_0);$$

$$\Delta x = 0,3 \quad \Delta f(x_0) \approx -0,13; \quad \Delta g(x_0) \approx -0,16; \quad \Delta f(x_0) > \Delta g(x_0).$$

2. $f(x) = x^3 + 2x^2 - 5x + 6;$

$$\Delta f(x_0) = \Delta x^3 + 3x_0^2 \Delta x + 3x_0 \Delta x^2 + 2\Delta x^2 + 4x_0 \Delta x - 5\Delta x;$$

$$\frac{\Delta f(x_0)}{\Delta x} = \Delta x^2 + 3x_0^2 + 3x_0 \Delta x + 2\Delta x + 4x_0 - 5;$$

$$\lim_{\Delta x \rightarrow 0} \frac{f(x_0)}{\Delta x} = 3x_0^2 + 4x_0 - 5.$$

C-22

1. $m(t) = 2 + t; \quad x(t) = t^2 - t; \quad V(t) = 2t - 1; \quad Vm = 2t^2 + 3t - 2;$
 $F = 4t + 3; \quad F(1) = 7 \text{ Н.}$

2. а) $f(x) = x^2 - 2\sqrt{x}; \quad f'(x) = 2x - \frac{1}{\sqrt{x}};$

б) $f(x) = \frac{x+2}{x+1} = 1 + \frac{1}{x+1}; \quad f'(x) = -\frac{1}{(x+1)^2}.$

C-23

1.

а) $(-1; 0); (2; \frac{1}{2}).$

б) $\lim_{x \rightarrow -1} f(x)$ не существует; $\lim_{x \rightarrow 2} f(x) = 1;$ в) $y \in (-1; 2,5)$

2.

$$f(x) = \frac{x+4}{\sqrt{x+5}-1} = \sqrt{x+5} + 1,$$

$$x \neq -4;$$

$$|f(x) - 2| = |\sqrt{x+5} - 1| < 0,2;$$

$$x \in (-4,36; -3,56); \quad \delta = 0,36.$$

C-24

1. $\lim_{x \rightarrow 2} f(x) = 3$; $\lim_{x \rightarrow 2} g(x) = -2$;
 - a) $\lim_{x \rightarrow 2} \frac{f(x) + g^2(x)}{4f(x) + 3g(x)} = \frac{3 + 4}{12 - 6} = \frac{7}{6}$;
 - б) $\lim_{x \rightarrow 2} \left(\sqrt{f(x)} + \sqrt{-(g(x))} \right)^2 + \left(\sqrt{f(x)} - \sqrt{-(g(x))} \right)^2 =$
 $= \lim_{x \rightarrow 2} (2f(x) - 2g(x)) = 6 + 4 = 10$.
2.
 - a) $\lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 + x - 12} = \lim_{x \rightarrow 3} \frac{(x-3)(x-2)}{(x+4)(x-3)} = \frac{1}{7}$
 - б) $\lim_{x \rightarrow -3} \left(\frac{x^2 - 9}{\sqrt{x+7} - 2} - 2x^2 \right) = \lim_{x \rightarrow -3} \left(\frac{(x+3)(x-3)(\sqrt{x+7} + 2)}{x+3} - 2x^2 \right) = -42$.

C-25

1.
 - a) $f(x) = \frac{1}{2\sqrt{x}} - \frac{1}{\sqrt{x^5}} + x^{101}$; $f'(x) = -\frac{3}{4x^{\frac{3}{2}}} + \frac{5}{2x^{\frac{7}{2}}} + 101x^{100}$;
 - б) $g(x) = (3x - x^2)\sqrt{x^3}$; $g'(x) = (3 - 2x)\sqrt{x^3} + \frac{3}{2}\sqrt{x}(3x - x^2)$.
2. $f(x) = \begin{cases} x^3, & x \geq 0 \\ -x^3, & x < 0 \end{cases} = x^2|x|$; $f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x} = 0$.

C-26

- a) $f(x) = 2x^3 + 3x^2 - 12x$; $f'(x) = 6(x^2 + x - 2) = 0$;
 при $x = -2$, $x = 1$;
 $f'(x) > 0$ при $x < -2$, $x > 1$; $f'(x) < 0$ при $x \in (-2; 1)$;
- б) $f(x) = \frac{3 - x^2}{x + 2}$; $f'(x) = \frac{-2x^2 - 4x - 3 + x^2}{(x+2)^2} = \frac{-x^2 - 4x - 3}{(x+2)^2}$;
 $f'(x) = 0$ при $x^2 + 4x + 3 = 0$; $x = -3$, $x = -1$;

$$f'(x) > 0 \text{ при } x \in (-3; -2) \cup (-2; -1);$$

$$f'(x) < 0, \quad x \in (-\infty; -3) \cup (-1; \infty).$$

C-27

$$1. \quad \text{а) } f(x) = \frac{1}{\sqrt{1 - \sqrt{x^2 - 4}}}; \text{ ОДЗ: } \begin{cases} x^2 - 4 \geq 0 \\ 1 - \sqrt{x^2 - 4} > 0 \end{cases};$$

$$x \in (-\sqrt{5}; -2] \cup [2; \sqrt{5});$$

$$\text{б) } f(x) = \sqrt{x - \sqrt{x - \sqrt{x}}}; \text{ ОДЗ: } \begin{cases} x \geq 0 \\ x \geq \sqrt{x} \\ x \geq x - \sqrt{x} \end{cases}; \begin{cases} x \geq 0 \\ x \leq -1, x \geq 1, \text{ значит,} \\ x^2 \geq x - \sqrt{x} \end{cases}$$

$$x \in [1; \infty) \cup \{0\}.$$

$$2. \quad f(x) = \frac{1}{1-x}; \quad f(f(x)) = \frac{1}{1 - \frac{1}{1-x}} = \frac{1-x}{-x} = \frac{x-1}{x} = 1 - \frac{1}{x};$$

$$f(f(f(x))) = 1 - 1 + x = x; \quad f(f(f(x))) = \frac{1}{1-x};$$

$$f_n(x) = \frac{1}{1-x}, \quad n = 3p - 2; \quad f_n(x) = x, \quad n = 3p;$$

$$f_n(x) = 1 - \frac{1}{x}, \quad n = 3p - 1;$$

ОДЗ: для $f(x): x \in (-\infty; 1) \cup (1; 0)$, для $f_n(x): x \in (-\infty; 0) \cup (0; 1) \cup (1; \infty)$ при $n \geq 2$.

$$3. \quad \text{а) } f(x) = \sqrt{2x^3 - 3x^2 + 7};$$

$$f'(x) = \frac{3x^2 - 3x}{\sqrt{2x^3 - 3x^2 + 7}};$$

$$\text{б) } f(x) = (x^2 + x\sqrt{7})^7;$$

$$f'(x) = 7(2x + \sqrt{7})(x^2 + x\sqrt{7})^6.$$

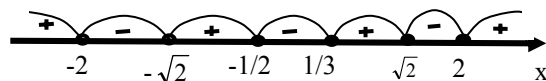
C-28

- a) $f(x) = \cos 3x \cos 2x - \sin 3x \sin 2x = \cos 5x$; $f'(x) = -5 \sin 5x$;
- б) $f(x) = \frac{1 - \operatorname{tg}^2(x+1)}{2 \operatorname{tg}(x+1)} = \operatorname{ctg}(2x+2)$; $f'(x) = \frac{-2}{\sin^2(2x+2)}$;
- в) $f(x) = \frac{1}{2} \cos^4(2x^2 - 3)$; $f'(x) = -2 \cos^3(2x^2 - 3) \sin(2x^2 - 3) (4x)$.

C-29

1. $\lim_{x \rightarrow a} \frac{x^2 - a^2}{\sqrt{x} - \sqrt{a}} = (\sqrt{x} + \sqrt{a})(x + a) \geq \frac{27}{2}$; $a\sqrt{a} \geq \frac{27}{8}$; $a \geq \frac{9}{4}$.

2. а) $\frac{6x^2 + x - 1}{x^4 - 6x^2 + 8} \geq 0$; $\frac{\left(x + \frac{1}{2}\right)\left(x - \frac{1}{3}\right)}{(x-2)(x+2)(x-\sqrt{2})(x+\sqrt{2})} \geq 0$;

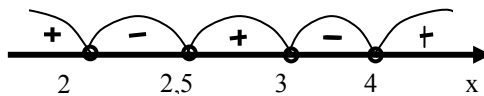


$x \in (-\infty; -2) \cup \left(-\sqrt{2}; -\frac{1}{2}\right] \cup \left[\frac{1}{3}; \sqrt{2}\right) \cup (2; +\infty)$.

б) $\frac{1}{x-2} + \frac{2}{x-3} > \frac{3}{x-4}$;
 $\frac{x^2 - 7x + 12 + 2x^2 - 12x + 16 - 3x^2 + 15x - 18}{(x-2)(x-3)(x-4)} > 0$;

$\frac{-4x+10}{(x-2)(x-3)(x-4)} > 0$;

$\frac{\left(x - \frac{5}{2}\right)}{(x-2)(x-3)(x-4)} < 0$;



C-30

1. $y = \sqrt{x}$; $y' = \frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{3}}$; $x = \frac{3}{4}$, значит, искомая точка $\left(\frac{3}{4}; \frac{\sqrt{3}}{2}\right)$.

2. $y = \cos\left(2x + \frac{\pi}{3}\right)$; $f\left(-\frac{\pi}{12}\right) = \frac{\sqrt{3}}{2}$; $y' = -2\sin\left(2x + \frac{\pi}{3}\right)$;
 $y'\left(-\frac{\pi}{12}\right) = -1$; $y_{kaa} = \frac{\sqrt{3}}{2} - x - \frac{\pi}{12}$.

C-31

1. $\left(\sqrt{3,99992} - \sqrt{1,00004}\right)^{60} \approx (2 - 0,00002 - 1 - 0,00002)^{60} \approx \approx 1 - 0,0024 = 0,9976$.

2. $\cos 33^0 \approx 0,8660 \cos 3^0 - \frac{1}{2} \sin 3^0 \approx 0,8399$.

C-32

1. $S(t) = -\frac{1}{6}t^2 + \frac{7}{2}t^2 - t$; $V(t) = -\frac{1}{2}t^2 + 7t - 1$; $a(t) = -t + 7$;

а) $7c$; б) $V(7) = -\frac{49}{2} + 48 = \frac{47}{2}$ м/с.

2. $S(t) = \frac{2}{2t-1}$; $V(t) = \frac{-4}{(2t-1)^2}$; $a(t) = \frac{16}{(2t-1)^3}$;

$F = \frac{16m_0}{(2t-1)^3} = 2m_0 S^3(t)$.

C-33

1. $f(x) = x^3 - 3x^2 + 2x - 7$; $f'(x) = 3x^2 - 6x + 2 = 0$ при

$$x = \frac{3 \pm \sqrt{3}}{3};$$

$$f(x) \text{ возрастает при } x \in \left(-\infty; \frac{3-\sqrt{3}}{3}\right] \cup \left[\frac{3+\sqrt{3}}{3}; +\infty\right);$$

$$\text{убывает при } x \in \left[\frac{3-\sqrt{3}}{3}; \frac{3+\sqrt{3}}{3}\right].$$

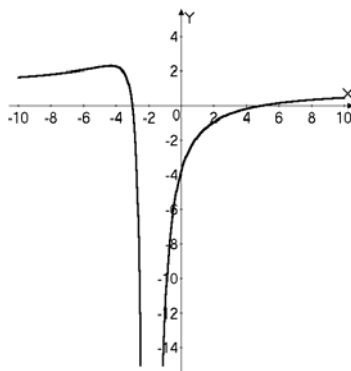
2. $f(x) = 8 \sin^2 x + 2 \cos 2x + 2 = 6 - 2 \cos 2x$;

$$f'(x) = 16 \sin x - 4 \sin 2x; f'(x) = 0$$

$$x_{\max} = \frac{\pi}{2} + \pi n; \quad x_{\min} = \pi n.$$

C-34

см.рис.



$$f(x) = \frac{(x-5)(x+3)}{(x+2)^2};$$

$$f'(x) = \frac{(x+3)(x+2)^2 + (x-5)(x+2)^2 - 2(x+2)(x+5)(x+3)}{(x+2)^4} =$$

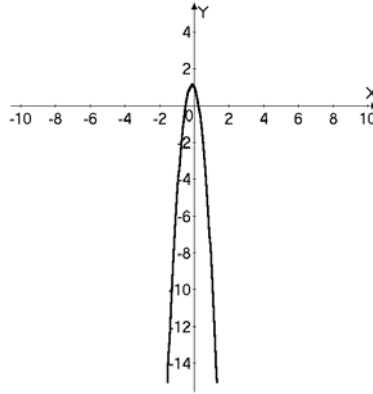
$$= \frac{x^2 + 5x + 6 + x^2 - 3x - 10 - 2x^2 + 4x + 30}{(x+2)^3} = \frac{6x + 26}{(x+2)^3};$$

$f(x)$ возрастает при $x < -\frac{13}{3}$, и $x > -2$; убывает при $x \in \left[-\frac{13}{3}; -2\right)$

$$x_{\max} = -\frac{13}{2}; x = -2 \text{ — не принадлежит ОДЗ}; f\left(-\frac{13}{3}\right) = \frac{44}{361}.$$

C-35

1.



$$h(x) = -8x^2 - 2x + 1; x_{\text{с}} = -\frac{1}{8};$$

$$h_{\text{с}} = -\frac{1}{8} + \frac{2}{8} + 1 = 1\frac{1}{8};$$

$$x \in \mathbb{R}, h(x) \in \left(-\infty; 1\frac{1}{8}\right];$$

$h(x)$ возрастает при $x \leq -\frac{1}{8}$; убывает при $x \geq -\frac{1}{8}$;

$$\text{Нули: } x = -\frac{1}{2}, x = \frac{1}{4}.$$

2.

$$3x^2 - 6x - 1 < 0;$$

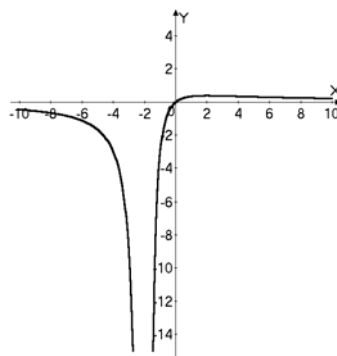
$$x \in \left(\frac{3 - 2\sqrt{3}}{3}; \frac{3 + 2\sqrt{3}}{3}\right).$$

3.

$$\frac{2}{3}x^2 - 2x^2 + 2x - \frac{2}{3} < 0; \quad x^3 - 3x^2 + 3x - 1 < 0;$$

$$(x-1)(x^2 - 2x + 1) < 0; \quad (x-1)^3 < 0; \quad \text{верно при } x < 1.$$

C-36



$$f(x) = \frac{3x}{x^2 + 4x + 4} = \frac{3x}{(x+2)^2};$$

$$f'(x) = \frac{3x^2 + 12x + 12 - 6x^2 - 12x}{(x+2)^4} = \frac{-3x^2 + 12}{(x+2)^4} = \frac{-3(x^2 - 4)}{(x+2)^4};$$

$$f'(x) = 0 \quad \text{при } x_{\max} = 2, \quad x = -2 \text{ не входит в ОДЗ};$$

$$f(x) \text{ возрастает при } x \in (-2; 2]; \quad \text{убывает при } x < -2, \quad x > 2.$$

C-37

1.

$$f(x) = \sqrt{2 - x - x^2}; \quad \text{ОДЗ: } x \in [-2; 1]$$

$$f'(x) = \frac{-1 - 2x}{2\sqrt{2 - x - x^2}}; \quad x_{\max} = -\frac{1}{2};$$

2.

Пусть больше осн. = $2x$;

$$H = \sqrt{400 - x^3 + 20x - 100} = \sqrt{300 + 20x - x^2};$$

$$S = (x + 10)\sqrt{300 + 20x - x^2};$$

$$S'(x) = \sqrt{300 + 20x - x^2} + \frac{(x + 10)(10 - x)}{\sqrt{300 + 20x - x^2}} = 0;$$

$$300 + 20x - x^2 - x^2 + 100 = 0; x^2 - 10x - 200 = 0;$$

$$x = 0 \text{ не подходит} \Rightarrow x = 40 \text{ см.}$$

C-38

1.

$$\cos \alpha = \frac{3}{5}, \sin \beta = \frac{4}{5}, \operatorname{tg} \alpha = \frac{4}{3}; 0 < \alpha < \pi, \alpha \in I \text{ четверти};$$

$$0 < \beta < \frac{\pi}{2}; 0 < \gamma < \pi, \gamma \in I \text{ четверти};$$

$$\sin \alpha = \frac{4}{5}, \cos \beta = \frac{3}{5}; \sin \gamma = \frac{4}{5}, \cos \gamma = \frac{3}{5};$$

$$\begin{aligned} \cos(\alpha + \beta + \gamma) &= \cos \alpha \cos \beta \cos \gamma - \sin \alpha \sin \beta \cos \gamma - \\ &- \sin \alpha \cos \beta \sin \gamma - \cos \alpha \sin \beta \sin \gamma = \frac{3}{5} \frac{3}{5} \frac{3}{5} - \frac{4}{5} \frac{4}{5} \frac{3}{5} - \\ &- \frac{4}{5} \frac{4}{5} \frac{3}{5} - \frac{3}{5} \frac{4}{5} \frac{4}{5} = -\frac{16}{25} \left(\frac{9}{5} \right) + \frac{27}{125} = -\frac{117}{125}. \end{aligned}$$

2.

$$\frac{\sin^2 4\alpha}{2 \cos \alpha + \cos 3\alpha + \cos 5\alpha} = 2 \sin \alpha \sin 2\alpha;$$

$$\frac{\sin^2 4\alpha}{2 \cos 2\alpha (\cos \alpha + \cos 3\alpha)} = \frac{4 \sin^2 \alpha \cos^2 2\alpha}{4 \cos^2 2\alpha \cos \alpha} = 2 \sin \alpha \sin 2\alpha.$$

3.

$$\text{a) } \frac{\operatorname{tg} 23^\circ - \operatorname{tg} 8^\circ}{1 + \operatorname{tg} 8^\circ \operatorname{tg} 23^\circ} = \operatorname{tg} 15^\circ = \sqrt{\frac{2 - \sqrt{3}}{2 + \sqrt{3}}};$$

$$\sin 15^\circ = \sqrt{\frac{1 - \sqrt{3}/2}{2}} = \frac{\sqrt{2 - \sqrt{3}}}{2};$$

$$\cos 15^\circ = \frac{\sqrt{2 + \sqrt{3}}}{2}.$$

C-39

а)

см.рис.

$$f(x) = \sin\left(\frac{x}{2} - \frac{\pi}{8}\right), \quad x \in R,$$

$$y \in [-1; 1]$$

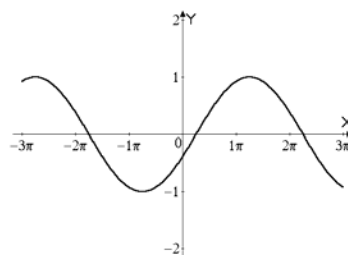
$$T = 4\pi$$

возрастает:

$$\left[-\frac{3\pi}{8} + 4\pi n; \frac{5\pi}{8} + 4\pi n\right]$$

$$\text{убывает: } x \in \left[\frac{5\pi}{8} + 4\pi n; \frac{13\pi}{8} + 4\pi n\right]$$

$$\max(\pi + 4\pi n; 1); \min(-\pi + 4\pi n; 1)$$



б) $f(x) = \cos\left(2x - \frac{\pi}{2}\right), \quad x \in R, \quad y \in [-1; 1]; \quad T = \frac{1}{\pi}$

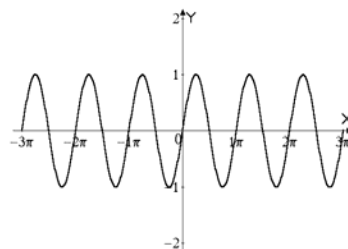
см.рис.

$$\text{возрастает: } \left[-\frac{\pi}{4} + \pi n; \frac{\pi}{4} + \pi n\right],$$

$$\max: \left(\frac{\pi}{4} + \pi n; 1\right)$$

$$\text{убывает: } \left[\frac{\pi}{4} + \pi n; \frac{3\pi}{4} + \pi n\right],$$

$$\min: \left(\frac{3\pi}{4} + \pi n; 1\right)$$



в) $y = \operatorname{tg}\left(\frac{1}{3}x + \frac{\pi}{4}\right);$

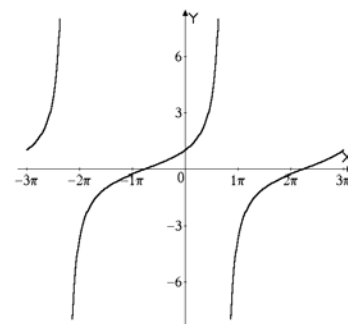
$$\cos\left(\frac{1}{3}x + \frac{\pi}{4}\right) \neq 0; \quad x \neq \frac{3\pi}{4} + 3\pi n,$$

$$y \in R$$

см.рис.

возрастает на R

$$\text{нули: } x = -\frac{3\pi}{4} + 3\pi n$$



C-40

1.

$$\text{a) } \cos\left(2 \arcsin \frac{2}{5}\right) = 1 - \frac{8}{25} = \frac{17}{25}$$

$$\text{б) } \operatorname{arctg} \sqrt{5} + \operatorname{arctg} \frac{1}{\sqrt{5}} = \operatorname{arctg} \sqrt{5} + \operatorname{arcctg} \sqrt{5} = \frac{\pi}{2}$$

2.

$$\text{a) } \cos x \cos 2x \cos 4x = 1$$

$$\begin{cases} \cos x = 1 \\ \cos 4x = 1 \\ \cos 2x = 1 \end{cases} \quad x = 2\pi n; \quad \begin{cases} \cos x = -1 \\ \cos 4x = -1 \\ \cos 2x = 1 \end{cases} \quad \emptyset;$$

$$\begin{cases} \cos x = -1 \\ \cos 4x = 1 \\ \cos 2x = -1 \end{cases} \quad \emptyset; \quad \begin{cases} \cos x = 1 \\ \cos 4x = 1 \\ \cos 2x = -1 \end{cases} \quad \emptyset;$$

$$\text{б) } 8 \cos^6 x = 3 \cos 4x + \cos 2x + 4;$$

$$1 - \cos^3 2x + 3 \cos^2 2x - 3 \cos 2x = 6 \cos^2 2x + \cos 2x + 1;$$

$$\cos 2x (+ \cos^2 2x + 3 \cos 2x + 4) = 0;$$

$$\Delta < 0;$$

$$x = \frac{\pi}{4} + \frac{\pi n}{2}.$$

3.

$$\text{a) } \cos x < \sin x; \sin\left(x - \frac{\pi}{4}\right) > 0;$$

$$x \in \left(\frac{\pi}{4} + \pi n; \frac{\sqrt{\pi}}{4} + \pi n\right)$$

$$\text{б) } \cos x \left(\sin x + \frac{1}{2}\right) \geq 0$$

$$x \in \left[-\frac{\pi}{6} + 2\pi n; \frac{\pi}{2} + 2\pi n\right] \cup \left[\frac{7\pi}{6} + 2\pi n; \frac{3\pi}{2} + 2\pi n\right]$$

C-42

1.

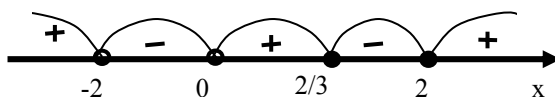
a) $x^2 - 8|x| + 12 \leq 0$; $|x| \in [2; 6]$; $x \in [-6; -2] \cup [2; 6]$

б) $1 + \frac{15}{x^2} > \frac{8}{x}$ ОДЗ: $x \neq 0$; $x^2 - 8x + 15 > 0$; $x \in (-\infty; 3) \cup (5; +\infty)$

2.

a) $\frac{1}{x-2} + \frac{1}{x} \leq \frac{2}{x+2}$; $\frac{x^2 + 2x + x^2 - 4 - 2x^2 + 4x}{x(x-2)(x+2)} \leq 0$

$\frac{x - \frac{2}{3}}{x(x-2)(x+2)} \leq 0$



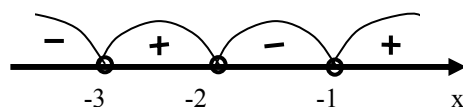
$x \in (-2; 0) \cup \left[\frac{2}{3}; 2\right]$

б) $\frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)} < 1$;

$\frac{(x^2 - 3x + 2)(x-3) - (x^2 + 3x + 2)(x+3)}{(x^2 + 3x + 2)(x+3)} < 0$;

$\frac{-6x^2 - 6x^2 - 6}{(x+1)(x+2)(x+1)} < 0$;

$\frac{x^2 + 1}{(x+1)(x+2)(x+3)} > 0$;



$x \in (-3; -2) \cup (-1; +\infty)$

C-43

1. а) $y = \frac{3}{x} - \frac{2}{x^2} + \frac{1}{x^3}$; $y' = -\frac{3}{x^2} + \frac{4}{x^3} - \frac{3}{x^4}$
 б) $y = \frac{2\sqrt{x}}{1-x^2}$; $y' = \frac{\left(\frac{1}{x}\right) - x + 4x\sqrt{x}}{(1-x^2)^4}$
 в) $y = (-2 + x^2)\sin x + 2x\cos x$
 $y' = 2x\sin x + (x^2 - 2)\cos x + 2\cos x - 2x\sin x = x^2\cos x$
 г) $y = (x^4 - x^3)^{42}$; $y' = 42(4x^3 - 3x^2)(x^4 - x^3)^{41}$
2. $f(x) = \frac{2}{x}$; $g(x) = x - x^3$; $f'(x) = -\frac{2}{x^2}$; $g'(x) = 1 - 3x^2$
 $f'(x) - g'(x) = -\frac{2}{x^2} - 1 + 3x^2 \leq 0$
 ОДЗ: $x \neq 0$; $3x^4 - x^2 - 2 \leq 0$; $D = 1 + 24 = 25$; $x^2 \in (0; 1]$
 $x \in [-1; 0) \cup (0; 1]$

C-44

1. $f(x) = x^2 - 2x + 2$; $f'(x) = 2x - 2$
 $y_{\text{vac}} = x_0^2 - 2x_0 + 2 + (2x_0 - 2)(x - x_0)$
 $1 = x_0 - 2x_0 + 2 - 2(x_0 - 1)(x_0 + 1)$
 $1 = -x_0^2 - 2x_0 + 4$; $x_0^2 + 2x_0 - 3 = 0$
 $D / 4 = 4$
 $x_0 = -3$, $x_0 = 1$; $y = 1$, $y = -8x - 7$
2. а) $\left(\sqrt{16,000032} - \sqrt{8,999982}\right)^{200} \approx (4 + 0,000004 - 3 + 0,000003)^{200} \approx$
 $\approx 1 + 0,000007 \cdot 200 = 1,0014$
 б) $\operatorname{tg} 48^\circ = \frac{1 + \operatorname{tg} 3^\circ}{1 - \operatorname{tg} 3^\circ} \approx 1,1047$

3.

$$S(t) = \frac{2t}{t^2 + 1}; V(t) = \frac{2t^2 + 2 - 4t^2}{(t^2 + 1)^2} = \frac{2 - 2t^2}{(t^2 + 1)^2}; V_0 = 2$$

$$V(t) = \frac{2 - 2t^2}{(t^2 + 1)^2} = 1; 2 - 2t^2 = (t^2 + 1)^2; t^2 + 4t - 1 = 0$$

$$D/4 = 5$$

$$t^2 = -2 + \sqrt{5}; t = \sqrt{\sqrt{5} - 2}$$

$$a(t) = \frac{-4t(t^2 + 1)^2 + 4(t^2 + 1)(t^2 - 1)}{(t^2 + 1)^4} =$$

$$= \frac{4t^2 - 4 - 4t^3 - 4t}{(t^2 + 1)^3} = \frac{-4(t^3 - t^2 + t + 1)}{(t^3 + 1)^3}$$

$$F = \frac{8\left((\sqrt{5} - 2)\left(\sqrt{\sqrt{5} - 2} - 1\right) + \sqrt{\sqrt{5} - 2} + 1\right)}{-(\sqrt{5} - 1)^3}$$

C-45

1) 3 корня

2) Пусть a - бок.стор, b - осн. $H = \sqrt{a^2 - \frac{b^2}{4}}$

$$\begin{cases} 2a + b = 2p \\ S = \frac{1}{2}b\sqrt{a^2 - \frac{b^2}{4}}; S = \frac{1}{2}b\sqrt{p^2 + \frac{b^2}{4} - pb - \frac{b^2}{4}} = \frac{1}{2}b\sqrt{p^2 - pb} \end{cases}$$

$$S'(b) = \frac{1}{2}\sqrt{p^2 - pb} + \frac{-bp}{4\sqrt{p^2 - pb}} = 0; 2p^2 - 2pb = bp$$

$$b = \frac{2p}{3}; a = p - \frac{p}{3} = \frac{2p}{3} \Rightarrow \text{треугольник правильный}$$

ПРОВЕРОЧНАЯ РАБОТА № 1 В1

1. $\cos \alpha = \frac{3}{5}, \sin \alpha = -\frac{4}{5}$

$\sin \alpha$ – ордината угла α на единичной окр.

$\cos \alpha$ – абсцисса угла α на единичной окр.

$\sin \pi = 0, \cos \pi = -1; \sin(-630^0) = \sin 90^0 = 1; \cos(-630^0) = 0$

2. $L = 2\pi r = 10\pi; \quad \overset{\cup}{AB} = \frac{10\pi}{22} = \frac{5\pi}{11}$

3. $\sin \alpha = \frac{1}{2}, \cos \alpha = \pm \frac{\sqrt{3}}{2}$

4. $(\sin \alpha + \cos \alpha)^2 + (\sin \alpha - \cos \alpha)^2 - 2 = 2 - 2 = 0$

5. $1 - \frac{1}{1 + \operatorname{tg}^2 \alpha} = \frac{1}{1 + \operatorname{ctg}^2 \alpha}; 1 - \cos^2 \alpha = \sin^2 \alpha$

6. $\cos 350^0 \sin \frac{5\pi}{4} < 0$

7. $y = x^3; y = \sin x; y = \operatorname{tg} x$

8. $\cos \alpha = -\frac{3}{4}; \cos(\pi - \alpha) = -\cos \alpha = \frac{3}{4}; \sin\left(\frac{\pi}{2} - \alpha\right) = \cos \alpha$

$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha; \sin(\pi - \alpha) = \sin \alpha; \cos(\pi - \alpha) = -\cos \alpha$

$\sin\left(\frac{3\pi}{2} - \alpha\right) = -\cos \alpha; \cos\left(\frac{3\pi}{2} - \alpha\right) = -\sin \alpha$

9. $\cos \alpha = 1, \sin \alpha = 0, \sin 2\alpha = 0$

10. $\sin 2\alpha - \sin 2\beta = 2 \sin(\alpha - \beta) \cos(\alpha + \beta)$

$\sin \alpha \pm \sin \beta = 2 \sin \frac{\alpha \pm \beta}{2} \cos \frac{\alpha \mp \beta}{2}$

B-2

1. $\sin \alpha = -\frac{3}{5}, \cos \alpha = -\frac{4}{5}, tg \alpha = \frac{3}{4}, ctg \alpha = \frac{4}{3}$
 $tg \alpha$ – отношение ординаты точки к ее абсциссе
 $ctg \alpha$ – отношение абсциссы точки к ее ординате
 $ctg \frac{\pi}{4} = tg \frac{\pi}{4} = 1; ctg(-450^0) = 0; tg 540^0 = 0$
2. $S = \pi r^2 = 7\pi; S = \frac{7\pi}{2\pi} \cdot 0,7 = 2,45$
3. $\cos \alpha = \frac{\sqrt{3}}{2}, \sin \alpha = \pm \frac{1}{2}, tg \alpha = \pm \frac{\sqrt{3}}{3}$
4. $(\sin \alpha + \cos \alpha)^2 - (\cos \alpha - \sin \alpha)^2 + \sin \alpha \cos \alpha =$
 $= 2 \sin 2\alpha + \frac{1}{2} \sin 2\alpha = \frac{5}{2} \sin 2\alpha$
5. $\frac{1}{1 + tg^2 \alpha} + \cos 2\alpha = (1 + tg^2 \alpha) \cos^2 \alpha$
 $\sin^2 \alpha + \cos^2 \alpha = \cos^2 \alpha + \sin \alpha = 1$
6. $\sin \frac{7\pi}{3} ctg 250^0 > 0$
7. $y = x^2, y = \cos x, y = ctgx$
8. $ctg\left(\frac{3\pi}{2} + \alpha\right) = -tg \alpha = 2,7$
9. $\cos \alpha = \frac{4}{5}, \alpha \in IV_{qu.}; \sin \frac{\alpha}{2} = -\sqrt{\frac{1 - 4/5}{2}} = -\frac{1}{\sqrt{10}}$
10. $(\cos 2\beta + \cos 2\alpha) = 2 \cos(\alpha + \beta) \cos(\alpha - \beta)$
 $\cos \alpha - \cos \beta = -2 \sin \frac{\alpha - \beta}{2} \sin \frac{\alpha + \beta}{2}$

ПРОВЕРОЧНАЯ РАБОТА № 2 В-1

1. $y = \frac{1}{x^2 + 1}, x \in R$

ф-ция – зависимость y от x , где каждому x ставится в соответствие единственное значение y .

обл. опр. ф-ции – мн-во значений которое может принимать x .

обл. зн. ф-ции – мн-во значений которое может принимать y .

2. $f(x) = x^2 - 2x + 1 = (x - 1)^2$

возрастает $x \geq 1$, убывает $x \leq 1$

функция наз. возраст. на мн-ве P , если для $\forall x, x_2 \in P, x_1 > x_2$,

$f(x_1) > f(x_2)$

3. а) $f(x) = \cos 2x; f(-x) = \cos(-2x) = \cos 2x = f(x)$

б) $f(x) = \sin^2 x; f(-x) = \sin^2(-x) = \sin^2 x = f(x)$

в) $f(x) = 2x^4 - 3x^2; f(-x) = 2(-x^4) - 3(-x^2) = 2x^4 - 3x^2 = f(x)$

4.

$f(x) = x^3 + x$

см.рис.

$f(x) = 0, x = 0$ – нули

$x \in R, y \in R$, из рис. видно,

что ф-ция возрастает на R .

Схема:

1) Обл. опр., обл. зн

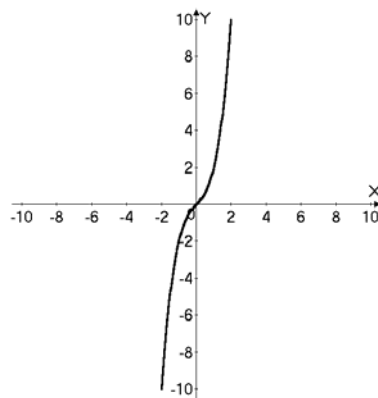
2) Нули

3) Промежуток

возрастания (убывания)

4) Экстремумы (из них

выбрать max и min ф-ции)



5. $\sin 2, \sin 4, \sin 6$

Ответ: $\sin 4, \sin 6, \sin 2$

6. а) $f(x) = \sin\left(3x + \frac{\pi}{7}\right); T = \frac{2\pi}{3}$

б) $f(x) = \operatorname{tg}^2\left(x - \frac{\pi}{2}\right) = \operatorname{ctg}^2 x; T = \pi$

7. а) $tg\sqrt{2} + tg(-\sqrt{2}) = 0$
 б) $tg \frac{22\pi}{7} ctg \frac{36\pi}{7} = tg \frac{22\pi}{7} ctg \frac{36\pi}{7} = 1$
 $tg(\alpha + \pi) = tg\alpha$; $tg\left(\frac{\pi}{2} - \alpha\right) = ctg\alpha$; $tg(\alpha + 2\pi) = tg\alpha$
8. $arccos(-1) = \pi$; $arccos \frac{\sqrt{3}}{2} = \frac{\pi}{6}$
 арккосинусом числа a наз. такое число $\in [0; \pi]$ cos , которого равен a
 $arccos a$, определен при $a \in [-1; 1]$
9. а) $tg\left(2x - \frac{\pi}{8}\right) = 1$; $x = \frac{3\pi}{16} + \frac{\pi n}{2}$
 б) $2 \cos\left(\frac{x}{2} + 1\right) = 1$; $\frac{x}{2} + 1 = \pm \frac{\pi}{3} + 2\pi n$; $x = \pm \frac{2\pi}{3} - 2 + 4\pi n$
 $\sin x = a$, $|a| \leq 1$; $x = (-1)^k \arcsin a + \pi k$
10. а) $tg 2x > 1$; $x \in \left(\frac{\pi}{8} + \frac{\pi n}{2}; \frac{\pi}{4} + \frac{\pi n}{2}\right)$
 б) $\sin x \leq -1$; $x = -\frac{\pi}{2} + 2\pi n$
11.
$$\begin{cases} \cos(x+y) = \frac{1}{2} \\ \sin(x-y) = 1 \end{cases}; \begin{cases} x+y = \pm \frac{\pi}{3} + 2\pi n \\ x-y = \frac{\pi}{2} + 2\pi k \end{cases}; \begin{cases} x = \frac{\pi}{4} \pm \frac{\pi}{6} + \pi n + \pi k \\ y = \pm \frac{\pi}{6} - \frac{\pi}{4} + \pi n - \pi k \end{cases}$$

В-2

1) см.рис.

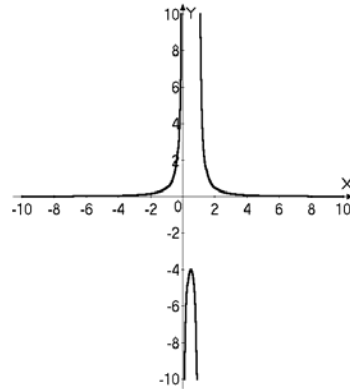
График ф-ции – мн-во точек на
 п-ти удовлетворяющих какому-либо
 ур-ю

2) допустим, что

$f(x) = a$ имеет 2 корня., тогда

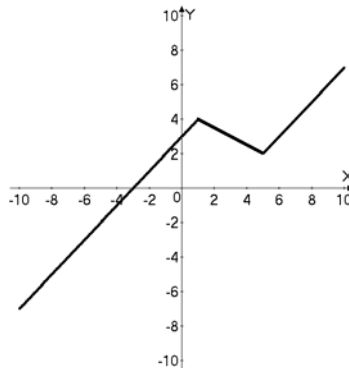
$f(x_1) = f(x_2) = a$, что не подходит

под опр. возрастающей
 (убывающей) ф-ции.



3. а) $f(x) = \sin \frac{x}{3}$; $f(-x) = \sin \frac{-x}{3} = -\sin \frac{x}{3} = -f(x)$
 б) $f(x) = x^2 \operatorname{tg} x$; $f(-x) = (-x)^2 \operatorname{tg}(-x) = -x^2 \operatorname{tg} x = -f(x)$
 в) $f(x) = x^7 - 5x^3$; $f(-x) = (-x)^7 - 5(-x)^3 = 5x^3 - x^7 = -f(x)$

4. См.рис.



Ф-ия ни четная ни нечетная, т.к. промежуток убывания не делится прямой $x=0$ пополам и экстремум ф-ции не находится на этой прямой.

5. $y = \cos\left(2x + \frac{\pi}{5}\right)$; $\max: \left(-\frac{\pi}{10} + \pi n; 1\right)$; $\min: \left(\frac{4\pi}{10} + \pi n; -1\right)$
 $y = \cos \alpha$, возрастает: $[-\pi + 2\pi n; 2\pi n]$; убывает: $[2\pi n; 2\pi n + \pi]$
 $x_1 = 2\pi n - \max$, $f(x_1) = 1$; $x_2 = \pi + 2\pi n - \min$, $f(x_2) = -1$

6. а) $f(x) = \cos\left(\frac{x}{2} - \frac{\pi}{4}\right)$, $T = 4\pi$; б) $f(x) = \sin^2 x + \operatorname{tg} x$
 $f_1(x) = \sin^2 x$, $T_1 = \pi$; $f_2(x) = \operatorname{tg} x$, $T_2 = \pi \Rightarrow T = \pi$
 ф-ция наз. периодической] $f(x) = f(x + T)$,
 где T - период, для $\forall x$.

7. $\operatorname{arctg}(-1) = -\frac{\pi}{4}$; $\operatorname{arctg} \frac{1}{\sqrt{3}} = \frac{\pi}{6}$
 $\operatorname{arctg} a$, определен при $\forall a$, $\operatorname{arctg} a \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

8. а) да ; б) нет, т.к. $\frac{3\pi}{2} \notin \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

$\arcsin a$ - такое число $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$, \sin которого равен a , $|a| \leq 1$

9. а) $4 \sin\left(\frac{x}{2} - 2\right) = 2$; $\frac{x}{2} - 2 = (-1)^k \frac{\pi}{6} + \pi k$; $x = (-1)^k \frac{\pi}{3} + 2\pi k + 4$

б) $\operatorname{tg}^3 3x = 3$; $\operatorname{tg} 3x = \pm\sqrt{3}$; $x = \pm \frac{\pi}{9} + \frac{\pi k}{3}$; $\cos x = a$, $|a| \leq 1$

$x = \pm \arccos a + 2\pi n$

10. а) $\cos x > \frac{1}{2}$; $x \in \left(-\frac{\pi}{3} + 2\pi n; \frac{\pi}{3} + 2\pi n\right)$

б) $\operatorname{tg} 2x \leq 1$; $x \in \left(-\frac{\pi}{4} + \frac{\pi k}{2}; \frac{\pi}{8} + \frac{\pi k}{2}\right]$

11. $\begin{cases} x + y = \frac{\pi}{2} \\ \sin x + \sin y = \sqrt{2} \end{cases}$; $\begin{cases} x = \frac{\pi}{2} - y \\ \sin y + \cos y = \sqrt{2} \end{cases}$; $\begin{cases} \sin\left(y + \frac{\pi}{4}\right) = 1 \\ x = \frac{\pi}{2} - y \end{cases}$

$y = \frac{\pi}{4} + 2\pi n$; $x = \frac{\pi}{4} - 2\pi n$

ПРОВЕРОЧНАЯ РАБОТА № 3 В-1

1.

а) $2x^2 - 3x + 1 \geq 0$

$\Delta = 9 - 8 = 1$

$x \in \left(-\infty; \frac{1}{2}\right] \cup [1; +\infty)$



б) $\frac{(x-1)(2x+3)}{(x^2-6x+8)} < 0$; $\frac{(x-1)(2x+3)}{(x-2)(x-4)} < 0$

$x \in \left(-\frac{3}{2}; 1\right) \cup (2; 4)$



$$2. \quad y = \frac{1}{x}; \quad y\left(-\frac{1}{2}\right) = -2; \quad y' = -\frac{1}{x^2}, \quad y'\left(-\frac{1}{2}\right) = -4$$

$$y_k = -2 - 4\left(x + \frac{1}{2}\right) = -4x - 4$$

$$3. \quad y = 3x^3 - 4,5x^2; \quad y' = 9x^2 - 9x; \quad y = \cos \frac{x}{2} - \sin 2x$$

$$y' = -\frac{1}{2} \sin \frac{x}{2} - 2 \cos 2x$$

4)

скорость в точке x_0

$$x(t) = 3x^4 - 2t^3 + 1$$

$$a) \quad V(t) = 12t^3 - 6t^2; \quad a(t) = 36t^2 - 12t$$

$$b) \quad V(2) = 72; \quad a(2) = 120$$

5.

$$g(x) = x\sqrt{x+1}; \quad g'(x) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$$

$$g'(3) = 2 + \frac{3}{4} = 2,75; \quad (g(x)f(x))' = g'(x)f(x) + g(x)f'(x)$$

6.

$$\sqrt{\sqrt{17}} \approx \left(2\left(1 + 0,0625 \cdot \frac{1}{4}\right)\right) \approx 2,03$$

7.

$$f(x) = x - 2\sqrt{x}; \quad f'(x) = 1 - \frac{1}{\sqrt{x}} = 0$$

$$x=1$$

убывает: $x \in [0;1]$; возрастает: $x \geq 1$

8.

$$y = x^3 - \frac{x}{3}; \quad y' = 3x^2 - \frac{1}{3} = 0; \quad x = \pm \frac{1}{3};$$

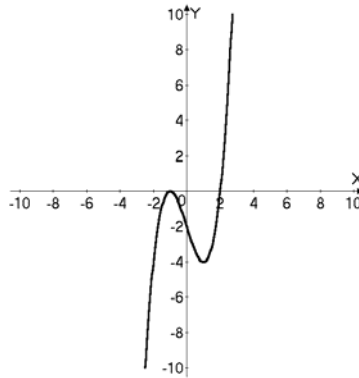
$$y\left(\frac{1}{3}\right) = \frac{1}{27} - \frac{1}{9} = -\frac{2}{27}; \quad y\left(-\frac{1}{3}\right) = \frac{2}{27};$$

$$\max : x = -\frac{1}{3}; \quad \min : x = \frac{1}{3}.$$

9.

$$f(x) = x^3 - 3x - 2$$

см.рис.



$$f'(x) = 3(x^2 - 1) = 0; x = \pm 1$$

возрастает: $x \leq -1$, $x \geq 1$; убывает: $x \in [-1; 1]$

$$x = -1: \max; f(-1) = 0; f(1) = \min = -4$$

10.

$$f(x) = x + \frac{4}{x}, x \in [1; 3]; f'(x) = 1 - \frac{4}{x^2} = 0, x = \pm 2$$

$$f(2) = 2 + 2 = 4; f(1) = 5, f(3) = 4\frac{1}{3}$$

$$\max: f(1) = 5; \min: f(2) = 4$$

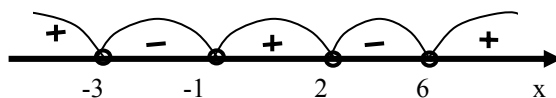
B-2

1. а) $3x - 7x^2 \leq 0; x(7x - 3) \geq 0$



$$x \in (-\infty; 0] \cup \left[\frac{3}{7}; +\infty\right)$$

$$6) \frac{x^2 - 5x - 6}{(x-2)(x+3)} > 0; \frac{(x-6)(x+1)}{(x-2)(x+3)} > 0$$



$$x \in (-\infty; -3) \cup (-1; 2) \cup (6; +\infty)$$

2.

$$y = 2x^2 - 1; y(3) = 17; y' = 4x; y'(3) = 12$$

$$y_k = 17 + 12(x - 3) = 12x - 19$$

3.

$$y = 2,5x^2 - x^5; y' = 5x - 5x^4; y = \operatorname{tg} 2x - 2 \operatorname{ctg} \frac{x}{2};$$

$$y' = \frac{2}{\cos^2 2x} + \frac{1}{\sin^2 \frac{x}{2}};$$

геометрич. смысл производной в т. x_0 -tg угла наклона касательной.

4.

$$\omega(t) = 2t^4 - t; \text{ а) } \omega'(t) = 8t^3 - 1; \text{ б) } \omega'(2) = 63$$

$$\omega(t) = 0, \text{ при } t = \frac{1}{2}$$

5.

$$f(x) = \frac{\sqrt{x-1}}{x}$$

$$\text{а) } f'(x) = \frac{\frac{x}{2\sqrt{x-1}} - \sqrt{x-1}}{x^2} = \frac{-x+2}{2x^2\sqrt{x-1}}; \text{ б) } f'(2) = 0$$

6.

$$f(x) = (2x^3 - 1)^{100}; f'(x) = 600x^2(2x^3 - 1)^{99}$$

$$(f(g(x)))' = f'(g(x)) \cdot g'(x)$$

7.

$$y = x^3 + x; y' = 3x^2 + 1 > 0 \Rightarrow \text{возрастает на } R$$

8.

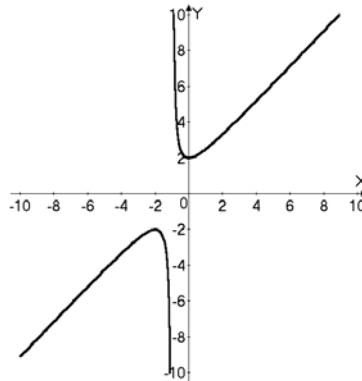
$$g(x) = \sqrt{x} - x; \quad g'(x) = \frac{1}{2\sqrt{x}} - 1 = 0$$

$$x = \frac{1}{4}; \quad x = \frac{1}{4} - \max f\left(\frac{1}{4}\right) = \frac{1}{4}$$

9.

$$y = \frac{x^2 + 2x + 2}{x + 1};$$

см.рис.



$$y = x + 1 + \frac{1}{x + 1}; \quad y' = 1 - \frac{1}{(x + 1)^2} = 0; \quad x = 0 \quad x = -2;$$

возрастает: $x \leq -2$, $x \geq 0$; убывает: $x \in [-2; 0]$, $x \neq -1$;

$$x_{\max} = -2, \quad x_{\min} = 0.$$

10.

$$\begin{cases} 12 = a + b \\ y = a^2 + b^2 \end{cases}; \quad \begin{cases} a = 12 - b \\ y = 2b^2 - 24b + 144 \end{cases}; \quad y'(x) = 4b - 24 = 0;$$

$$b = 6$$

$a = 6 \Rightarrow$ сумма квадратов \max

$$b = 0$$

$a = 12 \Rightarrow$ сумма квадратов \min

Примерные контрольные работы

КР № 1. В 1.

1. а) $\sin 240^\circ = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$; б) $\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$.
в) $\operatorname{ctg}(-\frac{\pi}{6}) = -\sqrt{3}$.
2. $\sin \alpha = -\frac{3}{5}$; $\pi < \alpha < \frac{3\pi}{2}$;
а) $\cos \alpha = -\frac{4}{5}$;
б) $\cos(\frac{\pi}{3} - \alpha) = \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha = -\frac{4}{10} - \frac{3\sqrt{3}}{10} = -\frac{4+3\sqrt{3}}{10}$.
3. $\frac{2 \sin^2 \alpha \cdot \operatorname{ctg} \alpha}{\cos^2 \alpha - \sin^2 \alpha} = \operatorname{tg} 2\alpha$; $\frac{\sin 2\alpha}{\cos 2\alpha} = \operatorname{tg} 2\alpha$.
4. $\sin x + \cos x = m$; $\sqrt{2}(\sin(x + \frac{\pi}{4})) = m$;
 $m \in [-\sqrt{2}; \sqrt{2}]$. $x = (-1)^k \arcsin \frac{m}{\sqrt{2}} - \frac{\pi}{4} + \pi k$.
 $2x = (-1)^k 2 \arcsin \frac{m}{\sqrt{2}} - \frac{\pi}{2} + 2\pi k$;
 $\sin 2x = \sin(2 \arcsin \frac{m}{\sqrt{2}} - \frac{\pi}{2}) = -\cos(2 \arcsin \frac{m}{\sqrt{2}}) =$
 $= 2 \cdot \frac{m^2}{2} - 1 = m^2 - 1$.

КР № 1. В 2.

1. а) $\cos 240^\circ = -\cos 60^\circ = -\frac{1}{2}$; б) $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$;
в) $\operatorname{tg}(-\frac{\pi}{3}) = -\sqrt{3}$.
2. $\cos \alpha = -\frac{15}{17}$; $\frac{\pi}{2} < \alpha < \pi$;

a). $\sin \alpha = \frac{8}{17}$;

б). $\sin(\frac{\pi}{3} + \alpha) = \frac{\sqrt{3}}{2} \cos \alpha + \frac{1}{2} \sin \alpha = -\frac{15\sqrt{3}}{34} + \frac{8}{34} = \frac{8-15\sqrt{3}}{34}$.

3. $\frac{2 \cos^2 \alpha \cdot \operatorname{tg} \alpha}{\sin^2 \alpha - \cos^2 \alpha} = -\operatorname{tg} 2\alpha$; $\frac{\sin 2\alpha}{-\cos 2\alpha} = -\operatorname{tg} 2\alpha$.

4. $\sin x - \cos x = n$; $\sin(x - \frac{\pi}{4}) = \frac{n}{\sqrt{2}}$; $n \in [-\sqrt{2}; \sqrt{2}]$;

$1 - 2 \sin x \cdot \cos x = n^2$; $\sin 2x = 1 - n^2$.

КР № 1. В 3.

1. а) $\operatorname{tg} 300^\circ = -\operatorname{tg} 60^\circ = -\sqrt{3}$;

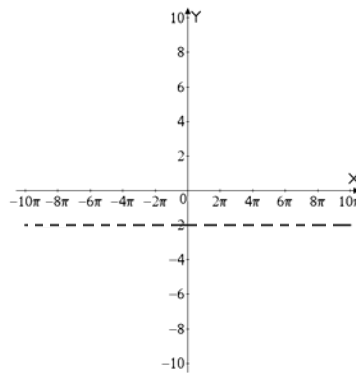
б) $\sin(-\frac{5\pi}{4}) = \frac{\sqrt{2}}{2}$; в) $\cos \frac{7\pi}{4} = \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$.

2. $\sin \alpha = \frac{4}{5}$; $\frac{\pi}{2} < \alpha < \pi$;

а) $\cos \alpha = -\frac{3}{5}$; $\operatorname{tg} \alpha = -\frac{4}{3}$; б) $\operatorname{tg}(\frac{\pi}{4} - \alpha) = \frac{1 - \operatorname{tg} \alpha}{1 + \operatorname{tg} \alpha} = -\frac{7}{3} \cdot \frac{3}{1} = -7$.

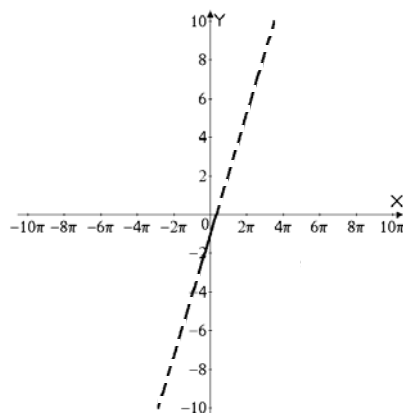
3. $\frac{\sin 3\alpha - \sin \alpha}{\cos 3\alpha + \cos \alpha} = \operatorname{tg} \alpha$; $\frac{\sin \alpha \cdot \cos \alpha}{\cos 2\alpha \cdot \cos \alpha} = \operatorname{tg} \alpha$.

4. $x \neq \frac{\pi n}{2}$.



КР № 1. В 4.

1. а) $\operatorname{ctg} 300^\circ = -\operatorname{ctg} 60^\circ = -\frac{1}{\sqrt{3}}$; б) $\cos \frac{4\pi}{3} = -\frac{1}{2}$;
 в) $\sin(-\frac{7\pi}{6}) = \sin \frac{5\pi}{6} = \frac{1}{2}$.
2. $\cos \alpha = -\frac{3}{5}$; $\pi < \alpha < \frac{3\pi}{2}$;
 а) $\sin \alpha = -\frac{4}{5}$; $\operatorname{tg} \alpha = \frac{4}{3}$. б) $\operatorname{tg}(\frac{\pi}{4} + \alpha) = \frac{1 + \operatorname{tg} \alpha}{1 - \operatorname{tg} \alpha} = -\frac{7}{3} \cdot \frac{3}{1} = -7$.
3. $\frac{\cos \alpha - \cos 5\alpha}{\sin 5\alpha + \sin \alpha} = \operatorname{tg} 2\alpha$; $\frac{\sin 3\alpha \cdot \sin 2\alpha}{\sin 3\alpha \cdot \cos 2\alpha} = \operatorname{tg} 2\alpha$.
4. $x \neq \frac{\pi n}{2}$.



КР № 2. В 1.

1. $y = \frac{\sqrt{x+2}}{x^2-9}$; ОДЗ: $\begin{cases} x \geq -2 \\ x \neq \pm 3 \end{cases}$; $x \in [-2; 3) \cup (3; +\infty)$.
2. $\sin(-750^\circ) + \operatorname{ctg} 945^\circ = -\sin 30^\circ + \operatorname{ctg} 45^\circ = -\frac{1}{2} + 1 = \frac{1}{2}$.
3. $f(x) = 2x^5 + 4\operatorname{tg} x$;
 $f(-x) = 2(-x)^5 + 4\operatorname{tg}(-x) = -2x^5 - 4\operatorname{tg} x = -f(x)$.

4.

$$y = 2 \sin x. \quad x \in R; \quad y \in [-2; 2].$$

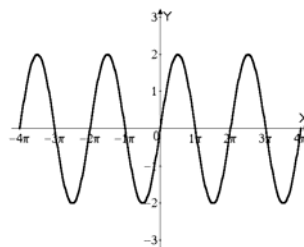
y возрастает на

$$x \in \left[-\frac{\pi}{2} + 2\pi; \frac{\pi}{2} + 2\pi\right];$$

$$\max: \left(\frac{\pi}{2} + 2\pi; 2\right);$$

$$y \text{ убывает на } x \in \left[-\frac{\pi}{2} + 2\pi; \frac{3\pi}{2} + 2\pi\right];$$

$$\min: \left(-\frac{\pi}{2} + 2\pi; -2\right).$$



$$5. \quad y = \frac{2\sqrt{x} + 3\sqrt{5-x}}{\cos x}; \quad \text{ОДЗ: } \begin{cases} x \geq 0 \\ x \leq 5 \\ \cos x \neq 0 \end{cases}; \quad x \in \left[0; \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}; \frac{3\pi}{2}\right) \cup \left(\frac{3\pi}{2}; 5\right].$$

КР № 2. В 2.

$$1. \quad y = \frac{\sqrt{2x+1}}{x^2-4} \quad \text{ОДЗ: } \begin{cases} x \geq -\frac{1}{2} \\ x \neq \pm 2 \end{cases}. \quad x \in \left[-\frac{1}{2}; 2\right) \cup (2; +\infty).$$

$$2. \quad \cos 1140^\circ + \operatorname{tg}(-495^\circ) = \cos 360^\circ - \operatorname{tg}135^\circ = \frac{1}{2} + 1 = \frac{3}{2}.$$

$$3. \quad f(x) = \frac{3x^2}{\sin x}; \quad f(-x) = \frac{3(-x)^2}{\sin(-x)} = \frac{-3x^2}{\sin x} = -f(x).$$

4.

$$y = 1,5 \cos x \quad x \in R$$

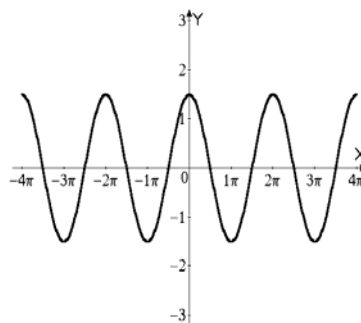
$$y \in \left[-\frac{3}{2}; \frac{3}{2}\right].$$

$$\text{нули: } x = \frac{\pi}{2} + \pi n.$$

$$y \text{ возрастает на } x \in [-\pi + 2\pi n; 2\pi n].$$

$$y \text{ убывает на } x \in [2\pi n; \pi + 2\pi n].$$

$$\max: \left(2\pi n; \frac{3}{2}\right) \quad \min: (\pi + 2\pi n; -1).$$



$$5. \quad y = \frac{3\sqrt{-x} + 2\sqrt{x+4}}{\sin x}; \text{ ОДЗ: } \begin{cases} x \leq 0 \\ x \geq -4; x \in [-4; -\pi) \cup (-\pi; 0) \\ \sin x \neq 0 \end{cases}$$

КР № 2. В 3.

$$1. \quad y = \frac{\sqrt{1-x}}{x^2 - 2x}; \text{ ОДЗ: } \begin{cases} x \leq 1 \\ x \neq 0 \\ x \neq 2 \end{cases} \quad x \in (-\infty; 0) \cup (0; 1].$$

$$2. \quad \sin(-660^\circ) + \cos 810^\circ = \sin 60^\circ + \cos(-90^\circ) = \frac{\sqrt{3}}{2}.$$

$$3. \quad h(x) = 3x^4 \operatorname{tg} x \quad h(-x) = 3(-x)^4 \operatorname{tg}(-x) = -3x^4 \operatorname{tg} x = -h(x).$$

4.

$$y = \sin \frac{1}{2}x \quad x \in \mathbb{R}; \quad y \in [-1; 1].$$

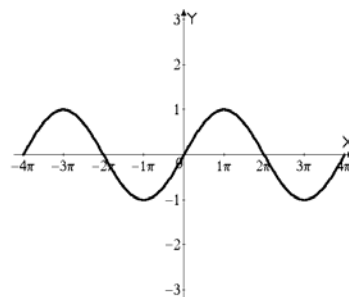
$$\text{нули: } x = 2\pi n;$$

$$\text{возрастает: } x \in [-\pi + 4\pi n; \pi + 4\pi n].$$

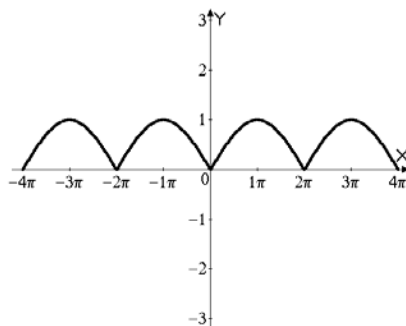
$$\text{убывает: } x \in [\pi + 4\pi n; 3\pi + 4\pi n]$$

$$\text{max: } (\pi + 4\pi n; 1);$$

$$\text{min: } (-\pi + 4\pi n; -1).$$



5.



$$y = \left| \sin \frac{x}{2} \right| \text{ возрастает на } x \in \left[\pi n; \frac{1}{2}\pi + \pi n \right].$$

КР № 2. В 4.

$$1. \quad y = \frac{\sqrt{-x-1}}{x^2+3x} \quad \text{ОДЗ:} \begin{cases} x \leq -1 \\ x \neq 0; \quad x \leq -1, \quad x \neq -3; \quad x \in (-\infty; -3) \cup (-3; -1] \\ x \neq -3 \end{cases}$$

$$2. \quad \cos 840^\circ + \operatorname{tg}(-585^\circ) = \cos 120^\circ + \operatorname{tg}135^\circ = -\frac{1}{2} - 1 = -\frac{3}{2}.$$

$$3. \quad \varphi(x) = \frac{5x^3}{\sin x} \quad \varphi(-x) = \frac{5(-x)^3}{\sin(-x)} = \frac{5x^3}{\sin x} = \varphi(x).$$

4.

$$y = \cos \frac{x}{2} = 0 \quad x = \pi + 2\pi n \quad \text{нули.}$$

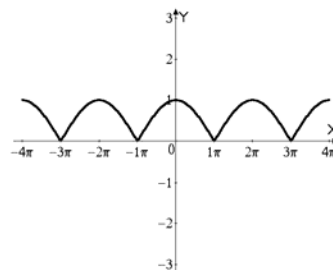
y возрастает на $[-2\pi + 4\pi n; \quad 4\pi n]$

max: $(4\pi n; 1)$.

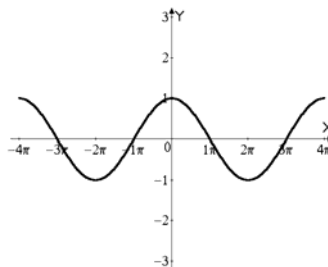
y убывает на $[4\pi n; \quad 2\pi + 4\pi n]$

min: $(2\pi + 4\pi n; \quad -1)$.

$x \in R. \quad y \in [-1; \quad 1]$.



5.



убывает на $[0; \quad \pi] \cup [2\pi; \quad 3\pi]$.

КР № 3. В 1.

$$1. \quad \text{а) } \sin x = -1; \quad x = -\frac{\pi}{2} + 2\pi n;$$

$$\text{б) } 2 \cos^2 x - \cos x - 1 = 0; \quad \cos x = 1; \quad x = 2\pi n;$$

$$\cos x = -\frac{1}{2}; \quad x = \pm 2\frac{\pi}{3} + 2\pi n.$$

- в), $\sin^2 x + \sqrt{3} \sin x \cdot \cos x = 0$; $\sqrt{3} \sin 2x - \cos 2x = -1$;
 $\sin(2x - \frac{\pi}{6}) = -\frac{1}{2}$; $x = (-1)^{k+1} \frac{\pi}{12} + \frac{\pi}{12} + \frac{\pi k}{2}$;
 2. $\sin x \geq -\frac{1}{2}$; $x \in [-\frac{\pi}{6} + 2\pi n; \frac{7\pi}{6} + 2\pi n]$.
 3. $\begin{cases} x + y = \pi \\ \sin x + \sin y = -\sqrt{2} \end{cases}$; $\begin{cases} x = \pi - y \\ \sin y = -\frac{\sqrt{2}}{2} \end{cases}$; $\begin{cases} y = (-1)^{k+1} \frac{\pi}{4} + \pi n \\ x = \pi - (-1)^{k+1} \frac{\pi}{4} - \pi n \end{cases}$.
 4. $|2 \sin x - 1| \leq 1$; $\begin{cases} \sin x \leq 1 \\ \sin x \geq 0 \end{cases}$; $x \in [2\pi n; \pi + 2\pi n]$.

КР № 3. В 2.

1. а) $\cos x = -1$; $x = \pi + 2\pi n$;
 б) $2 \sin^2 x - \sin x - 1 = 0$;
 $\sin x = 1$ $x = \frac{\pi}{2} + 2\pi n$;
 $\sin x = -\frac{1}{2}$ $x = (-1)^{k+1} \frac{\pi}{6} + \pi k$.
 в) $\cos^2 x - \sqrt{3} \sin x \cdot \cos x = 0$; $\cos x = 0$; $x = \frac{\pi}{2} + \pi n$.
 $\cos x \neq 0$; $\operatorname{tg} x = \frac{1}{\sqrt{3}}$; $x = \frac{\pi}{6} + \pi k$.
 2. $\cos x \leq -\frac{1}{2}$; $x \in [\frac{2\pi}{3} + 2\pi n; \frac{4\pi}{3} + 2\pi n]$.
 3. $\begin{cases} x + y = \pi \\ \cos x - \cos y = \sqrt{2} \end{cases}$; $\begin{cases} x = \pi - y \\ \cos y = -\frac{\sqrt{2}}{2} \end{cases}$; $y = \pm \frac{3\pi}{4} + 2\pi n$.
 4. $|2 \cos x + 1| \leq 1$; $\begin{cases} \cos x \leq 0 \\ \cos x \geq -1 \end{cases}$; $x \in [\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n]$.

КР № 3. В 3.

1. а). $\sin x = \frac{\sqrt{2}}{2}$; $x = (-1)^k \frac{\pi}{4} + \pi k$.

$$\text{б)} 2 \sin^2 x = \cos x + 1; \quad 2 \cos^2 x + \cos x - 1 = 0;$$

$$\cos x = -1. \quad x = \pi + 2\pi n; \quad \cos x = \frac{1}{2}, \quad x = \pm \frac{\pi}{3} + 2\pi n;$$

$$\text{в)} \sin^2 x - 2 \sin x \cdot \cos x = 3 \cos^2 x \quad \cos x \neq 0;$$

$$\operatorname{tg}^2 x - 2 \operatorname{tg} x - 3 = 0. \quad \operatorname{tg} x = 3 \quad x = \operatorname{arctg} 3 + \pi k.$$

$$\operatorname{tg} x = -1 \quad x = -\frac{\pi}{4} + \pi k.$$

$$2. \quad \operatorname{tg} x \geq -1 \quad x \in \left[-\frac{\pi}{4} + \pi n; \frac{\pi}{2} + \pi n\right).$$

$$3. \quad \begin{cases} x + y = \frac{\pi}{2} \\ \sin x + \sin y = -\sqrt{2} \end{cases}; \quad \begin{cases} x = \frac{\pi}{2} - y \\ \sin(y + \frac{\pi}{4}) = -1 \end{cases}; \quad \begin{cases} y = -\frac{3\pi}{4} + 2\pi n \\ x = \frac{5\pi}{4} - 2\pi n \end{cases}.$$

$$4. \quad 2 \sin^2 x + \sin x - 1 \leq 0. \quad \sin x \in [-1; \frac{1}{2}].$$

$$x \in [-\frac{7\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n].$$

КР № 3. В 4.

$$1. \quad \text{а)} \cos x = \frac{\sqrt{2}}{2}; \quad x = \pm \frac{\pi}{4} + 2\pi k.$$

$$\text{б)} 2 \cos^2 x - 1 = \sin x; \quad 2 \sin^2 x + \sin x - 1 = 0$$

$$\sin x = -1; \quad x = -\frac{\pi}{2} + 2\pi n; \quad \sin x = \frac{1}{2}; \quad x = (-1)^k \frac{\pi}{6} + \pi k.$$

$$\text{в)} \sin^2 x + \sin x \cdot \cos x = 2 \cos^2 x, \quad \cos x \neq 0.$$

$$\operatorname{tg}^2 x + \operatorname{tg} x - 2 = 0 \quad \operatorname{tg} x = -2 \quad x = -\operatorname{arctg} 2 + \pi k.$$

$$\operatorname{tg} x = 1 \quad x = \frac{\pi}{4} + \pi k.$$

$$2. \quad \operatorname{tg} x \leq \sqrt{3}. \quad x \in (-\frac{\pi}{2} + \pi n; \frac{\pi}{3} + \pi n].$$

$$3. \quad \begin{cases} x - y = \frac{\pi}{2} \\ \cos x - \cos y = -\sqrt{2} \end{cases}; \quad \begin{cases} x = \frac{\pi}{2} + y \\ \sin y + \cos y = \sqrt{2} \end{cases}; \quad \begin{cases} \sin(y + \frac{\pi}{4}) = 1 \\ x = \frac{\pi}{2} + y \end{cases}.$$

$$\begin{cases} y = \frac{\pi}{4} + 2\pi n. \\ x = \frac{3\pi}{4} + 2\pi n. \end{cases}$$

$$4. \quad 2 \cos^2 x - \cos x - 1 \leq 0 \quad \cos x \in \left[-\frac{1}{2}; 1\right].$$

$$x \in \left[-\frac{2\pi}{3} + 2\pi n; \frac{2\pi}{3} + 2\pi n\right].$$

КР № 4. В 1.

1. $y = x^2 \quad \Delta y = x_0^2 + 2x_0 \Delta x + (\Delta x)^2 - x_0^2 = (\Delta x)^2 + 2x_0 \Delta x.$
 $x_0 = 1 \quad \Delta x = 0,6 \quad \Delta y = 0,36 + 1,2 = 1,56.$
2. а) $f(x) > \frac{1}{3}x^3 + x^2 + 2x \quad f'(x) = x^2 + 2x + 2.$
 б) $\varphi(x) = \frac{2}{x^3} - x \quad \varphi'(x) = -\frac{6}{x^4} - 1$
 в) $g(x) = 4 \sin x \quad g'(x) = 4 \cos x \quad g'(-\frac{2\pi}{3}) = -2.$
 г) $h(x) = \frac{2-3x}{x+2} \quad h'(x) = \frac{-8}{(x+2)^2} \quad h'(-1) = -8.$
3. $f(x) = \frac{1}{3}x^3 - 4x; f'(x) = x^2 - 4 \quad g(x) = \sqrt{x} \quad g'(x) = \frac{1}{2\sqrt{x}}$
 $\frac{f'(x)}{g'(x)} = 2\sqrt{x}(x^2 - 4) = 0 \quad x = 0 \quad x = \pm 2, \text{ но } x \neq 0.$
 $x > 0 \Rightarrow x = 2.$
4. $f(x) = -0,5x|x| \quad \text{Да} \quad f'(0) = 0.$

КР № 4. В 2.

1. $y = \frac{1}{2}x^2 \quad \Delta y = \Delta x x_0 + (\Delta x)^2 \frac{1}{2} \quad x_0 = 1 \quad \Delta x = 0,8; \Delta y = 0,8 + 0,32 = 1,12.$
2. а) $f(x) = -\frac{2}{3}x^3 + 2x^2 - x \quad f'(x) = -2x^2 + 4x - 1.$
 б) $f(x) = \frac{4}{x^2} + x \quad f'(x) = -\frac{8}{x^3} + 1.$

$$\text{в)} g(x)=3\cos x \quad g'(x)=-3\sin x \quad g'(-\frac{5}{6}\pi)=\frac{3}{2}.$$

$$\text{г)} f(x)=\frac{3+2x}{x-2}; \quad h'(x)=\frac{-7}{(x-2)^2}; \quad h'(1)=-7.$$

$$3. \quad f(x)=\frac{2}{3}x^3-18x; \quad f'(x)=2x^2-18; \quad g(x)=2\sqrt{x}; \quad g'(x)=\frac{1}{\sqrt{x}};$$

$$\frac{f'(x)}{g'(x)}=2\sqrt{x}(x^2-9)x=0; \quad x=\pm 3, \text{ но } x>0 \Rightarrow x=3.$$

$$4. \quad f(x)=2x|x|, \quad \text{да, } f'(0)=0.$$

КР № 4. В 3.

$$1. \quad y=x^3 \quad f\left(\frac{1}{2}\right)=\frac{1}{8}; f(x_0+\Delta x)=8.; \begin{cases} 8=2\kappa+\vartheta. \\ \frac{1}{8}=\frac{1}{2}\kappa+\vartheta. \end{cases}; \frac{3}{2}\kappa=6\frac{3}{8}; \kappa=5,25.$$

$$2. \quad \text{а)} f(x)=\frac{2}{3}x^3-x^2-7x; \quad f'(x)=2x^2-2x-7.$$

$$\text{б)} \varphi(x)=\frac{1}{2x^3}+7; \quad \varphi'(x)=-\frac{1}{2x^4}.$$

$$\text{в)} g(x)=2\operatorname{tg}x; \quad g'(x)=2/\cos^2x; \quad g'(-\frac{3\pi}{4})=4.$$

$$\text{г)} h(x)=\frac{4x+1}{x+3}; \quad h'(x)=\frac{11}{(x+3)^2}; \quad h'(-2)=11.$$

$$3. \quad f(x)=x^3-6x^2; \quad f'(x)=3(x^2-4x); \quad g(x)=\frac{5x}{3}; \quad g'(x)=\frac{1}{6\sqrt{x}}.$$

$$f'(x)g'(x)=\frac{x^2-4x}{2\sqrt{x}}=0; \quad x=0 \text{ и } x=4, \text{ но } x>0 \Rightarrow x=4.$$

$$4. \quad f(x)=x^2+1; \quad f(g(x))=g^2(x)+1=x; \quad g(x)=\sqrt{x-1}.$$

КР № 4. В 4.

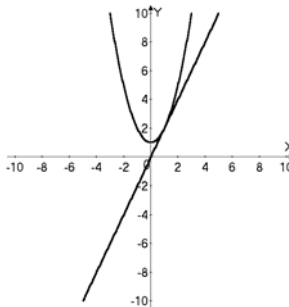
$$1. \quad y=\frac{1}{2}x^3; \quad y(0,6)=0,108; \quad y(2)=4.$$

$$\begin{cases} 4=2\kappa+\vartheta \\ 0,108=0,6\kappa+\vartheta \end{cases}; \quad \begin{matrix} 1,4\kappa=3,892. \\ \kappa=2,78. \end{matrix}$$

2. а) $f(x) = -\frac{1}{3}x^3 + 4x^2 + 2x$; $f'(x) = -x^2 + 8x + 2$.
 б) $\varphi(x) = \frac{2}{x^2} - 10$; $\varphi'(x) = -\frac{4}{x^3}$.
 в) $g(x) = 4 \operatorname{ctgx}$; $g'(x) = -\frac{4}{\sin^2 x}$; $\varphi'(-\frac{2n}{3}) = -\frac{16}{3}$.
 г) $h(x) = \frac{3x+4}{x-3}$; $h'(x) = \frac{-13}{(x-3)^2}$; $h'(4) = -13$.
3. $f(x) = x^3 - 3x^2$; $f'(x) = 3(x^2 - 2x)$; $g(x) = \frac{2}{3}\sqrt{x}$; $g'(x) = \frac{1}{3\sqrt{x}}$.
 $f(x)g'(x) = \frac{x^2 - 2x}{\sqrt{x}} = 0$; $x=0$ и $x=2$, но $x>0 \Rightarrow x=2$.
4. $f(x) = x^2 - 2$; $g(x^2 - 2) = x$; $g(x) = \sqrt{x+2}$.

КР. № 5. В1.

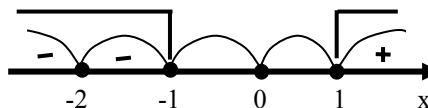
1. $\frac{x^2 - 9}{x - 5} < 0$;
 $x \in (-\infty; -3) \cup (3; 5)$.
2. $x(t) = t^2 + 5$; $v(t) = 2t$; $v(3) = 6$.
3. $f(x) = 2 - \frac{1}{x}$; $f'(x) = \frac{1}{x^2}$;
 $f'(1) = 1$; $\alpha = \frac{\pi}{4}$.



4. $f(x) = x^2 + 1$; $f(1) = 2$; $f'(x) = 2x$; $f'(1) = 2$;
 $y_k = 2 + 2(x-1) = 2x$.

5. $x(x^2 + 4x + 4)\sqrt{x^2 - 1} \leq 0;$

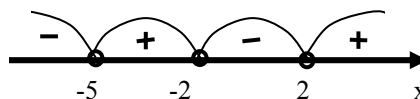
$x \in (-\infty; -1] \cup \{1\}.$



КР. № 5 В2.

1. $\frac{x^2 - 4}{x + 5} > 0;$

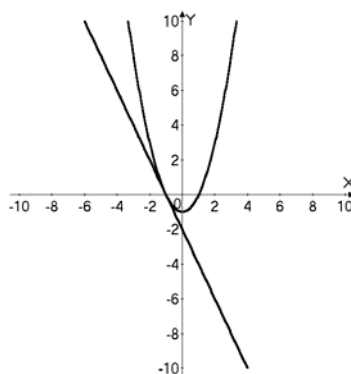
$x \in (-5; -2) \cup (2; +\infty).$



2. $x(t) = 3t^3 + 2t + 1;$
 $v(t) = 9t^2 + 2 \quad v(2) = 38.$

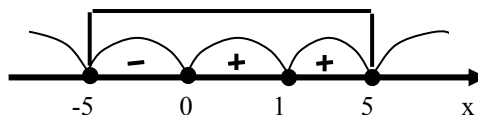
3. $f(x) = 3 - \frac{4}{x}; \quad f'(x) = \frac{4}{x^2}; \quad f'(2) = 1; \quad \alpha = \frac{\pi}{4}.$

4. $f(x) = x^2 - 1;$



$f(-1) = 0; \quad f'(x) = 2x; \quad f'(-1) = -2; y = -2x - 2.$

5. $x(x^2 - 2x + 1)\sqrt{25 - x^2} \geq 0; \quad x(x - 1)^2\sqrt{25 - x^2} \geq 0;$

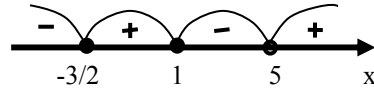


$x \in \{-5\} \cup [0; 5].$

КР. № 5 В3.

1. $\frac{(x-1)(2x+3)}{x-5} \leq 0;$

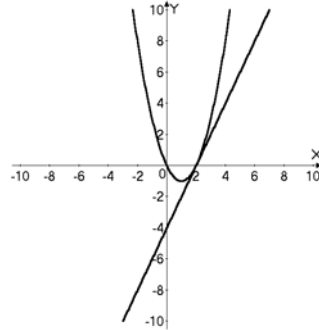
$x \in \left(-\infty; -\frac{3}{2}\right] \cup [1; 5).$



2. $x(t) = 3t^3 + 2t + 1;$ $v(t) = 9t^2 + 2;$
 $a(v) = 18t;$ $a(2) = 36.$

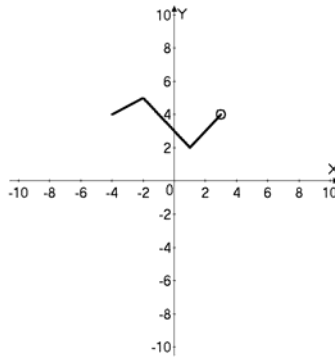
3. $f(x) = 1 - \frac{\sqrt{3}}{x};$ $f'(x) = \frac{\sqrt{3}}{x^2};$ $f'(-1) = \sqrt{3};$ $\alpha = \frac{\pi}{3}.$

4.



$f(x) = x^2 - 2x;$ $f(2) = 0;$ $f'(x) = 2x - 2;$
 $f'(2) = 2.;$ $y_{\text{кас}} = 2x - 4.$

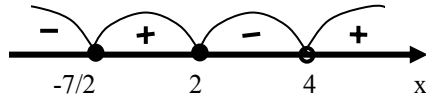
5.



КР. № 5 В4.

1. $\frac{(x-2)(2x+7)}{x-4} \geq 0;$

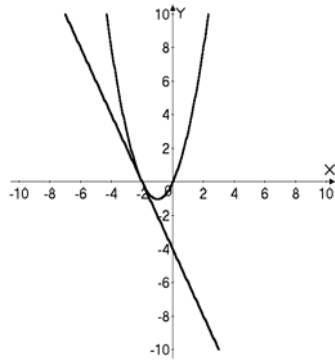
$x \in \left[-\frac{7}{2}; 2\right] \cup (4; +\infty).$



2. $x(t) = 2t^3 + 3t + 1; \quad v(t) = 6t^2 + 3; \quad a(t) = 12t; \quad a(3) = 36 \text{ м/с}^2.$

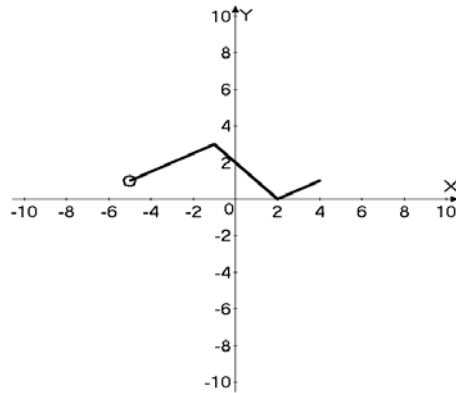
3. $f(x) = 2 - \frac{\sqrt{3}}{x}; \quad f'(x) = \frac{\sqrt{3}}{x^2}; \quad f'(1) = \sqrt{3}; \quad \alpha = \frac{\pi}{3}.$

4.



$f(x) = x^2 + 2x; \quad f'(x) = 2x + 2; \quad f(-2) = 0 \quad f'(-2) = -2.$
 $y = -2x - 4.$

5.



КР. № 6 В1.

1.

$$f(x) = x^3 - 3x^2 + 4;$$

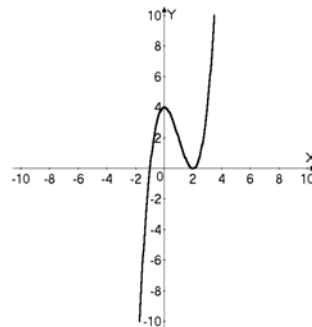
$$f'(x) = 3(x^2 - 2x) = 0;$$

$$x_{\max} = 0; \quad x_{\min} = 2; \quad f(0) = 4$$

$$f(2) = 0;$$

возрастает $x \in (-\infty; 0] \cup [2; +\infty)$

убывает $x \in [0; 2]$.



2.

$$\begin{cases} a + b = 12. \\ 2a^2b = y. \end{cases}; \begin{cases} b = 12 - a. \\ y = 24a^2 - 2a^3 \end{cases}$$

$$y' = 6a(8 - a) = 0$$

$$\begin{cases} a = 0 \\ b = 12. \\ y = 0. \end{cases}; \begin{cases} a = 8 \\ b = 4 \\ y = 512. \end{cases} \quad 8+4=12.$$

3.

$$\varphi(x) = -4,3x \cos^2 x + \sin x = -4,3x - \cos 2x.$$

$$\varphi'(x) = -4,3 + 2 \sin 2x < 0.$$

КР. № 6 В2.

1.

$$f(x) = -x^3 + 3x^2 - 4$$

$$f'(x) = -3x(x - 2) = 0.$$

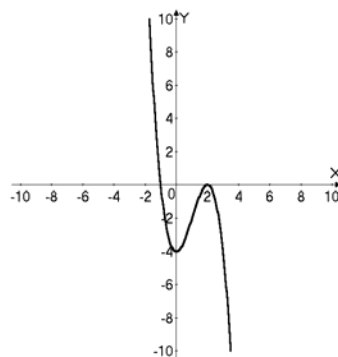
$$x = 0 \quad x = 2.$$

возрастает: $x \in [0; 2]$.

убывает: $x \leq 0, \quad x \geq 2$

$$f(0) = \min = -4$$

$$f(2) = \max = 0.$$



2.

$$\begin{cases} a + b = 9 \\ y = a^2 \cdot 3b \end{cases};$$

$$\begin{cases} b = 9 - a \\ y = 27a^2 - 3a^3 \end{cases}; \quad y' = 9a(6 - a) \quad a = 6 \quad b = 3. \quad 6 + 3 = 9$$

$$3. \quad f(x) = 2 \sin x \cdot \sin\left(\frac{\pi}{2} + x\right) + 3, 2x = \sin 2x + 3, 2x.$$

$$f'(x) = 2 \cos 2x + 3, 2 > 0.$$

КР. № 6 В3.

1.

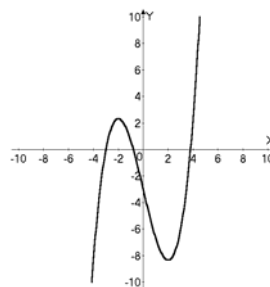
$$f(x) = \frac{1}{3}x^3 - 4x - 3;$$

$$f'(x) = x^2 - 4 = 0; \quad x = \pm 2.$$

$$\max = f(-2) = \frac{7}{3}; \quad \min = f(2) = -\frac{25}{3};$$

$f(x)$ убывает на $x \in [-2; 2]$

возрастает на $x \leq -2$ и $x \geq 2$.



$$2. \quad \begin{cases} a + b = 8. \\ a^3 b = y. \end{cases}; \quad \begin{cases} b = 8 - a. \\ y = 8a^3 - a^4 \end{cases}; \quad \begin{matrix} y' = 4a^2(6 - a) \\ a = 6 \quad b = 2 \quad 6 + 2 = 8. \end{matrix}$$

$$3. \quad f(x) = c \text{ — одно решение, тогда, } c < -8\frac{1}{3}, \quad c > 2\frac{1}{3}.$$

$$2 \text{ решения } c = -8\frac{1}{3}, \quad c = 2\frac{1}{3}; \quad 3 \text{ решения, тогда, } c \in \left(-8\frac{1}{3}; 2\frac{1}{3}\right).$$

КР. № 6 В4.

1.

$$f(x) = -\frac{1}{3}x^3 + 4x + 3;$$

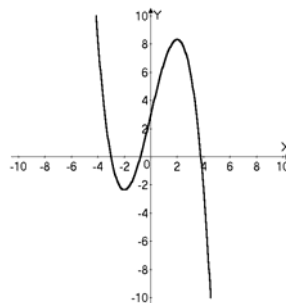
$$f'(x) = -x^2 + 4 = 0;$$

$$x = \pm 2 \quad \min = f(-2) = -\frac{7}{3};$$

$$\max = f(2) = \frac{25}{3};$$

$f(x)$ возрастает на $x \in [-2; 2]$

убывает на $x \leq -2, \quad x \geq 2$.

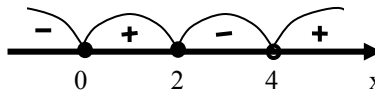


$$2. \quad \begin{cases} a + b = 12. \\ y = 2a^2 b \end{cases}; \quad \begin{cases} b = 12 - a. \\ y = 24a^3 - 2a^4 \end{cases}; \quad \begin{matrix} y' = 8a^2(9 - a); \\ a = 9 \quad b = 3; \quad 12 = 9 + 3. \end{matrix}$$

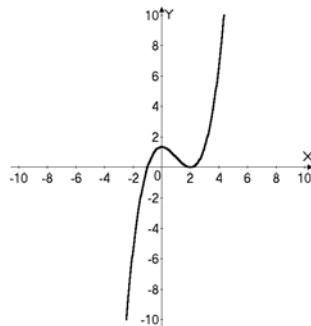
3. $f(x) = m - 1$ решение, тогда, $m > 8\frac{1}{3}$, $m < -2\frac{1}{3}$;
 2 корня $m = 8\frac{1}{3}$ $m = -2\frac{1}{3}$; 3 корня $m \in \left(-2\frac{1}{3}; 8\frac{1}{3}\right)$.

КР. № 7 В1.

1. а) $2\sin^2 - 1 = 0$; $\sin x = \pm \frac{\sqrt{2}}{2}$; $x = \frac{\pi}{4} + \frac{\pi n}{2}$.
 б) $\sin 2x + \sqrt{3} \cos 2x = 0$; $\cos 2x \neq 0$; $\operatorname{tg} 2x = -\sqrt{3}$; $x = -\frac{\pi}{6} + \frac{\pi n}{2}$.
1. $f(x) = \frac{2x}{2+x} - 3\sin x$; $f'(x) = \frac{4}{(2+x)^2} - 3\cos x$; $f'(0) = -2$.
2. а) $2\cos x - \sqrt{2} > 0$; $\cos x > \frac{\sqrt{2}}{2}$; $x \in \left(-\frac{\pi}{4} + 2\pi n; \frac{\pi}{4} + 2\pi n\right)$.
- б) $\frac{2x-x^2}{x-4} \geq 0$;
 $\frac{x(x-2)}{x-4} \leq 0$; $x \in (-\infty; 0] \cup [2; 4)$.



4.



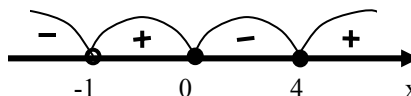
$$-1 \leq x \leq 3.$$

5. $(4x^2 - 9)(x^2 + x + 1) < 0$; $x \in \left(-\frac{3}{2}; \frac{3}{2}\right)$;
 $\cos x > 0$; $x \in \left(-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n\right)$, но $\frac{\pi}{2} > \frac{3}{2} \Rightarrow$

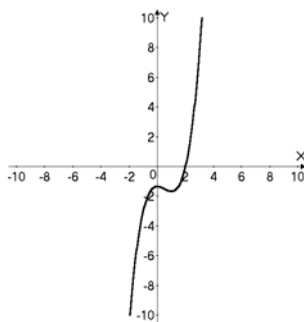
при $x \in \left(-\frac{3}{2}; \frac{3}{2}\right)$; $\cos x > 0$.

КР. № 7 В2.

1. а) $2 \cos^2 x - 1 = 0$; $\cos x = \pm \frac{\sqrt{2}}{2}$; $x = \frac{\pi}{4} + \frac{\pi n}{2}$;
 б) $3 \sin 2x - \sqrt{3} \cos 2x, \cos 2x \neq 0$; $\operatorname{tg} 2x = \frac{\sqrt{3}}{3}$; $x = \frac{\pi}{12} + \frac{\pi k}{2}$.
2. $f(x) = \frac{3x}{x+3} + 7 \cos x$; $f'(x) = \frac{9}{(x+3)^2} - 7 \sin x$; $f'(0) = 1$.
3. а) $2 \sin x - \sqrt{3} > 0$; $\sin x > \frac{\sqrt{3}}{2}$; $x \in \left(\frac{\pi}{3} + 2\pi n; \frac{2\pi}{3} + 2\pi n\right)$;
 б) $\frac{4x-x^2}{x+1} \leq 0$; $\frac{x(x-4)}{x+1} \geq 0$;
 $x \in (-1; 0] \cup [4; +\infty)$.



4.



- $f(x) \in [-3; 0]$ при $x \in [-1; 2]$.
5. $(x^2 - 3x)(x^2 - x + 1) < 0$; $x \in (0; 3)$;
 $\sin x > 0$, при $x \in (2\pi n; \pi + 2\pi n)$;
 т.к. $\pi > 3$, то $\sin x > 0$, при $x \in (0; 3)$.

КР. № 7 В3.

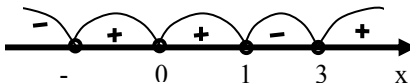
1. а) $4 \sin^2 x - 3 = 0$; $\sin x = \pm \frac{\sqrt{3}}{2}$; $x = (-1)^k \frac{\pi}{3} + \pi k$;
 $x = (-1)^{k+1} \frac{\pi}{3} + \pi n$.

б) $\sin\left(2x + \frac{\pi}{3}\right) + \cos\left(2x + \frac{\pi}{3}\right) = 0$; $\cos\left(2x + \frac{\pi}{3}\right) \neq 0 \Rightarrow$
 $\operatorname{tg}\left(2x + \frac{\pi}{3}\right) = -1$; $x = -\frac{7\pi}{24} + \frac{\pi n}{2}$.

2. $f(x) = \frac{x^2 + 1}{x + 1} + 2 \cos x$; $f'(x) = \frac{2x^2 + 2x - x^2 - 1}{x + 1} - 2 \sin x$;
 $f'(0) = -1$.

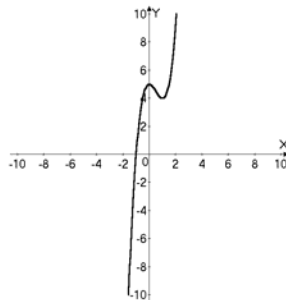
3. а) $2 \cos x + \sqrt{2} \leq 0$; $\cos x \leq \frac{\sqrt{2}}{2}$; $x \in \left[\frac{\pi}{4} + 2\pi n; \frac{7\pi}{4} + 2\pi n\right]$.

б) $\frac{x^2(x^2 - 1)}{x - 3} > 0$;



$x \in (-1; 0) \cup (0; 1) \cup (3; +\infty)$.

4. $y = 2x^3 - 3x^2 + 5$; $y \geq 0$ при $x \geq -1$.



5. $(x^2 + 1)(x^2 - 5x + 6) < 0$; $x \in (2; 3)$; $\sin \frac{x}{2} > 0$;
 $x \in (4\pi n; 2\pi + 4\pi n)$;

т.к. $2 > 0$, $2\pi > 3 \Rightarrow \sin \frac{x}{2} > 0$ при $x \in (2; 3)$.

КР. № 7 В4.

1. а) $4 \cos^2 x - 3 = 0$; $\cos x = \pm \frac{\sqrt{3}}{2}$; $x = \pm \frac{\pi}{3} + 2\pi n$; $x = \pm \frac{2\pi}{3} + 2\pi n$;

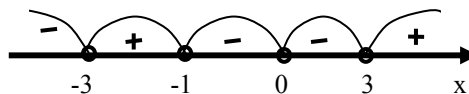
б) $\sin\left(2x - \frac{\pi}{4}\right) - \cos\left(2x - \frac{\pi}{4}\right) = 0$; $\sin\left(2x - \frac{\pi}{4}\right) = 0$; $x = \frac{\pi}{4} + \frac{\pi n}{2}$.

2. $f(x) = \frac{x^2 + 2}{x + 2} - 2 \sin x$; $f'(x) = \frac{2x^2 + 4x - x^2 - 2}{(x + 2)^2} - 2 \cos x$;

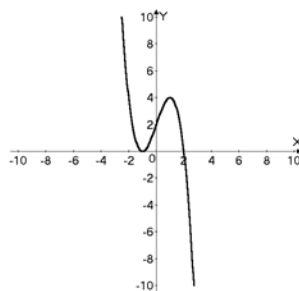
$f'(0) = -\frac{1}{2} - 2 = -\frac{5}{2}$.

3. а) $2 \sin x + \sqrt{3} \leq 0$; $\sin x \leq -\frac{\sqrt{3}}{2}$; $x \in \left[-\frac{2\pi}{3} + 2\pi n, -\frac{\pi}{3} + 2\pi n\right]$.

б) $\frac{x^2(x^2 - 9)}{x + 1} < 0$; $x \in (-\infty; -3) \cup (-1; 0) \cup (0; 3)$.



4.



$x > 2$.

5. $(x^2 + 3)(x^2 - 10x + 24) < 0$; $x \in (4; 6)$; $\cos \frac{x}{2} < 0$;

$x \in (\pi + 4\pi n, 3\pi + 4\pi n)$; т.к. $4 > \pi$, $6 < 3\pi$, то $\cos \frac{x}{2} < 0$;

при $x \in (4; 6)$.

Материалы для итогового повторения

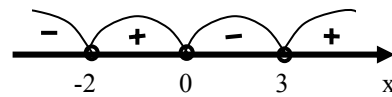
В1.

$$1. \quad 2 \cos^2 x + \cos x = 0; \quad \begin{array}{l} \cos x = 0 \\ \cos x = \frac{1}{2} \end{array} \quad \begin{array}{l} x = \frac{\pi}{2} + \pi n; \\ x = \pm \frac{2\pi}{3} + 2\pi n. \end{array}$$

$$2. \quad f(x) = x^{-2} + \frac{1}{2} \sin 2x; \quad f'(x) = \cos 2x - \frac{2}{x^3}.$$

$$3. \quad y = \frac{\sqrt{9-x^2}}{\sin x - 1}; \quad \text{ОДЗ: } \begin{cases} x^2 - 9 \leq 0 \\ \sin x \neq 1 \end{cases}; \quad \begin{cases} x \in [-3; 3] \\ x \neq \frac{\pi}{2} + 2\pi n \end{cases}; \quad \begin{cases} x \in [-3; 3] \\ x \neq \frac{\pi}{2} \end{cases}.$$

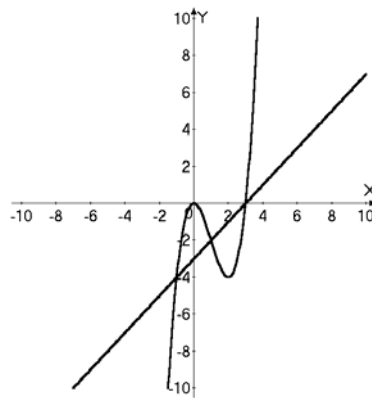
$$4. \quad \frac{x(x+2)}{x-3} \leq 0; \quad x \in (-\infty; -2] \cup [0; 3).$$



5.

$$x^2(x-3) = x-3; \quad x=3 \quad x=\pm 1.$$

3 точки пересечения.



B2.

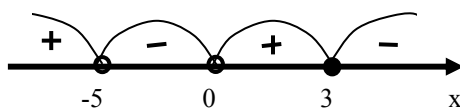
$$1. \quad 2 \sin x - 1 < 0; \quad \sin x < \frac{1}{2}; \quad x \in \left(-\frac{7\pi}{6} + 2\pi n; -\frac{\pi}{6} + 2\pi n \right).$$

$$2. \quad f(x) = x^{-1} - 2 \cos \frac{x}{2}; \quad f'(x) = \sin \frac{x}{2} - \frac{1}{x^2}.$$

$$3. \quad \sqrt{x-5}(\sin^2 x - 3 \sin x) > 0; \quad \begin{cases} x \geq 5 \\ x = 5 \\ \sin x = 0. \end{cases}$$

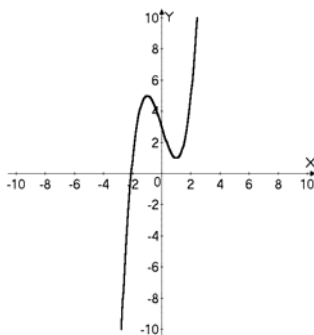
$$x = \pi n, \quad n \geq 2, \quad x = 2.$$

$$4. \quad \frac{3-x}{x(x+5)} \geq 0;$$



$$x \in (-\infty; -5) \cup (0; 3]$$

5.



$$y = x^3 - 3x + 3; \quad y' = 3(x^2 - 1) = 0;$$

$$x = \pm 1; \quad y(1) = 1; \quad y\left(-\frac{1}{2}\right) = \frac{9}{2} - \frac{1}{8} = \frac{35}{8}; \quad y(3) = 21;$$

$$\max -f(3) = 21.$$

$$\min -f(1) = 1.$$

B3.

$$1. \quad f(x) = 2 \sin x \cdot \sin\left(\frac{\pi}{2} - x\right) = \sin 2x; \quad f'(x) = 2 \cos 2x;$$

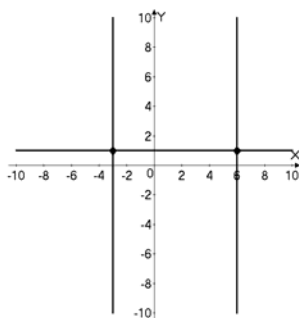
$$f'(\pi) = 2.$$

$$2. \quad f(x) = \frac{2x+5}{3-x}; \quad f'(x) = \frac{11}{(x-3)^2} > 0 \text{ при } x \neq 3, \text{ то есть при}$$

$$x \in (-\infty; 3) \cup (3; \infty).$$

$$3. \quad \sin\left(\frac{5}{3}\pi + x\right) - \sin\left(\frac{4}{3}\pi + x\right) = 2 \sin \frac{\pi}{6} \cos\left(\frac{3\pi}{2} + x\right) = \sin x.$$

$$4. \quad (y-1)(x^2 - 3x - 18) = 0; \quad \begin{cases} y = 1 \\ x = 6 \\ x = -3 \end{cases}.$$



5.

$$y = 4x^2(x-2)^2 = 4x^4 - 16x^3 + 16x^2;$$

$$y' = 16x(x^2 - 3x + 2) = 0;$$

$$x = 0, \quad x = 2, \quad x = 1;$$

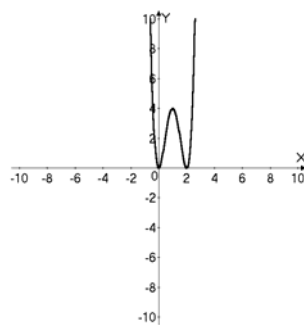
$$y_{\max} = y(1) = 4;$$

$$y_{\min} = y(0) = y(2) = 0;$$

$$\text{нули: } x = 0 \quad x = 2.$$

$$\text{убывает: } x \leq 0, \quad x \in [1; 2]$$

$$\text{возрастает: } x \in [0; 1] \cup x \geq 2.$$



B4.

$$1. \quad f(x) = 2 \cos x \cdot \cos\left(\frac{\pi}{2} - x\right) = \sin 2x; \quad f'(x) = 2 \cos 2x; \quad f'\left(\frac{\pi}{2}\right) = -2.$$

$$2. \quad x(t) = 3t^4 + 2t^3 + 6; \quad v(t) = 6t^2(2t + 1); \quad a(f) = 36t^2 + 12t.$$

$$v(2) = 120 \quad a(2) = 168.$$

$$3. \quad \cos\left(\frac{4\pi}{3} + x\right) + \cos\left(\frac{2\pi}{3} + x\right) = 2 \cos(\pi + x) \cos \frac{\pi}{3} = -\cos x.$$

4.

$$y = \frac{1}{4}x^2(x-4)^2 = \frac{1}{4}x^4 - 2x^3 + 4x^2;$$

$$y' = x^3 - 6x^2 + 8x = 0;$$

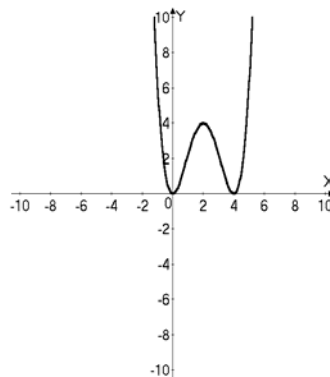
$$x = 0 \quad x = 4 \quad x = 2.$$

$$y_{\max} = y(2) = 4$$

$$y_{\min} = y(0) = y(4) = 0.$$

возрастает: $x \in [0; 2] \cup x \geq 4$

убывает: $x \leq 0, \quad x \in [2; 4]$



$$5. \quad \begin{cases} x + y = \frac{\pi}{2} \\ \sin x - \sin y = \sqrt{2} \end{cases};$$

$$\begin{cases} y = \frac{\pi}{2} - x \\ \sin\left(x - \frac{\pi}{4}\right) = 1 \end{cases}; \quad \begin{cases} x = \frac{3\pi}{4} + 2\pi n \\ y = -\frac{\pi}{4} - 2\pi n \end{cases}.$$

B5.

$$1. \quad \frac{\sin 2\alpha}{1 + \cos 2\alpha} = \frac{2 \sin \alpha \cdot \cos \alpha}{2 \cos^2 \alpha} = \operatorname{tg} \alpha.$$

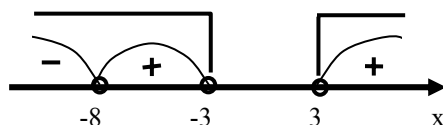
$$2. \quad \operatorname{tg}^2 x + 3 \operatorname{tg} x - 4 = 0; \quad \begin{matrix} \operatorname{tg} x = -4 \\ \operatorname{tg} x = 1 \end{matrix};$$

$$x = \operatorname{arctg}(-4) + \pi n$$

$$x = \frac{\pi}{4} + \pi n.$$

$$3. \quad f(x) = (3 - 2x)^6; \quad f'(x) = -12(3 - 2x)^5; \quad f'(1) = -12.$$

4. $\sqrt{x^2 - 9}(x + 8) > 0; \quad x \in (-8; -3) \cup (3; +\infty).$



5. $y = (x - 1)^2(2x + 4);$

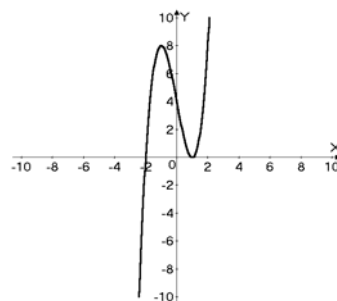
$$\begin{aligned} y' &= 2(x - 1)(2x + 4) + 2(x - 1)^2 = \\ &= 2(x - 1)(2x + 4 + x - 1) = \\ &= 2(x - 1)(3x + 3) = 0. \end{aligned}$$

$$x_{\min} = 1 \quad x_{\max} = -1;$$

$$y(1) = 0; \quad y(-1) = 8;$$

y возрастает на $x \in (-\infty; -1) \cup (1; \infty);$

убывает на $x \in [-1; 1]$



B6.

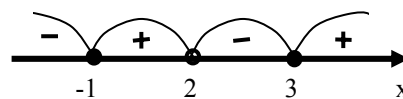
1. $\sin \alpha = \frac{\sqrt{3}}{2}; \quad 0 < \alpha < 90^\circ; \quad \cos \alpha = \frac{1}{2};$

$$\sin(30^\circ + \alpha) = \frac{1}{2} \cos \alpha + \frac{\sqrt{3}}{2} \sin \alpha = \frac{1}{4} + \frac{3}{4} = 1.$$

2. $y = 0,5x^2 - 2x; \quad y(4) = 0; \quad y' = x - 2; \quad y'(4) = 2;$

$$y_{\text{кас}} = 2(x - 4) = 2x - 8.$$

3. $\frac{3}{x-2} \geq x; \quad \frac{3-x^2+2x}{x-2} \geq 0; \quad \frac{x^2-2x-3}{x-2} \leq 0;$
 $x \in (-\infty; -1] \cup (2; 3].$



4. $1 - \cos^2 x = \sin 2x;$

$$\sin^2 x - 2 \sin x \cdot \cos x - 3 \cos^2 x = 0, \quad \cos x \neq 0;$$

$$\begin{aligned} \operatorname{tg}^2 x - 2\operatorname{tg} x - 3 &= 0; & \operatorname{tg} x &= 3; & x &= \operatorname{arctg} 3 + \pi n; \\ \operatorname{tg} x &= -1; & x &= -\frac{\pi}{4} + \pi n. \end{aligned}$$

5.

$$g(x) = x^4 - 2x^2 + 3;$$

$$g'(x) = 4x(x^2 - 1) = 0;$$

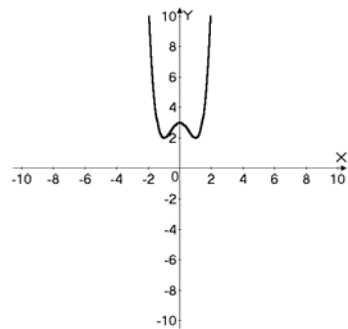
$$x_{\max} = 0 \quad x_{\min} = \pm 1. \quad g(0) = 3; \quad .$$

$$g(\pm 1) = 2;$$

убывает на $x \in (-\infty; -1) \cup (0; 1)$;

возрастает на:

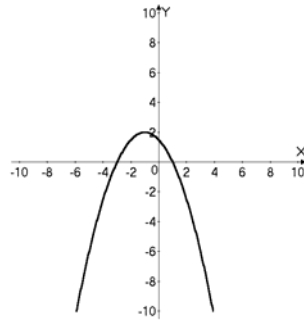
$$x \in [-1; 0] \cup [1; +\infty).$$



B7.

$$1. \quad \cos \alpha = -\frac{3}{5}; \quad \frac{\pi}{2} < \alpha < \pi; \quad \sin \alpha = \frac{4}{5}; \quad \sin 2\alpha = -\frac{24}{25}.$$

2.



$$3. \quad \frac{\cos\left(\alpha + \frac{5\pi}{4}\right) - \cos\left(\alpha - \frac{5\pi}{4}\right)}{\sqrt{2} \sin(\pi + \alpha)} = \frac{-2 \sin \alpha \cdot \sin \frac{5\pi}{4}}{-\sqrt{2} \sin \alpha} = -\frac{\sqrt{2}}{2} \cdot \frac{2}{\sqrt{2}} = -1.$$

$$4. \quad y = \frac{x - 2,5}{x^2 - 4} \quad \text{а) ОДЗ: } x \neq \pm 2, \text{ значит, } y \text{ непрерывна на}$$

$$x \in (-\infty; -2) \cup (2; \infty);$$

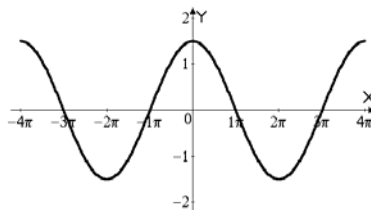
$$6) y = \frac{x-2,5}{x^2-4}; \quad y' = \frac{x^2-4-2x^2+5x}{(x^2-4)^2} = \frac{-x^2+5x-4}{(x^2-4)^2} = 0;$$

$x=4 \quad x=1$; возрастает на $x \in (1; 2) \cup (2; 4)$.

$$5. \quad \begin{cases} a+b+c=54 \\ a=2b \\ y=abc. \end{cases}; \quad \begin{cases} c=54-3b \\ y=108b^2-6b^3 \\ a=2b. \end{cases}; \quad \begin{cases} y'=18b(12-b)=0 \\ b=12 \\ a=24 \\ c=18. \end{cases}$$

В 8.

1. $\sin(-840^\circ) + \operatorname{tg}(-855^\circ) = -\sin 120^\circ - \operatorname{tg} 135^\circ = -\frac{\sqrt{3}}{2} + 1 = \frac{2-\sqrt{3}}{2}$.
2. $f(x) = (2x-4)(x+1)^2 \quad f'(x) = 2(x+1)(2x-4) + 2(x+1)^2 = 2(x+1)(2x-4+x+1) = 2(x+1)(3x-3) = 0; \quad x = \pm 1.$
 $f(1) = \max = -8 \quad f(-1) = \min = 0.$
 возрастает $x \leq -1, \quad x \geq 1.$ убывает: $x \in [-1; 1]$
3. $2\sin^2 x - 1 = \sin x. \quad 2\sin^2 x - \sin x - 1 = 0.$
 $\sin x = 1 \quad x = \frac{\pi}{2} + 2\pi n \quad \sin x = -\frac{1}{2} \quad x = (-1)^{k+1} \frac{\pi}{6} + \pi k.$
- 4.



$$5. \quad \begin{cases} a+b+c=48 \\ a=b \\ y=abc. \end{cases}; \quad \begin{cases} c=48-2b \\ y=48b^2-2b^3 \end{cases}; \quad \begin{cases} y'=6b(16-b)=0 \\ b=16. \\ a=16 \\ c=16. \end{cases}$$

Карточки-задания для проведения зачетов

Зачет № 1 .

Карточка 1.

1. ф-ия – зависимость y от x , при котором для каждого допустимого x ставится в соответствие зн. y .

обл. опр. ф-ции допустимые зн. x ; обл. зн. ф-ции допустимые зн. y

Схема исследования ф-ции:

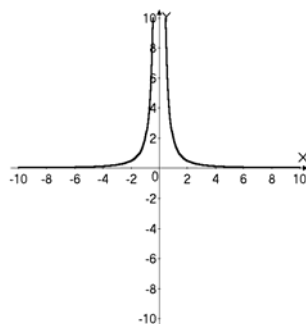
1) обл. зн., обл. опр.; 2) нули; 3) экстремумы; 4) \max , \min ;

5) промежутки возраст., убыв.

2. а) $\sin(-1830^\circ) = -\sin 30^\circ = -\frac{1}{2}$; б) $\cos(-1140^\circ) = \cos 60^\circ = \frac{1}{2}$;

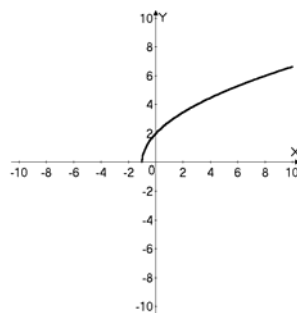
в) $\operatorname{tg}(-585^\circ) = \operatorname{tg} 135^\circ = -1$.

3.

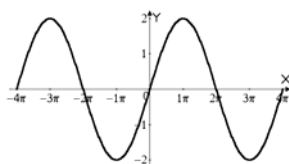


$f(x) = \frac{2}{x^2}$ возрастает на $x < 0$; убывает на $x > 0$

4.



5.



Карточка 2.

1. Четная функция, когда $f(-x) = f(x)$;
 например $y = x^2$; $y = x^4$ - график симметричен относительно ОУ;
 нечетная функция, когда $f(-x) = -f(x)$;
 например: $y = x$, $y = x^3$, график симметричен относ. О.

2. $f(x) = \frac{\sqrt{9-x^2}}{x+2}$; ОДЗ: $\begin{cases} 9-x^2 \geq 0 \\ x \neq -2 \end{cases}$; $x \in [-3; -2) \cup (-2; 3]$.

3. $f(x) = \frac{3}{\sin \frac{x}{2}} > 0$; $\sin \frac{x}{2} > 0$; $x \in (4\pi n; 2\pi + 4\pi n)$;

$f(x) < 0$ $x \in (2\pi + 4\pi n; 4\pi + 4\pi n)$.

4.

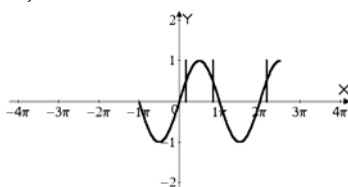
а)

$\sin x = 1$; $x = \frac{\pi}{2} + 2\pi n$;

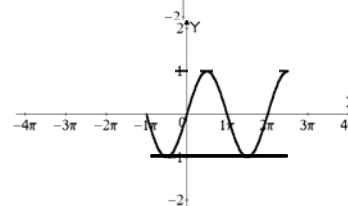
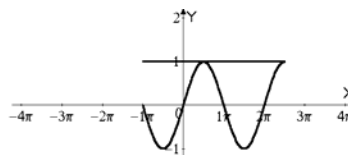
б)

$\sin x = -1$; $x = -\frac{\pi}{2} + 2\pi n$

в)

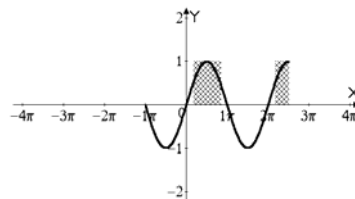


$\sin x = \frac{1}{2}$; $x = (-1)^k \frac{\pi}{6} + \pi k$

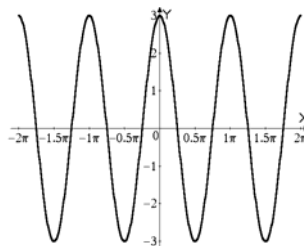


$$r) \sin x > \frac{1}{2};$$

$$x \in \left(\frac{\pi}{6} + 2\pi n; \frac{5\pi}{6} + 2\pi n \right).$$



5.



Карточка 3.

1. Пусть функция является периодич. и T - ее период то

$$f(x) = f(x + T); \sin x, \cos x \quad T = 2\pi; \operatorname{tg} x, \operatorname{ctg} x \quad T = \pi.$$

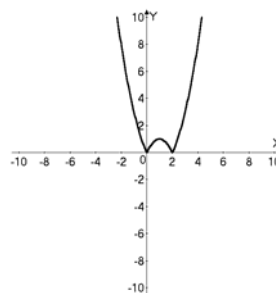
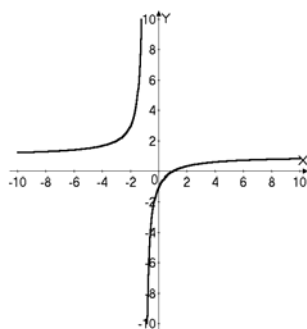
$$2. \quad f(x) = \frac{2x^2 + 1}{2 \cos x}; \quad f(-x) = \frac{2(-x)^2 + 1}{\cos(-x)} = \frac{2x^2 + 1}{\cos x} = f(x) \Rightarrow \text{четная}.$$

3.

$$y = |x^2 - 2x|; \quad \min: (0;0); (2;0); \quad \max (1;1)$$

$$4. \quad y = 2 \sin 2x \cos 2x = \sin 4x; \quad T = \frac{\pi}{2}.$$

5.

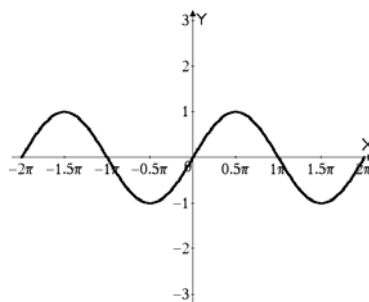


КАРТОЧКА 4.

$$1. \quad y = \sin x$$

$$x \in R$$

$$y \in$$



$[-1; 1]$ возрастает на

$$\left[-\frac{\pi}{2} + 2\pi n; \frac{\pi}{2} + 2\pi n \right]$$

убывает на $\left[\frac{\pi}{2} + 2\pi n; \frac{3\pi}{2} + 2\pi n \right]$

нули: $x = \pi n$;

$$\min: \left(-\frac{\pi}{2} + 2\pi n; -1 \right);$$

$$\max: \left(\frac{\pi}{2} + 2\pi n; 1 \right).$$

$$2. f(x) = \frac{\sqrt{2x-1}}{2x^2-3x-5}; \quad \text{ОДЗ: } \begin{cases} x \geq \frac{1}{2} \\ 2x^2-3x-5 \neq 0 \end{cases};$$

$$\begin{cases} x \neq \frac{5}{2} & x \neq -1 \\ x \geq \frac{1}{2} \end{cases}; \quad \text{Итого: } x \in \left[\frac{1}{2}; \frac{5}{2} \right) \cup \left(\frac{5}{2}; +\infty \right).$$

$$3. f(x) = 3 \operatorname{tg}(2x-4); \quad T = \frac{\pi}{2}.$$

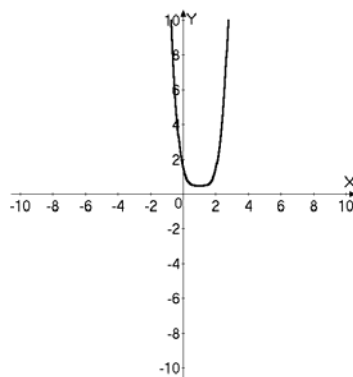
4.

$$f(x) = (x-1)^4 + \frac{1}{2};$$

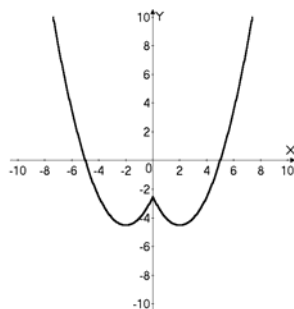
$$x_{\min} = 1; \quad f(1) = \frac{1}{2};$$

возрастает на $x \in (1; \infty)$;

убывает на $x \in (-\infty; 1)$.



5.



КАРТОЧКА 5.

1.

$$y = \cos x; \quad \text{нули: } x = \frac{\pi}{2} + \pi k;$$

$$x \in R; \quad y \in [-1; 1];$$

y возрастает на $x \in [-\pi + 2\pi n; 2\pi n];$

убывает на $x \in [2\pi n; \pi + 2\pi n];$

max: $(2\pi n; 1);$

min: $(-\pi + 2\pi n; -1).$

$$2. f(x) = -2x^2 + 3x + 4;$$

$$f(-1) = -2 - 3 + 4 = -1; f(x+1) = -2x^2 - 4x - 2 + 3x + y = -2x^2 - x + 5 = -1;$$

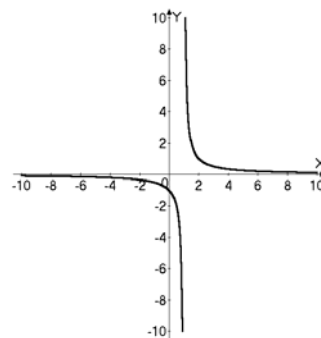
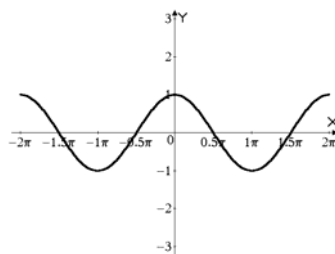
$$2x^2 + x - 6 = 0; x_1 = -2, \quad x_2 = \frac{3}{2}.$$

$$3. f(x) = \operatorname{tg}\left(2x - \frac{\pi}{3}\right); \cos\left(2x - \frac{\pi}{3}\right) \neq 0; \quad x \neq \frac{5\pi}{12} + \frac{\pi n}{2};$$

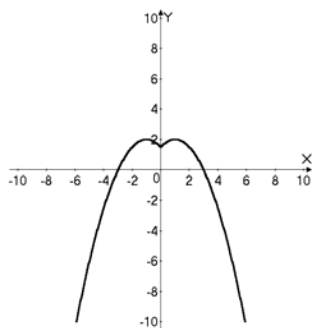
возрастает на области определения

$$4. f(x) = \frac{1}{x-1}$$

убывает: $x \neq 1$



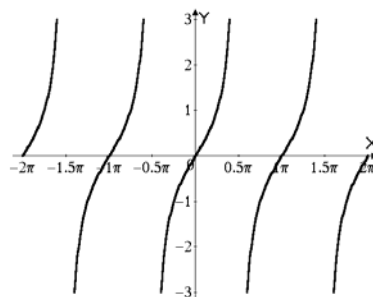
5.



КАРТОЧКА 6.

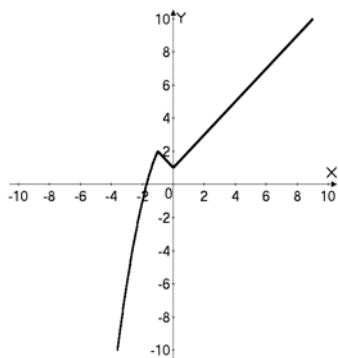
1.

$y = \operatorname{tg} x$; $y \in R$; $x \neq \frac{\pi}{2} + \pi k$;
возрастает на области
определения; нули: $x = \pi k$



2.

возрастает на



$x \in (-\infty; -1) \cup (0; \infty)$;

убывает на $x \in [-1; 0]$

$x = -1$; $f(-1) = 2$; $x = 0$; $f(0) = 1$.

3. $y = \frac{\sqrt{x^2 - 16}}{x + 4}$; ОДЗ: $\begin{cases} x^2 - 16 \geq 0; \\ x \neq -4 \end{cases}$;

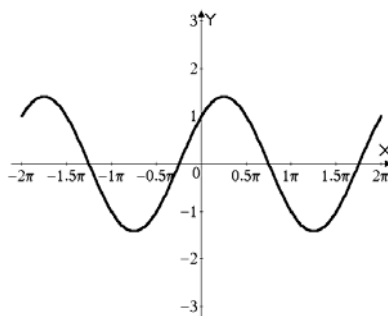
$x \in (-\infty; -4) \cup [4; +\infty)$.

4. $y = 2\sin x + 3$;

max: $(\frac{\pi}{2} + 2\pi n; 5)$;

min: $(-\frac{\pi}{2} + 2\pi n; 1)$.

5.



ЗАЧЕТ № 2

КАРТОЧКА 1.

1. \arcsin числа a – такое число из $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$, \sin которого равен a .

2. а) $2\cos^2 x + 3\cos x + 1 = 0$;

$$\cos x = -1; \quad x = \pi + 2\pi n; \quad \cos x = -\frac{1}{2}; \quad x = \pm \frac{2\pi}{3} + 2\pi n;$$

б) $\sin^2 x + \sqrt{3} \sin x \cos x = 0$;

$$\sin x(\sin x + \sqrt{3} \cos x) = 0; \quad x = \pi n; \quad x = -\frac{\pi}{3} + \pi n.$$

3. $\operatorname{tg} 3x < -1$; $x \in \left(-\frac{\pi}{6} + \frac{\pi n}{3}; -\frac{\pi}{12} + \frac{\pi n}{3}\right)$.

$$4. \begin{cases} x - y = \pi \\ \sin(x + y) = -1 \end{cases}; \begin{cases} x + y = -\frac{\pi}{2} + 2\pi n; \\ x - y = \pi \end{cases}; \begin{cases} x = \frac{\pi}{4} + \pi n \\ y = -\frac{3\pi}{4} + \pi n \end{cases}.$$

$$5. |2\sin x + 4| \leq 5; \quad \begin{cases} \sin x \leq \frac{1}{2} \\ \sin x \geq -\frac{9}{2} \end{cases}; \quad x \in \left[-\frac{7\pi}{6} + 2\pi n; \frac{\pi}{6} + 2\pi n\right].$$

КАРТОЧКА 2.

1. $\arccos a$ – такое число из $[0; 2\pi]$, \cos которого равен a .
2. $\operatorname{tg} x + \operatorname{ctg} x = 2$; $\operatorname{tg} x = 1$; $x = \frac{\pi}{4} + \pi n$.
3. $2\sin^2 x + 5\sin x \cos x - 7\cos^2 x = 0$; $\cos x \neq 0$; $2\operatorname{tg}^2 x + 5\operatorname{tg} x - 7 = 0$;
 $\operatorname{tg} x = \frac{7}{2}$; $x = -\operatorname{arctg} \frac{7}{2} + \pi n$; $\operatorname{tg} x = 1$; $x = \frac{\pi}{4} + \pi n$.
3. $\cos\left(\frac{\pi}{2} + x\right) < -\frac{\sqrt{3}}{2}$; $\sin x > \frac{\sqrt{3}}{2}$; $x \in \left(\frac{\pi}{3} + 2\pi n; \frac{2\pi}{3} + 2\pi n\right)$.
4. $\begin{cases} x + y = \frac{\pi}{2} \\ \cos x + \sin y = -1 \end{cases}$; $\begin{cases} x = \frac{\pi}{2} - y \\ \sin y = -\frac{1}{2} \end{cases}$; $\begin{cases} y = (-1)^{k+1} \frac{\pi}{6} + \pi k \\ x = \frac{\pi}{2} - (-1)^{k+1} \frac{\pi}{6} - \pi k \end{cases}$.
5. $2\sin^2 x - |\sin x| = 0$; $\sin x = 0$; $x = \pi n$; $\sin x = \frac{1}{2}$; $x = (-1)^k \frac{\pi}{6} + \pi k$.

КАРТОЧКА 3.

1. $\operatorname{arctg} a$ – такое число из $\left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$, tg которого равен a .
2. а) $\frac{2}{\operatorname{ctg} x + 1} = 2 - \operatorname{ctg} x$; ОДЗ: $\begin{cases} x \neq -\frac{\pi}{4} + \pi n \\ x \neq \pi n \end{cases}$;
 $\operatorname{ctg}^2 x - \operatorname{ctg} x = 0$; $\operatorname{ctg} x = 0$; $x = \frac{\pi}{2} + \pi n$; $\operatorname{ctg} x = 1$; $x = \frac{\pi}{4} + \pi n$;
- б) $1 - 2\sin 2x + 2\cos^2 x = 0$, $\cos x \neq 0$; $\sin^2 x - 4\sin x \cos x + 3\cos^2 x = 0$;
 $\operatorname{tg}^2 x - 4\operatorname{tg} x + 3 = 0$; $\operatorname{tg} x = 3$; $x = \operatorname{arctg} 3 + \pi n$; $\operatorname{tg} x = 1$; $x = \frac{\pi}{4} + \pi n$.
3. $\cos 2x \geq -\frac{\sqrt{2}}{2}$; $x \in \left[-\frac{3\pi}{8} + \pi n; \frac{3\pi}{8} + \pi n\right]$.
4. $\begin{cases} x + y = \pi \\ \sin^2 x + \sin^2 y = 1 \end{cases}$; $\begin{cases} x = \pi - y \\ \sin y = \pm \frac{\sqrt{2}}{2} \end{cases}$; $\begin{cases} y = \frac{\pi}{4} + \frac{\pi n}{2} \\ x = \frac{\pi}{4} - \frac{\pi n}{2} \end{cases}$.
5. $2\sin^2 x + \sin x \geq 0$; $\sin\left(x - \frac{\pi}{4}\right) \geq -\frac{1}{\sqrt{2}}$; $x \in \left[2\pi n; \frac{3\pi}{2} + 2\pi n\right]$.

КАРТОЧКА 4

1. $\cos t = a$; $|a| \leq 1$; $t = \pm \arccos a + 2\pi n$.
2. а) $1 + \cos x = 2\sin^2 x$; $\cos 2x + \cos x = 0$;
 $\cos \frac{3x}{2} \cos \frac{x}{2} = 0$; $x = \frac{\pi}{3} + \frac{2\pi n}{3}$; $x = \pi + 2\pi n$;
- б) $\sin 2x + 2\sqrt{3} \cos^2 x = 0$; $\cos x(\sin x + \sqrt{3} \cos x) = 0$; $x = \frac{\pi}{2} + \pi k$; $x = -\frac{\pi}{3} + \pi k$.
3. $\sin\left(x + \frac{\pi}{4}\right) \leq \frac{\sqrt{2}}{2}$; $x \in \left[-\frac{3\pi}{2} + 2\pi n; 2\pi n\right]$.
4. $\begin{cases} x + y = \frac{\pi}{2} \\ \sin^2 x + \cos^2 y = 1 \end{cases}$; $\begin{cases} x = \frac{\pi}{2} - y \\ \cos y = \pm \frac{\sqrt{2}}{2} \end{cases}$; $\begin{cases} y = \frac{\pi}{4} + \frac{\pi n}{2} \\ x = \frac{\pi}{4} - \frac{\pi n}{2} \end{cases}$.
5. $\sqrt{2x - \pi} (\sin x - 1) = 0$; ОДЗ: $x \geq \frac{\pi}{2}$; $\sin x = 1$; $x = \frac{\pi}{2} + 2\pi n$; $n = 0; 1; 2; 3$.

КАРТОЧКА 5.

1. $\sin t = a$; $|a| \leq 1$; $t = (-1)^k \arcsin a + \pi k$.
2. а) $1 - \cos 2x + \sin x = 0$; $2\sin^2 x + \sin x = 0$;
 $\sin x = 0$; $x = \pi n$; $\sin x = -\frac{1}{2}$; $x = (-1)^{k+1} \frac{\pi}{6} + \pi k$;
- б) $5\sin 2x - 6\cos x = 6$; $12\cos^2 \frac{x}{2} - 10\sin \frac{x}{2} \cos \frac{x}{2} = 0$ $\cos \frac{x}{2} = 0$;
 $x = 2\pi n + \pi$; $\sin \frac{x}{2} = \frac{6}{5} \cos \frac{x}{2}$; $x = 2\arctg \frac{6}{5} + 2\pi n$.
3. $\operatorname{tg} 2x \geq -\sqrt{3}$; $x \in \left[-\frac{2\pi}{3} + 2\pi n; \pi + 2\pi n\right]$.
4. $\begin{cases} x + y = \pi \\ \sin x + \sin y = -1 \end{cases}$; $\begin{cases} x = \pi - y \\ \sin y = \frac{1}{2} \end{cases}$; $\begin{cases} y = (-1)^k \frac{\pi}{6} + \pi k \\ x = \pi - (-1)^k \frac{\pi}{6} - \pi k \end{cases}$
5. $|x|\sin x + x = 0$, т.к. $x > 0$, то $\sin x = -1$; $x = -\frac{\pi}{2} + 2\pi n$; $n \in \mathbb{N}$.

КАРТОЧКА 6.

1. $\operatorname{tg} t = a$; $t = \operatorname{arctg} a + \pi n$.
2. а) $\cos 2x = \cos x$; $\sin \frac{x}{2} \sin \frac{3x}{2} = 0$; $x = \frac{2\pi n}{3}$;
б) $\sqrt{3} \sin x + \cos x = -1$;
 $2\cos^2 \frac{x}{2} + 2\sqrt{3} \sin \frac{x}{2} \cos \frac{x}{2} = 0$; $\cos \frac{x}{2} = 0$; $x = \pi + 2\pi n$;
 $\cos \frac{x}{2} + \sqrt{3} \sin \frac{x}{2} = 0$; $x = -\frac{\pi}{3} + 2\pi n$.
3. $\sin\left(\frac{3\pi}{2} + x\right) > -\frac{1}{2}$; $\cos x < \frac{1}{2}$; $x \in \left(\frac{\pi}{3} + 2\pi n; \frac{5\pi}{3} + 2\pi n\right)$.
4. $\begin{cases} x - y = \frac{\pi}{2} \\ \cos x - \cos y = -\sqrt{2} \end{cases}$; $\begin{cases} x = \frac{\pi}{2} + y \\ \cos y = \frac{\sqrt{2}}{2} \end{cases}$; $\begin{cases} y = \pm \frac{\pi}{4} + 2\pi n \\ x = \frac{\pi}{2} \pm \frac{\pi}{4} + 2\pi n \end{cases}$.
5. $2\cos^2 x + \cos x - 1 \leq 0$;
 $\cos x \in [-1; \frac{1}{2}]$; $x \in \left[\frac{\pi}{3} + 2\pi n; \frac{5\pi}{3} + 2\pi n\right]$.

ЗАЧЕТ № 3.

КАРТОЧКА 1.

1. Производной функции в точке x_0 называется
$$\lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x};$$

пример: $f(x) = x$; $\Delta f(x_0) = \Delta x$; $\lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta x} = 1$.
2. $f(x) = x^4 - 6x^3 + 8x - 7$; $f'(x) = 4x^3 - 18x^2 + 8$;
 $f'(-1) = -4 - 18 + 8 = -14$.
3. $\varphi(x) = \frac{6-x}{x} = \frac{6}{x} - 1$; $\varphi'(x) = -\frac{6}{x^2}$; $\varphi'(x) < 0$, $x \neq 0$.
4. $h(x) = (6 + 5x)^7$; $h'(x) = 35(6 + 5x)^6$; $h(-1) = 35$.
5. $f(x) = \sin^2 3x$; $f'(x) = 6\sin 3x \cos 3x = 3$; $\sin 6x = 1$; $x = \frac{\pi}{12} + \frac{\pi k}{3}$.

КАРТОЧКА 2.

- $$(f(x) + g(x))' = \lim_{\Delta x \rightarrow 0} \left(\frac{\Delta f(x_0)}{\Delta x} + \frac{\Delta g(x_0)}{\Delta x} \right) =$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\Delta f(x_0)}{\Delta x} + \lim_{\Delta x \rightarrow 0} \frac{\Delta g(x_0)}{\Delta x} = f'(x) + g'(x);$$

$$f(x) = x^7; \quad g(x) = \frac{1}{x}; \quad (f(x) + g(x))' = 7x^6 - \frac{1}{x^2}.$$
- $$f(x) = x^3 - 2x^2 + x + 10; \quad f'(x) = 3x^2 - 4x + 1;$$

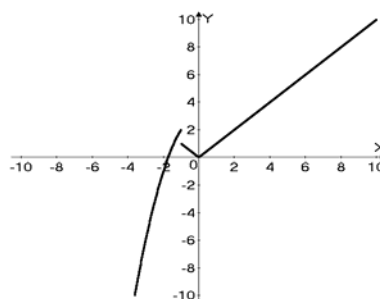
$$f'(-2) = 12 + 8 + 1 = 21; \quad f'(x) \leq 0; \quad 3x^2 - 4x + 1 \leq 0;$$

$$x \in \left[\frac{1}{3}; 1 \right].$$
- $$g(x) = \sin\left(2x - \frac{\pi}{4}\right); \quad g'(x) = 2\cos\left(2x - \frac{\pi}{4}\right) = 0; \quad x = \frac{3\pi}{8} + \frac{\pi k}{2}; \quad g'(\pi) = \sqrt{2}.$$
- $$f(x) = x\sqrt{x-5}; \quad f'(x) = \sqrt{x-5} + \frac{x}{2\sqrt{x-5}}; \quad f'(6) = 1 + \frac{3}{1} = 4.$$
- $$f(x) = \begin{cases} |x| & x \geq -1 \\ -x^2 + 3 & x < -1 \end{cases};$$

a)

б) $x = -1$;

в) нет.



КАРТОЧКА 3.

- $$(f(x)g(x))' = f'(x)g(x) + g'(x)f(x);$$

$$(\lambda f(x))' = \lim_{\Delta x \rightarrow 0} \lambda \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} = \lambda \lim_{\Delta x \rightarrow 0} \frac{\Delta f(x_0)}{\Delta x} = \lambda f'(x);$$

$$y = 2x^2; \quad y' = 4x; \quad y = 7x^5; \quad y' = 35x^4.$$
- $$f(x) = (2x - 3)(4x^2 + 6x + 9); \quad f'(x) = 8x^2 + 12x + 18 + (2x - 3)(8x + 6);$$

$$f'(-2) = 32 - 24 + 18 + 7 \cdot 10 = 96.$$
- $$f(x) = \operatorname{tg} 3x; \quad f'(x) = \frac{3}{\cos^2 x}; \quad f'\left(-\frac{\pi}{4}\right) = 6.$$

$$4. f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 4x; \quad f'(x) = x^2 - 3x - 4; \quad g(x) = 2\sqrt{x};$$

$$g'(x) = \frac{1}{\sqrt{x}}; \quad f(x)g'(x) = \frac{x^2 - 3x - 4}{\sqrt{x}} = 0; \quad x = 4; \quad x = -1, \text{ но } x > 0 \Rightarrow x = 4.$$

$$5. f(x) = \frac{x-1}{\sqrt{2-\sqrt{2-x}}}; \quad \text{ОДЗ: } \begin{cases} x \leq 2 \\ \sqrt{2-x} < 2 \end{cases}; \quad \begin{cases} x > -2 \\ x \leq 2 \end{cases}.$$

КАРТОЧКА 4.

$$1. \left(\frac{f(x)}{g(x)} \right)' = \frac{f'(x)g(x) - g'(x)f(x)}{g^2(x)}; \quad y = \frac{x-2}{x+4}; \quad y' = \frac{6}{(x+4)^2}.$$

$$2. f(x) = 2\sqrt{x} + \frac{3}{x^2}; \quad f'(x) = \frac{1}{\sqrt{x}} - \frac{6}{x^3}; \quad f'(1) = 1 - 6 = -5.$$

$$3. h(x) = \cos 2x; \quad h'(x) = -2\sin 2x; \quad h'\left(-\frac{\pi}{3}\right) = \sqrt{3};$$

$$-2\sin 2x = 1; \quad \sin 2x = -\frac{1}{2}; \quad x = (-1)^{k+1} \frac{\pi}{12} + \frac{\pi k}{2}.$$

$$4. f(x) = \frac{2x+7}{x-4}; \quad f'(x) = \frac{-15}{(x-4)^2}; \quad f'(5) = -15; \quad f'(x) < 0 \text{ при } x \neq 4.$$

$$5. f(x) = 1 - 2x; \quad f(g(x)) = 1 - 2g(x) = x; \quad g(x) = \frac{1}{2} - \frac{x}{2}.$$

КАРТОЧКА 5.

$$1. (f^n(x))' = n f'(x) f^{n-1}(x); \quad y = x^{100}; \quad y' = 100x^{99}; \quad y = (2x)^{100}; \quad y' = 200(2x)^{99}.$$

$$2. f(x) = \operatorname{ctg} 4x; \quad f'(x) = \frac{-4}{\sin^2 4x}; \quad f'\left(-\frac{\pi}{6}\right) = -\frac{8}{3}.$$

$$3. f(x) = 2x^3 + 3x^2 - 12x; \quad f'(x) = 6(x^2 + x - 2) = -12; \quad x = 0 \text{ и } x = -1; \quad f'(x) > 0, \quad x \in (-2; 1).$$

$$4. f(x) = (x-1)\sqrt{x}; \quad f'(x) = \sqrt{x} + \frac{x-1}{2\sqrt{x}}; \quad f'(1) = 1.$$

$$5. f(x) = x^2 - 1; \quad f(f(x)) = (x^2 - 1)^2 - 1 = x^4 - 2x^2 > 0; \quad x^2(x^2 - 2) > 0; \quad x \in (-\sqrt{2}; 0) \cup (0; \sqrt{2}).$$

КАРТОЧКА 6.

1. $y = \sin x$; $y' = \cos x$.
2. $f(x) = (3x^2 - 2)(3x^2 + 2) = 9x^4 - 4$; $f'(x) = 36x^3$; $f'(-1) = -36$.
3. $f(x) = \cos 3x \cos x - \sin 3x \sin x = \cos 4x$; $f'(x) = -4\sin 4x$;
 $f\left(-\frac{\pi}{3}\right) = -2\sqrt{3}$.
4. $f(x) = \frac{1}{3}x^3 - x^2$; $g(x) = \frac{1}{3}x^3 + x$; $f'(x) = x^2 - 2x$;
 $g'(x) = x^2 + 1$; $\frac{x^2 - 2x}{x^2 + 1} \leq 0$; $x \in [0; 2]$.
5. $f(x) = \frac{2}{2-x}$; $f(f(x)) = \frac{2}{2 - \frac{2}{2-x}} = \frac{4-2x}{2-2x} = 1 + \frac{2}{2-2x}$;
 $f(f(f(x))) = 1 + \frac{2}{2 - \frac{4}{2-x}} = 1 + \frac{4-2x}{-2x} = 2 - \frac{2}{x}$.

КАРТОЧКА 7.

1. $y = \cos x$; $y' = -\sin x$; $y = \operatorname{tg} x$; $y' = \frac{1}{\cos^2 x}$; $y = \operatorname{ctg} x$; $y' = -\frac{1}{\sin^2 x}$.
2. $f(x) = (2x^2 - 5)(x^2 - 4) = 2x^4 - 13x^2 + 20$; $f'(x) = 8x^3 - 26x$.
3. $f(x) = \frac{2x-7}{x+3}$; $f'(x) = \frac{13}{(x+3)^2}$; $f'(-2) = 13$; $f'(x) > 0$; при $x \neq -3$.
4. $f(x) = \sin x \cos x + 1$; $f'(x) = \cos 2x$; $f'\left(-\frac{\pi}{3}\right) = -\frac{1}{2}$.
5. $f(x) = \sin^2 x$; $f'(x) = \sin 2x > -\frac{1}{2}$; $x \in \left(-\frac{\pi}{12} + \pi n; \frac{7\pi}{12} + \pi n\right)$.

ЗАЧЕТ № 4

КАРТОЧКА 1.

1. Геометрический смысл производной в точке x_0 – tg угла наклона касательной в точке x_0 .
 $y = f'(x_0)x + b$. Но нам нужно, чтобы $y(x_0) = f(x_0) \Rightarrow$
 $f(x_0) = f'(x_0)x_0 + b \Rightarrow b = -f'(x_0)x_0 + f(x_0) \Rightarrow$
 $y_{\text{кас}} = f'(x_0)x + f(x_0) - f'(x_0)x_0$.

2. $f(x) = 6x + 5\cos x$; $f'(x) = 6 - 5\sin x > 0 \Rightarrow$ возрастает.

3. $\begin{cases} a+b=15 \\ a^3+3b=y \end{cases}; \begin{cases} b=15-a \\ y=a^3-3a+45 \end{cases}; y'=3a^2-3=0; 15=1+14.$

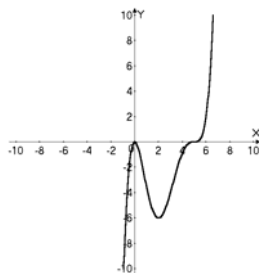
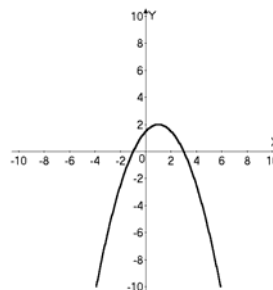
4. а)

$$f(x) = -\frac{1}{2}x^2 + x + \frac{3}{2}; \quad x_B = 1;$$

$$f(1) = -\frac{1}{2} + 1 + \frac{3}{2} = 2; x \in R; f \leq 2.$$

$f(x)$ убывает на $x \geq 1$; возрастает на $x \leq 1$;
 $x_{\max} = 1 -$

б) $f(x) = \frac{1}{18}x^2(x-5)^3;$



$$f'(x) = \frac{x(x-5)^3}{9} + \frac{x^2(x-5)^2}{6} = 0;$$

$$x(x-5)^2(2x-10+3x) = 0;$$

$$x = 0; \quad x = 5; \quad x = 2; \quad f(0) = 0;$$

$$f(2) = -\frac{2}{9} \cdot 27 = -6;$$

$f(x)$ возрастает на $x < 0, \quad x > 2$
 убывает на $x \in (0; 2).$

КАРТОЧКА 2.

1. производная от перемещения – скорость.

производная от скорости – ускорение.

2. $f(x) = -\sin x$; $f(0) = 0$; $f'(x) = -\cos x$; $f'(0) = -1$; $y_k = -x$.

3. $f(x) = -x^3 + 2x^2 - 8x + 1$; $x \in [-2; 1]$; $f'(x) = -3x^2 + 4x - 8 = 0$;

$$\frac{D}{4} = 4 - 24 < 0 \Rightarrow \text{убывает на } R$$

$$\max: f(-2) = 8 + 8 + 16 + 1 = 33$$

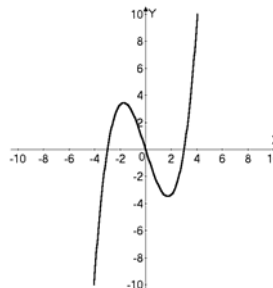
$$\min: f(1) = -1 + 2 - 8 + 1 = -6$$

4. а)

$$f(x) = \frac{1}{3}x^3 - 3x$$

$$f'(x) = x^2 - 3 = 0 \quad x = \pm\sqrt{3}$$

$$\max: f(-\sqrt{3}) = -\sqrt{3} + 3\sqrt{3} = 2\sqrt{3}$$

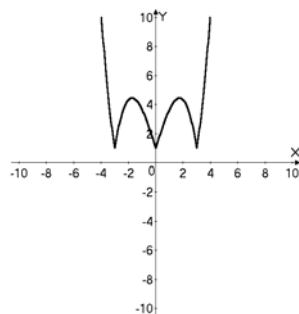


$$\min: f(\sqrt{3}) = \sqrt{3} - 3\sqrt{3} = -2\sqrt{3};$$

$f(x)$ возрастает на $x < -\sqrt{3}$, $x > \sqrt{3}$;

убывает на $x \in [-\sqrt{3}; \sqrt{3}]$;

б)



КАРТОЧКА 3.

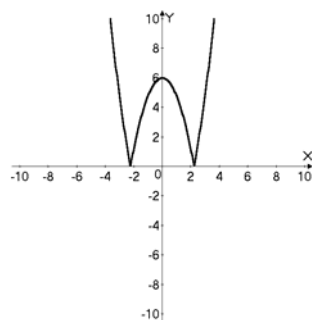
1. возрастание: производная > 0 ; убывание: производная < 0 .

$$2. s(t) = 6t^3 + 5t + 2 \quad v(t) = 18t^2 + 5 \quad v(2) = 77$$

$$a(t) = 36t \quad a(2) = 72$$

$$3. a) f(x) = -0,5x^2 + 6 \quad f'(x) = -x \quad f'(1) = -1$$

б)



По рисунку видно, что в точках $x = \pm 2\sqrt{3}$ производной не существует, а в точке $x = 0$ она равна нулю.

4. Пусть вершина прямоугольника, лежащая правее нуля, равна x_0 , тогда

$$S = 2x_0 \cdot f(x_0) = -x_0^3 + 12x_0$$

$$S' = -3x_0^2 + 12 = 0 \quad x_0 = \pm 2,$$

но $x_0 > 0 \Rightarrow$ длина = 4, высота = 4.

КАРТОЧКА 4.

1. Критические точки – точки, в которых производная равна нулю или не существует.

Пусть в этой точке производная меняет знак с «больше» на «меньше», то это точка \max .

Если с «меньше» на «больше» $\Rightarrow \min$.

2. $f(x) = -0,5x^2 + 2x$; $f(0) = 0$; $f'(x) = -x + 2$; $f'(0) = 2$; $y_{\text{кас}} = 2x$.

3. $f(x) = -7x - 6\sin x$; $\left[-\frac{\pi}{6}; \frac{7\pi}{6}\right]$; $f'(x) = -7 - 6\cos x < 0$ всегда

max: $f\left(-\frac{\pi}{6}\right) = \frac{7\pi}{6} + 3$; min: $f\left(\frac{7\pi}{6}\right) = -\frac{49\pi}{6} + 3$.

4. а)

$f(x) = 2x\sqrt{3-x}$

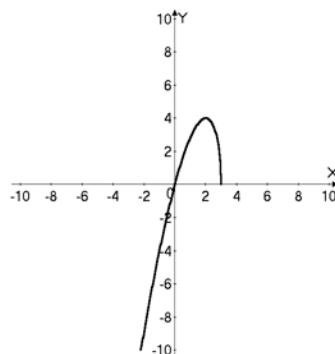
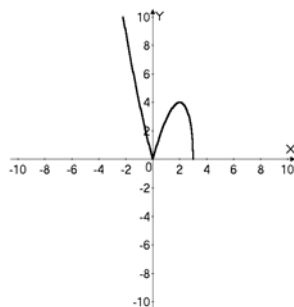
$f'(x) = 2\sqrt{3-x} - \frac{x}{\sqrt{3-x}} = 0$

$6 - 2x = x$ $x = 2$ — max $f(2) = 4$

возрастает: $x \leq 2$,

убывает $x \in [2; 3]$

б) $1 \leq x \leq 3$



КАРТОЧКА 5.

1. а) находим $P(f)$ и $E(f)$; б) нули; в) критические точки;

г) max и min; д) промежутки возрастания, убывания

Для квадратичной функции $y = ax^2 + bx + c$ находим вершину ($y'(x_0) = 0$).

Если $a > 0$, то $x \leq x_0$ убывает, $x \geq x_0$ возрастает, а x_0 — min;

если $a < 0$, то наоборот.

2. $f(x) = 2\sin x - x$; $f'(x) = 2\cos x - 1$; $y = 2\sin x_0 - x_0 + (2\cos x_0 - 1)(x - x_0)$;

$2\cos x_0 - 1 = 0$; $x_0 = \pm \frac{\pi}{3} + 2\pi n$

3. $f(x) = \frac{1}{3}x^3 - 9x + 10$; $x \in [0; 6]$; $f'(x) = x^2 - 9$;

$x = \pm 3$; $f(0) = 10$; $f(3) = \min = -8$; $f(6) = \max = 28$.

4. а)

$$f(x) = 10 \frac{x-2}{x^2+5};$$

$$f'(x) = \frac{10x^2 + 50 - 20x^2 + 40x}{(x^2+5)^2} =$$

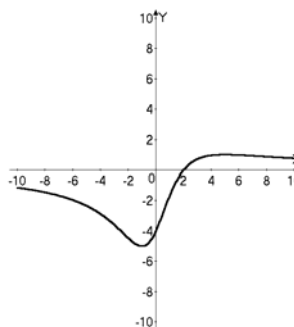
$$= \frac{-10(x^2 - 4x - 5)}{(x^2+5)^2} = 0; x_{\max} = 5, \quad x_{\min} = -1;$$

$f(x)$ возрастает на $x \in [-1; 5]$;

убывает на $x < -1, x > 5$;

$$f(5) = \frac{30}{30} = 1; \quad f(-1) = \frac{-30}{6} = -5;$$

б) из рисунка видно, что $f(x) > -4$ при $x < -2,5, x > 0$.



КАРТОЧКА 6.

1. находите экстремумы, смотрите значения в них и в концевых точках отрезка. Что больше, то max. Что меньше, то min.

$$2. f(x) = \frac{1}{5}x^5 - x^3 - 4x + 1 \quad f'(x) = x^4 - 3x^2 - 4 = 0; x^2 = 4 \quad x = \pm 2;$$

$f(x)$ убывает: $x \in (-2; 2)$ возрастает на $x < -2, x > 2$.

$$3. \begin{cases} 2a + b = 24 \\ ab = y \end{cases}; \quad \begin{cases} b = 24 - 2a \\ y = 24a - a^2 \end{cases}; \quad y' = 24 - 4a = 0$$

$$a = 6 \quad b = 12 \quad S = 72$$

$$4. а) f(x) = \frac{4x^2 - 8x}{4x^2 - 8x + 5} = 1 - \frac{5}{(4x^2 - 8x + 5)};$$

$$f'(x) = \frac{5(8x - 8)}{(4x^2 - 8x + 5)^2} = 0; \quad x = 1;$$

$f(x)$ возрастает: $x > 1$;

убывает: $x < 1$;

$$x_{\min} = 1; \quad f(1) = \frac{4-8}{-4+5} = -4;$$

$$б) f(x) = \left| \frac{4x^2 - 8x}{4x^2 - 8x + 5} \right|;$$

Из рисунка видно, что $x = 1$ – критическая точка; $(1; 4)$ – max.

